MAE 140, Winter 2014, HW #1 Solns:

problem 1-14:

Let IL stand for Incandescent Lamp, and CFL stand for Compact Fluorescent Lamp; then based on the problem statement, the following holds:

$$IL: \begin{cases} P_{IL} = 100W \\ V_{IL} = 120V \end{cases}, \quad CFL: \begin{cases} P_{CFL} = 16W \\ V_{CFL} = 120V \end{cases}.$$

Moreover, provided the total time-duration; T = 1000 hrs, and that electricity costs $C = 7.8$ ¢/kWh, one can compute the following:

$$C_{IL} = P_{IL} \times T \times C = \underbrace{100 \times 10^3}_{100\ kWh} \times 7.8 \Rightarrow \boxed{C_{IL} = 780\ ¢}$$

$$C_{CFL} = P_{CFL} \times T \times C = \underbrace{16 \times 10^3}_{16\ kWh} \times 7.8 \Rightarrow \boxed{C_{CFL} = 124.8¢}$$

Hence, comparing C_{IL} and C_{CFL}; one infers that $C_{IL} - C_{CFL} = 655.2¢$ and so CFL is 655.2 cents cheaper compared to IL.

problem 1-16:

According to the problem statement, the maximum current that the fuse can tolerate is 5A; i.e., $i_{fuse} = 5A$.

Besides, each string has 25 bulbs each of which rated at 7W, therefore, $P_{string} = 25 \times 7 \Rightarrow \boxed{P_{string} = 175W}$. On the other hand, provided that the strings are connected to a 120V circuit, and that $i_{fuse} = 5A$, the maximum power can be computed as: $P_{max} = 120 \times 5 = 600W$ and therefore, maximum # of possible strings:

$$n_{strings} = \frac{P_{max}}{P_{string}} = \frac{600}{175} = 3.43.$$

Therefore, max number of strings = 3.

Problem 1-22:

According to the problem statement and the current/voltage values provided for every device, we can compute the following, where P_i, v_i and i_i stand for the power, voltage and current of the i^{th} device, respectively:

$P_1 = v_1 i_1 = 15 \times -1 \Rightarrow P_1 = -15W < 0 \Rightarrow$ Delivering power

$P_2 = v_2 i_2 = 5 \times 1 \Rightarrow P_2 = +5W > 0 \Rightarrow$ Absorbing power

$P_3 = v_3 i_3 = 10 \times 2 \Rightarrow P_3 = +20W > 0 \Rightarrow$ Delivering power

$P_4 = v_4 i_4 = -10 \times -1 \Rightarrow P_4 = +10W > 0 \Rightarrow$ Absorbing power

$P_5 = v_5 i_5 = 20 \times -3 \Rightarrow P_5 = -60W < 0 \Rightarrow$ Delivering power

$P_6 = v_6 i_6 = 20 \times 2 \Rightarrow P_6 = +40W > 0 \Rightarrow$ Absorbing power

Moreover, one can perform the power balance as follows:

$$\sum_{i=1}^{6} P_i = P_1 + P_2 + P_3 + P_4 + P_5 + P_6 = -15 + 5 + 20 + 10 - 60 + 40 = 0,$$

which further validates the computations.

problem 2-1:

Provided the information in the problem statement, $R = 56k\Omega = 5.6\times10^4\Omega$, in addition, $i = 2.2mA = 2.2\times10^{-3}A$, and therefore voltage across the resistor: $v = Ri = (5.6\times10^4)\times(2.2\times10^{-3}) \Rightarrow \boxed{v = 123.2\ V}$.

problem 2-6:

According to figure P2-6 and the information therein:

$i = 10mA = 10^{-2}A$, $v = 100V \Rightarrow R_x = \frac{v}{i} = \frac{100}{10^{-2}} \Rightarrow \boxed{R_x = 10k\Omega}$.

In addition the power delivered to the resistor can be computed as follows:

$P = vi = 10^2\times10^{-2} \Rightarrow \boxed{P = 1W}$.

problem 2-9:

According to the problem statement, $\boxed{R = 100k\Omega = 10^5\Omega}$, in addition, $\boxed{P_{max} = 0.125W}$; therefore, the maximum voltage can be computed as follows:

$P_{max} = \frac{v_{max}^2}{R} \Rightarrow v_{max} = \sqrt{P_{max}\cdot R} = \sqrt{0.125\times10^5} \Rightarrow \boxed{v_{max} = \pm 111.80\ V}$.

Problem 2-12:

let R and T stand for the resistance and temperature, respectively. Then, according to the problem statement we know:

$$\begin{cases} R_1 = 5\times10^3\,\Omega \\ T_1 = 25^\circ C \end{cases}, \quad \begin{cases} R_2 = 340\,\Omega \\ T_2 = 100^\circ C \end{cases},$$

and that there is a linear relationship between R and T. Therefore, in other words, we are looking for unknown parameters, a and b in $R = aT + b$ equation. Provided, R_1, T_1 & R_2, T_2 values we can perform the following:

$$\begin{cases} R_1 = aT_1 + b \\ R_2 = aT_2 + b \end{cases} \Rightarrow \begin{cases} 5\times10^3 = 25a + b \\ 340 = 100a + b \end{cases} \Rightarrow a = \frac{340 - 5\times10^3}{75}$$

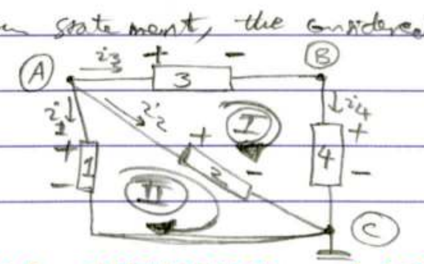
$$\Rightarrow \boxed{a = -62.133} \Rightarrow R_1 = -62.133\,T_1 + b \Rightarrow b = 5\times10^3 + 62.133\times25$$
$$\Rightarrow \boxed{b = 6.55\times10^3}$$

therefore, $\boxed{R = -62.133\,T + 6.55\times10^3}$. Provided this latter eq'n, we can find T^* at which $R = 10^3\,\Omega$:

$$10^3 = -62.133\,T^* + 6.55\times10^3 \Rightarrow T^* = \frac{5.55\times10^3}{62.133} \Rightarrow \boxed{T^* = 89.32^\circ C}$$

Problem 2-15:

According to the problem statement, the considered circuit is as follows:



(a) The nodes on this circuit are Ⓐ, Ⓑ and Ⓒ. In additi, we have drawn loops Ⓘ and Ⓘⅰ on this circuit.

(b) According to the def's of elements be connected in series or parallel, it can be observed that:

Elements 3 and 4 are connected in series.
Elements 1 and 2 are connected in parallel
Element 1 is in parallel w/ the equivalent element of 3 and 4.
Element 2 is in parallel w/ the equivalent element of 3 and 4.

(c) We shall first write KCL eqns for each node, Ⓐ, Ⓑ, Ⓒ as follows:

node Ⓐ: $i_1 + i_2 + i_3 = 0$,
node Ⓑ: $i_3 - i_4 = 0$,
node Ⓒ: $i_1 + i_2 + i_4 = 0$.

Referring to the circuit and provided the signs assigned to each element, KVL eqns can be written for loops Ⓘ and Ⓘⅰ as follows:

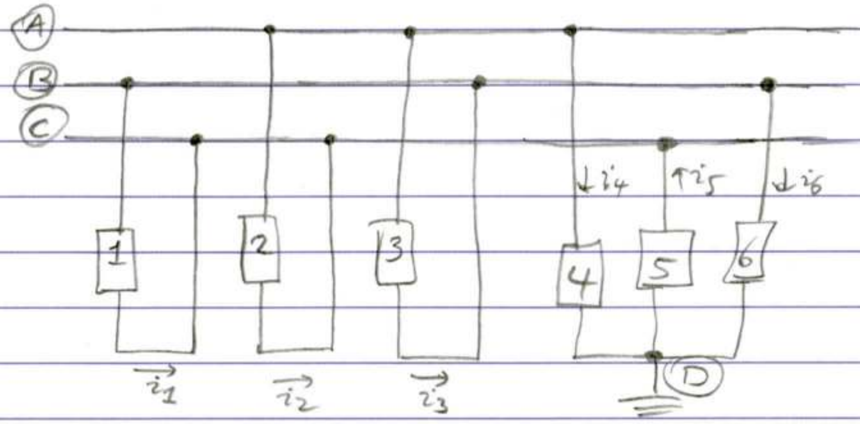
loop Ⓘ: $v_3 + v_4 - v_2 = 0$,

loop Ⓘⅰ: $v_2 - v_1 = 0$,

where v_i declares the voltage across the element i.

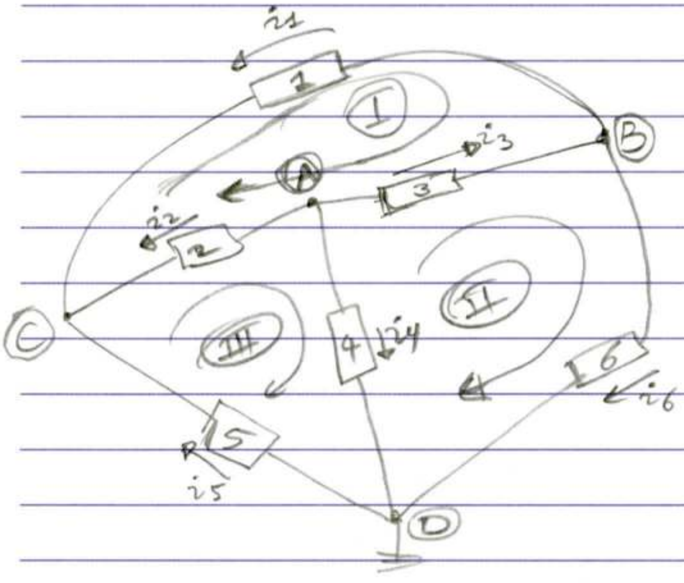
problem 2-20:

let us first recall the circuit provided in Fig. P2-20:



(Circuit 1)

then, by rearranging different elements of the circuit, in order to avoid overcrossing of various wires, we obtain the following circuit:



(Circuit 2)

In the rest of our analysis, we shall focus on Circuit 2, provided circuits 1 and 2 are equivalent.

(a) Referring to circuit 2, our nodes are A, B, C and D; in addition, we consider loops I, II and III.

(b) KCL eq'ns for our designated nodes yield:

node (A): $i_2+i_3+i_4=0$

node (B): $i_1+i_6=i_3$

node (C): $i_1+i_2+i_5=0$

node (D): $i_4+i_6=i_5$

(c) Given $i_1=-20mA$, $i_2=-12mA$, and $i_3=50mA$ and by KCL eq'ns obtained in step (b), we find i_4, i_5 and i_6 as follows:

$$i_2+i_3+i_4=0 \Rightarrow -12+50+i_4=0 \Rightarrow \boxed{i_4=-38mA}$$

$$i_1+i_6=i_3 \Rightarrow -20+i_6=50 \Rightarrow \boxed{i_6=70mA}$$

$$i_1+i_2+i_5=0 \Rightarrow -20-12+i_5=0 \Rightarrow \boxed{i_5=32mA}$$

(d) Provided our rearrangement on circuit 1 which resulted in circuit 2, we conclude that these two circuits are equivalent. On the other hand, it is easy to verify that circuit 2 and the one drawn in Fig. P2-17 are equivalent; therefore, we conclude that circuit 1, i.e, Fig. P2-20 and Fig. P2-17 are indeed identical.

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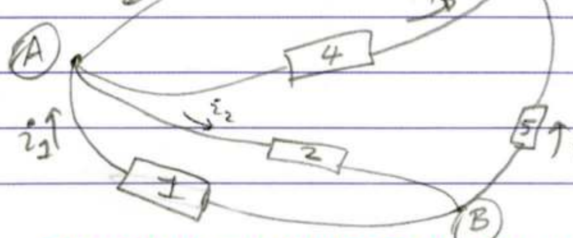
Problem 2-22:

Referring to the circuit drawn in Fig P2-22, that is recalled below:



we first write KCL eq'ns associated to each node (A), (B) and (C):

$$\begin{cases} \text{node (A)}: & i_3 + i_1 - i_2 - i_4 = 0, \\ \text{node (B)}: & i_2 - i_1 - i_5 = 0, \\ \text{node (C)}: & i_4 + i_5 - i_3 = 0, \end{cases}$$

then, given $i_1 = 25\,mA$, $i_2 = 10\,mA$ and $i_3 = -15\,mA$, we obtain i_4 and i_5 as follows:

$$\begin{cases} i_3 + i_1 - i_2 - i_4 = 0 \Rightarrow -15 + 25 - 10 - i_4 = 0 \Rightarrow \boxed{i_4 = 0\,mA} \\ i_2 - i_1 - i_5 = 0 \Rightarrow 10 - 25 - i_5 = 0 \Rightarrow \boxed{i_5 = -15\,mA} \\ i_4 + i_5 - i_3 = 0 \Rightarrow 0 + (-15) - (-15) = 0 \checkmark \end{cases}$$

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END of HW #1 Sol'ns