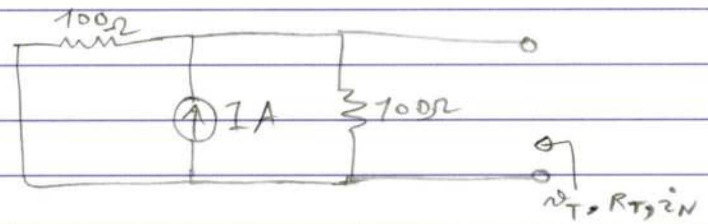
MAE 140, Winter 2014, HW#3 Sol'ns:

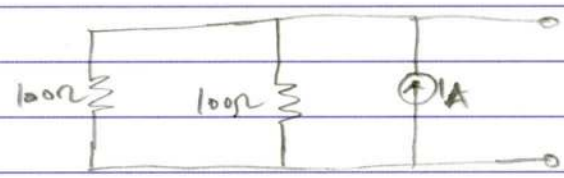
problem 3-48:

We would like to find Thévinin and Norton equivalent circuits for the following circuit:

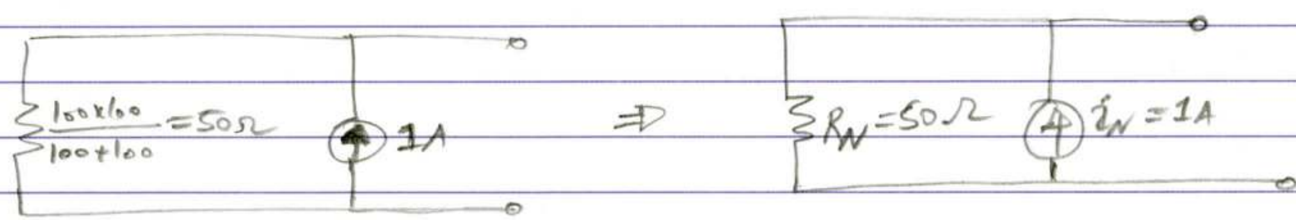


Norton equivalent circuit:

In order to find Norton equivalent circuit, we swap the wires containing 1A current source and 100Ω resistance. Therefore, we obtain the following circuit:



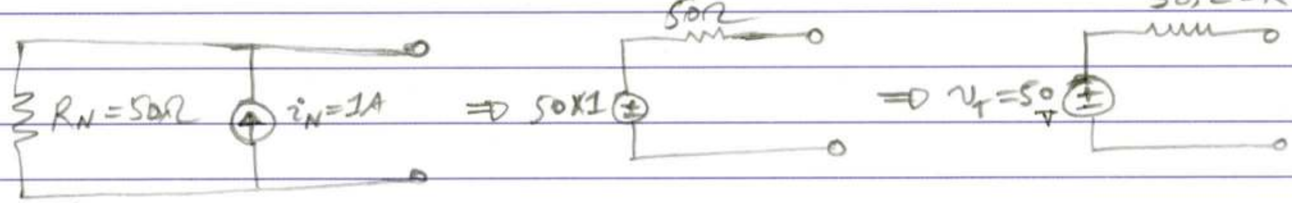
in which the two 100Ω resistances are in parallel, therefore the equivalent resistance can be obtained as in the following circuit:



Indeed, this latter circuit represents the Norton equivalent circuit, w/ $\boxed{R_N=50\Omega}$ and $\boxed{i_N=1A}$.

Thévinin equivalent circuit:

In order to find Thévinin equivalent circuit, we shall apply source transformation technique on the derived Norton equivalent circuit, which then results the following circuit:

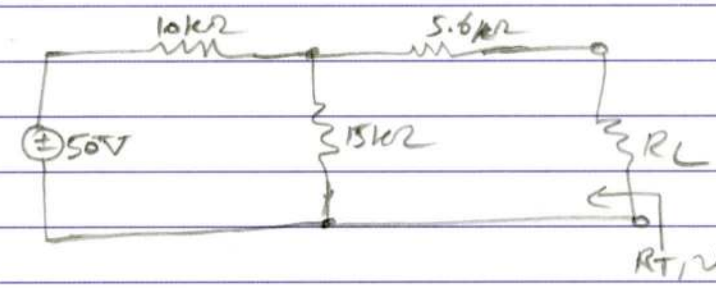


Indeed, this last circuit represents Thévinin equivalent circuit w/

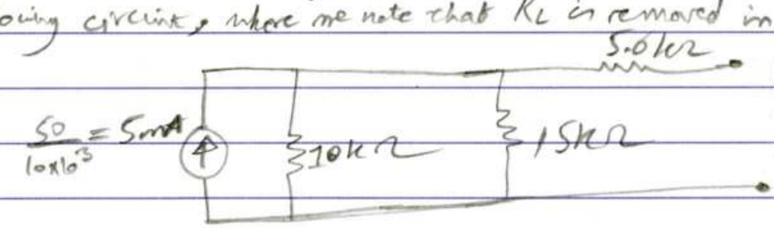
$\boxed{v_T = 50V}$ and $\boxed{R_T = 50\Omega}$.

problem 3-49:

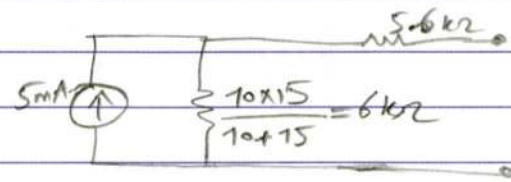
(a) we shall derive the Thévinin equivalent circuit for the following circuit, seen from R_L:



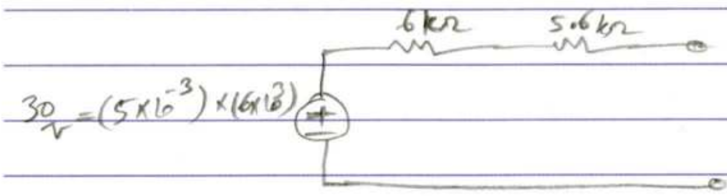
we shall first apply source transformation technique where we obtain the following circuit, where we note that R_L is removed in order to avoid potential confusion:



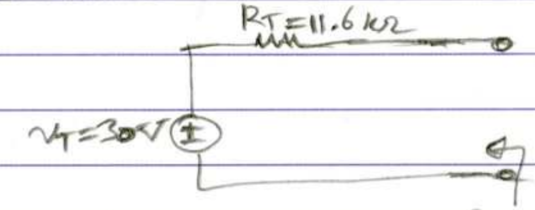
Then computing the equivalent resistance for $10k\Omega$ and $15k\Omega$ resistances, results in the following circuit:



Once more, applying source transformation technique results in the following circuit:



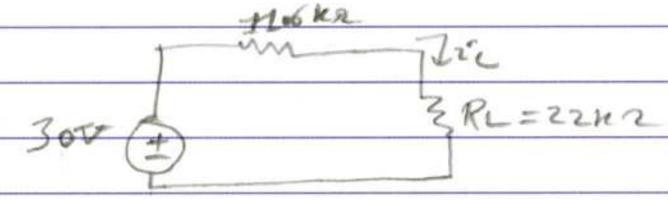
where then provided 6 kΩ and 5.6 kΩ resistances are in series, we obtain the following circuit:



$R_T = 11.6\,k\Omega$, $v_T = 30V$

where the Thévenin equivalent circuit is drawn w/ $\boxed{R_T = 11.6\,k\Omega}$ and $\boxed{v_T = 30V}$.

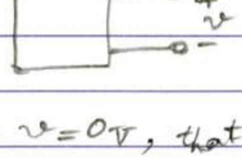
(b) For $R_L = 22\,k\Omega$, we can find i_L, referring to the Thévenin equivalent circuit obtained above:



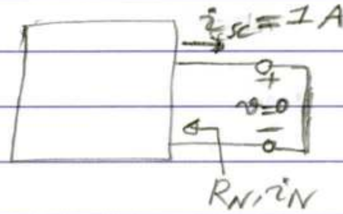
$$\Rightarrow \quad i_L = \frac{30V}{(22+11.6)\times 10^3} \Rightarrow \boxed{i_L = 0.893\ mA}$$

problem 3-58

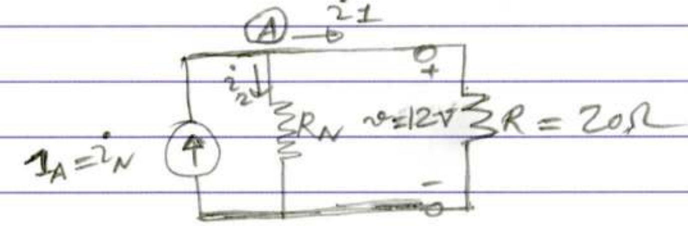
According to the problem statement, we know that for the circuit shown below:



we have $i = 1A$, when $v = 0V$, that is to say the short-circuit current is 1A, as shown in the following circuit:



this is indeed the value of the Norton equivalent of the circuit in the box, i.e., $i_N = i_{sc} = 1A$. We shall then find R_N according to the fact that $v = 12V$ when a 20Ω resistance is connected, this is shown in the following circuit:

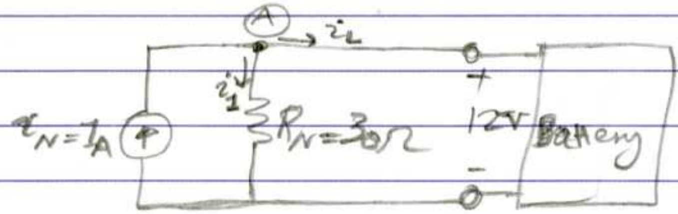


First: given $R = 20\Omega$ and $v = 12V \Rightarrow i_1 = \frac{12}{20} \Rightarrow \boxed{i_1 = 0.6\,A}$

Second, KCL at node A: $i_N = i_1 + i_2 \Rightarrow i_2 = i_N - i_1 = 1 - 0.6 \Rightarrow \boxed{i_2 = 0.4A}$

Third: provided $R = 20\Omega$ and R_N are in parallel, thus the voltage drop across R_N is also $12V \Rightarrow R_N = \frac{12V}{i_2 = 0.4A} \Rightarrow \boxed{R_N = 30\Omega}$

Therefore, having obtained Norton equivalent circuit w/ $i_N = 1A$ and $R_N = 30\Omega$, we can find the current delivered to a 12-V battery as in the following:

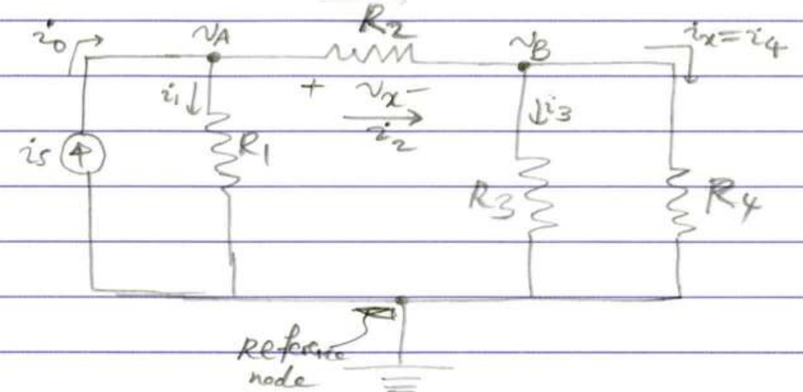


voltage drop across $R_N = 30\Omega$ is $12V \Rightarrow i_1 = \frac{12}{30} \Rightarrow \boxed{i_1 = 0.4A}$

KCL at node A: $i_L = i_N - i_1 \Rightarrow i_L = 1 - 0.4 \Rightarrow \boxed{i_L = 0.6A}$; which is the answer.

problem 3-2:

(a) Given the following circuit, we shall formulate the node-voltage eq'ns as follows:



The reference node, node voltages and the currents associated to each branch are shown on the circuit. We shall then proceed as follows:

First: we note that there are 2 nodes in the circuit, A and B, therefore, we shall employ the method by inspection in order to obtain the final result. In order to do so, we need to develop conductance matrix $G = [G_{ij}]$ w/ the following properties:

(a) G_{ii} = Sum of conductances connected to node i,

(b) G_{ij} = − Conductance b/w node i and node j.

Moreover, in $Gv = i$ eq'n, the vector i constitutes current sources entering node i, and vector v constitutes the node voltages. In the next step, these points are elaborated.

Second:

$$Gv = i \Rightarrow \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} i_S \\ 0 \end{bmatrix}$$

provided i_S enters node A and no current source enters node B.

inspecting the circuit:
$$\begin{cases} G_{11} = \dfrac{1}{R_1} + \dfrac{1}{R_2} \;;\; G_{12} = -\dfrac{1}{R_2} \\ G_{21} = -\dfrac{1}{R_2} \;;\; G_{22} = \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4} \end{cases} \Rightarrow G = \begin{bmatrix} \dfrac{1}{R_1} + \dfrac{1}{R_2} & -\dfrac{1}{R_2} \\ -\dfrac{1}{R_2} & \dfrac{1}{R_2} + \dfrac{1}{R_3} + \dfrac{1}{R_4} \end{bmatrix}$$

$$\Rightarrow \boxed{\underbrace{\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & +\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} v_A \\ v_B \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} i_s \\ 0 \end{pmatrix}}_{b}} \quad (*)$$

(b) We then solve (*) for v_A and v_B, on this way we have used Cramer's rule+ which is discussed below:

$$A = \begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & +\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) \end{pmatrix} \Rightarrow \boxed{\det(A) = +\left(\frac{1}{R_1R_2}+\frac{1}{R_1R_3}+\frac{1}{R_1R_4}+\frac{1}{R_2R_3}+\frac{1}{R_2R_4}\right)}$$

$$A_A = \begin{pmatrix} i_s & -\frac{1}{R_2} \\ 0 & +\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right) \end{pmatrix} \Rightarrow \boxed{\det(A_A) = +i_s\left(\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}\right)}$$

$$A_B = \begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2} & i_s \\ -\frac{1}{R_2} & 0 \end{pmatrix} \Rightarrow \boxed{\det(A_B) = +i_s\left(\frac{1}{R_2}\right)}$$

$$\Rightarrow v_A = \frac{\det(A_A)}{\det(A)} \Rightarrow \boxed{v_A = i_s\left(\frac{\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}}{\frac{1}{R_1R_2}+\frac{1}{R_1R_3}+\frac{1}{R_1R_4}+\frac{1}{R_2R_3}+\frac{1}{R_2R_4}}\right)}$$

and,

$$v_B = \frac{\det(A_B)}{\det(A)} \Rightarrow \boxed{v_B = i_s\left(\frac{\frac{1}{R_2}}{\frac{1}{R_1R_2}+\frac{1}{R_1R_3}+\frac{1}{R_1R_4}+\frac{1}{R_2R_3}+\frac{1}{R_2R_4}}\right)}$$

+ We would like to note that Cramer's rule is employed here, nonetheless other approaches, e.g., Gaussian elimination may be used, as well.

(C) v_x and i_x can be then obtained as follows:

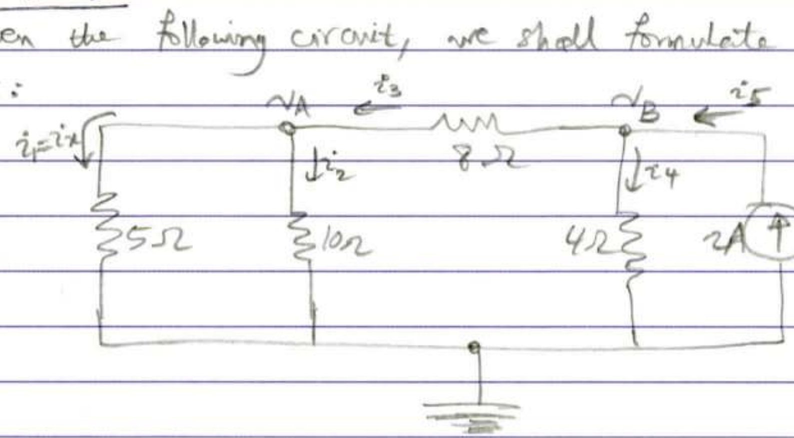
$$v_x = v_A - v_B \Rightarrow v_x = i_s\left(\frac{\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}-\frac{1}{R_2}}{\frac{1}{R_1R_2}+\frac{1}{R_1R_3}+\frac{1}{R_1R_4}+\frac{1}{R_2R_3}+\frac{1}{R_2R_4}}\right)$$

$$\Rightarrow \boxed{v_x = \left(\frac{\frac{1}{R_3}+\frac{1}{R_4}}{\frac{1}{R_1R_2}+\frac{1}{R_1R_3}+\frac{1}{R_1R_4}+\frac{1}{R_2R_3}+\frac{1}{R_2R_4}}\right) i_s}$$

$$i_x = i_4 = \frac{v_B}{R_4} \Rightarrow \boxed{i_x = i_s\left(\frac{\frac{1}{R_2R_4}}{\frac{1}{R_1R_2}+\frac{1}{R_1R_3}+\frac{1}{R_1R_4}+\frac{1}{R_2R_3}+\frac{1}{R_2R_4}}\right)}$$

problem 3-3:

(a) Given the following circuit, we shall formulate the node-voltage eq'ns as follows:



The reference node, node voltages, and the currents associated to each branch are shown on the circuit. We shall then proceed as follows:

First: we note that there are 2 nodes in the circuit, A and B, therefore we employ the method by inspection in order to obtain the final result. On this way, we need to develop matrix G as in $GV = i$. The properties of this matrix has been discussed in problem 3-2.

Second:

$$Gv = i \Rightarrow \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

provided there is no current source connected to A and there is 2A current source connected to B.

inspecting the circuit:
$$\begin{cases} G_{11} = \frac{1}{5} + \frac{1}{10} + \frac{1}{8} = \frac{17}{40} \; ; \; G_{12} = -\frac{1}{8} \\ G_{21} = -\frac{1}{8} \; ; \; G_{22} = \frac{1}{8} + 1/4 = 3/8 \end{cases}$$

$$\Rightarrow G = \begin{bmatrix} \frac{17}{40} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{3}{8} \end{bmatrix} ; v = \begin{bmatrix} v_A \\ v_B \end{bmatrix} ; i = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \boxed{\underbrace{\begin{pmatrix} \frac{17}{40} & -\frac{1}{8} \\ -\frac{1}{8} & \frac{3}{8} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} v_A \\ v_B \end{pmatrix}}_{X} = \underbrace{\begin{pmatrix} 0 \\ 2 \end{pmatrix}}_{b}} \quad (*)$$

(b) we then solve (*) for v_A and v_B, by using Cramer's rule, as discussed below:

$$A = \begin{pmatrix} \frac{17}{40} & -1/8 \\ -1/8 & 3/8 \end{pmatrix} \Rightarrow \det(A) = \frac{17}{40} \times 3/8 - (-1/8)(-1/8) \Rightarrow \boxed{\det(A) = \frac{23}{160}}$$

$$A_A = \begin{pmatrix} 0 & -1/8 \\ 2 & 3/8 \end{pmatrix} \Rightarrow \boxed{\det(A_A) = 1/4} \qquad A_B = \begin{pmatrix} 17/40 & 0 \\ -1/8 & 2 \end{pmatrix} \Rightarrow \boxed{\det(A_B) = 17/20}$$

$$\Rightarrow v_A = \frac{\det(A_A)}{\det(A)} = \frac{1/4}{23/160} \Rightarrow \boxed{v_A = \frac{40}{23} v}$$

and,
$$v_B = \frac{\det(A_B)}{\det(A)} = \frac{17/20}{23/160} \Rightarrow \boxed{v_B = \frac{136}{23} v}$$

(c) v_x and i_x can be then obtained as follows:

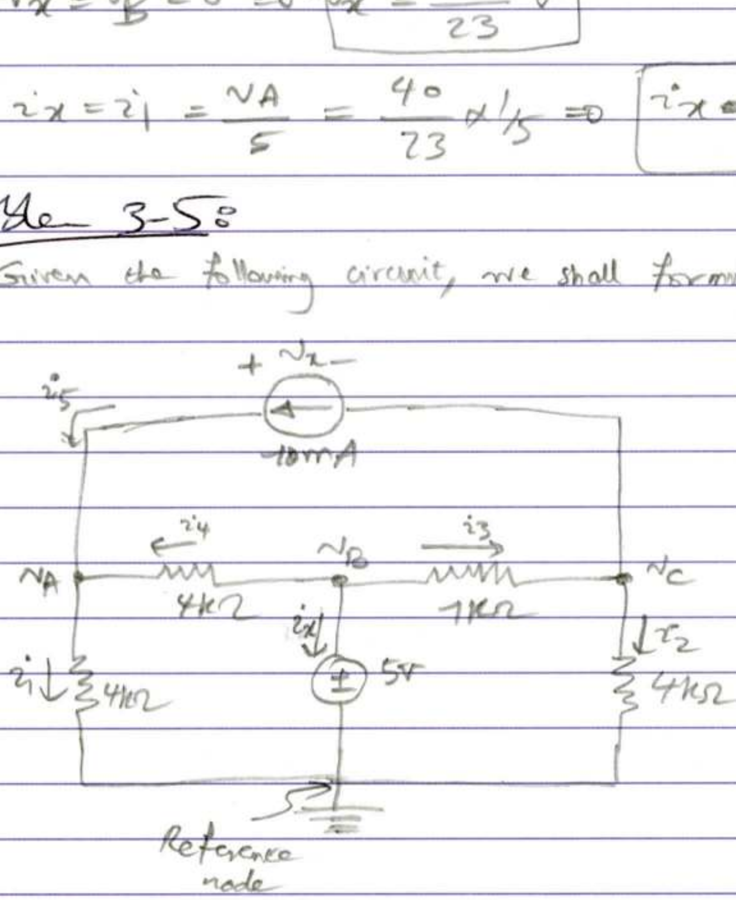
$v_x = v_B - 0 \Rightarrow \boxed{v_x = \frac{136}{23}\,V}$

$i_x = i_1 = \frac{v_A}{5} = \frac{40}{23} \times \frac{1}{5} \Rightarrow \boxed{i_x = \frac{8}{23}\,A}$

■

prob. 3-5:

(a) Given the following circuit, we shall formulate the node-voltage eqns as follows:



The reference node, node voltages, and the currents associated to each branch are shown on the circuit. We note that provided there is a voltage source of 5V in the circuit, we shall refer to the second method, ie, we consider $\boxed{v_B = 5V}$ in our subsequent analysis as discussed in the following:

First: we derive $Gv = i$ eqn for nodes A, B, and C, considering i_x as our current source leaving node B. Therefore, one obtains:

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{31} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} 10m \\ -i_x \\ -10m \end{bmatrix}$$

$G_{11} = \frac{1}{4k} + \frac{1}{4k}$; $G_{12} = -\frac{1}{4k}$; $G_{13} = 0$
$G_{21} = -\frac{1}{4k}$; $G_{22} = \frac{1}{4k} + \frac{1}{1k} + 0$; $G_{23} = -\frac{1}{1k}$
$G_{31} = 0$; $G_{32} = -\frac{1}{1k}$; $G_{33} = \frac{1}{1k} + \frac{1}{4k}$
and $v_B = 5V$.

Accordingly, we shall obtain the following eq'ns:

$$\begin{cases} \left(\frac{1}{4k}+\frac{1}{4k}\right)v_A + \left(-\frac{1}{4k}\right)5 = 10m \\ -\frac{1}{4k}v_A + \left(\frac{1}{4k}+\frac{1}{1k}\right)5 - \frac{1}{1k}v_C = -i_x \\ \left(\frac{-1}{1k}\right)5 + \left(\frac{1}{1k}+\frac{1}{4k}\right)v_C = -10m \end{cases} \Rightarrow \begin{cases} \left(\frac{1}{4k}+\frac{1}{4k}\right)v_A = 10m + \frac{5}{4}m \quad (*) \\ \left(\frac{1}{1k}+\frac{1}{4k}\right)v_C = -10m + 5m \quad (**) \\ i_x = \left[v_C + \frac{1}{4}v_A - \left(\frac{1}{4}+1\right)5\right]_{mA} \quad (***) \end{cases}$$

Therefore, provided eq'ns (*) and (**), we get the following matrix form, wherein we have simplified both RHS and LHS of the eq'ns:

$$\Rightarrow \boxed{\underbrace{\begin{pmatrix} +0.5 & 0 \\ 0 & +1.25 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} v_A \\ v_C \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} +11.25 \\ -5 \end{pmatrix}}_{b}} \quad (\star)$$

(b) we then solve ($\star$) for v_A and v_C:

$$v_A = \frac{11.25}{0.5} = 2 \times 11.25 \Rightarrow \boxed{v_A = 22.5\,V}$$

$$v_C = \frac{5}{-1.25} \Rightarrow \boxed{v_C = -4\,V}$$

(c) v_x and i_x can be then obtained as follows:

$$v_x = v_A - v_C = 22.5 - (-4) \Rightarrow \boxed{v_x = 26.5\,V}$$

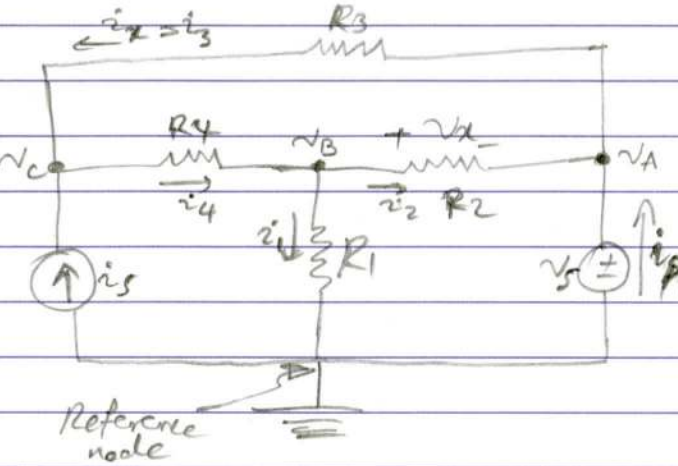
recalling (***) for i_x:

$$i_x = \left[v_C + \frac{1}{4}v_A - 5(1+1/4)\right]_{mA} = \left[-4 + \frac{1}{4}(22.5) - 5(5/4)\right]_{mA}$$

$$\Rightarrow \boxed{i_x = -4.625\ mA}$$

problem 3-6:

(a) Given the following circuit, we shall formulate the node-voltage eq'ns as follows:



The reference node, node voltages, and the currents associated to each branch are shown on the circuit. We also note that provided there is a voltage source of v_s in the circuit, we shall refer to the second method, i.e., we consider $v_A = v_s$ in our subsequent analysis, as discussed in the following:

First: we derive $GV = i$ eq'n for nodes A, B, and C, considering i_A as the current source approaching node A. Therefore, one obtains:

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} \\ G_{21} & G_{22} & G_{23} \\ G_{23} & G_{32} & G_{33} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \end{bmatrix} = \begin{bmatrix} i_A \\ 0 \\ i_s \end{bmatrix}$$

$G_{11} = \frac{1}{R_2} + \frac{1}{R_3}$; $G_{12} = \frac{-1}{R_2}$; $G_{13} = \frac{-1}{R_3}$;
$G_{21} = -\frac{1}{R_2}$; $G_{22} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}$;
$G_{23} = \frac{-1}{R_4}$; $G_{31} = \frac{-1}{R_3}$; $G_{32} = \frac{-1}{R_4}$;
$G_{33} = \frac{1}{R_3} + \frac{1}{R_4}$ and $v_A = v_s$

Accordingly, we shall obtain the following eq'ns:

$$\begin{cases} \left(\frac{1}{R_2} + \frac{1}{R_3}\right) v_s - \frac{1}{R_2} v_B - \frac{1}{R_3} v_C = i_A \\ -\frac{1}{R_2} v_s + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_4}\right) v_B - \frac{1}{R_4} v_C = 0 \\ -\frac{1}{R_3} v_s - \frac{1}{R_4} v_B + \left(\frac{1}{R_3} + \frac{1}{R_4}\right) v_C = i_s \end{cases}$$

where thereby some rearrangement of terms, we shall obtain:

$$\begin{cases} \left(\frac{1}{R_2}+\frac{1}{R_3}\right) v_S - \frac{1}{R_2} v_B - \frac{1}{R_3} v_C = i_A & (*) \\ \left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}\right) v_B - \frac{1}{R_4} v_C = \frac{1}{R_2} v_S & (**) \\ -\frac{1}{R_4} v_B + \left(\frac{1}{R_3}+\frac{1}{R_4}\right) v_C = i_S + \frac{1}{R_3} v_S & (***) \end{cases}$$

Therefore writing eq'ns (**) and (***) in matrix form, we get:

$$\boxed{\Rightarrow \underbrace{\begin{pmatrix} \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4} & -\frac{1}{R_4} \\ -\frac{1}{R_4} & \frac{1}{R_3}+\frac{1}{R_4} \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} v_B \\ v_C \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} \frac{1}{R_2} v_S \\ i_S + \frac{v_S}{R_3} \end{pmatrix}}_{b}} \quad (*)$$

(b) Having derived (*) and for $R_1 = R_2 = R_3 = R_4 = 10\,k\Omega$, $v_S = 25\,V$ and $i_S = 1\,mA$, we solve (*) for v_B and v_C:

$$\begin{pmatrix} 3\times10^{-4} & -10^{-4} \\ -10^{-4} & 2\times10^{-4} \end{pmatrix} \begin{pmatrix} v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 2.5\times10^{-3} \\ 10^{-3} + 2.5\times10^{-3} \end{pmatrix} = \begin{pmatrix} 2.5\times10^{-3} \\ 3.5\times10^{-3} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 3 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} v_B \\ v_C \end{pmatrix} = \begin{pmatrix} 25 \\ 35 \end{pmatrix} \Rightarrow \begin{cases} \boxed{v_B = 17\,V} \\ \boxed{v_C = 26\,V} \end{cases}$$

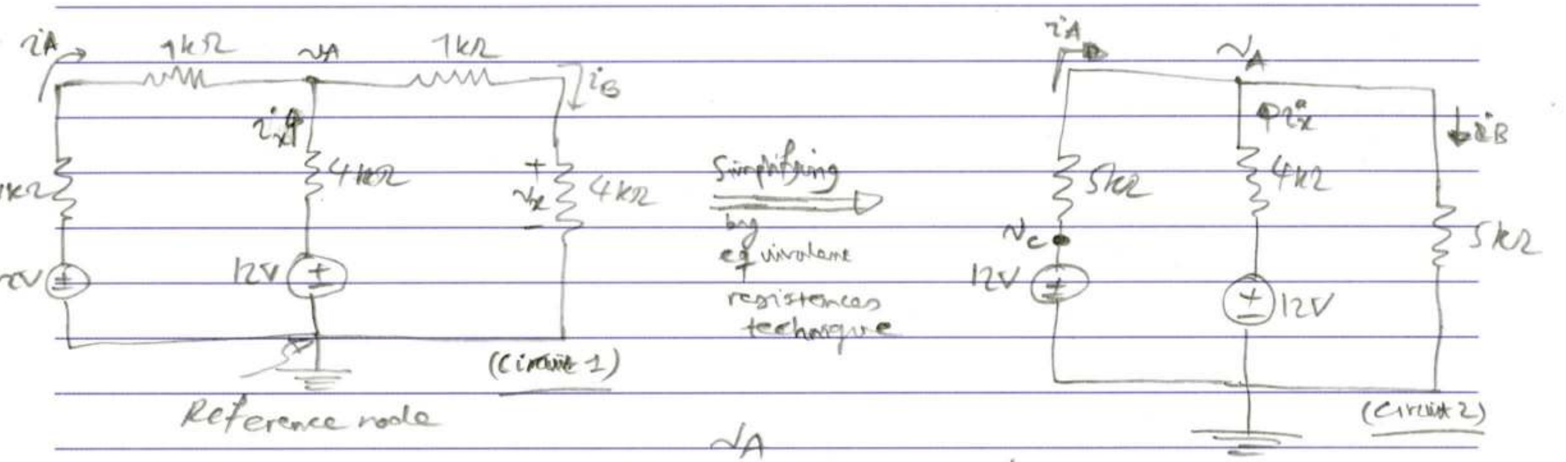
where then referring to the circuit, i_x and v_x can be computed as follows:

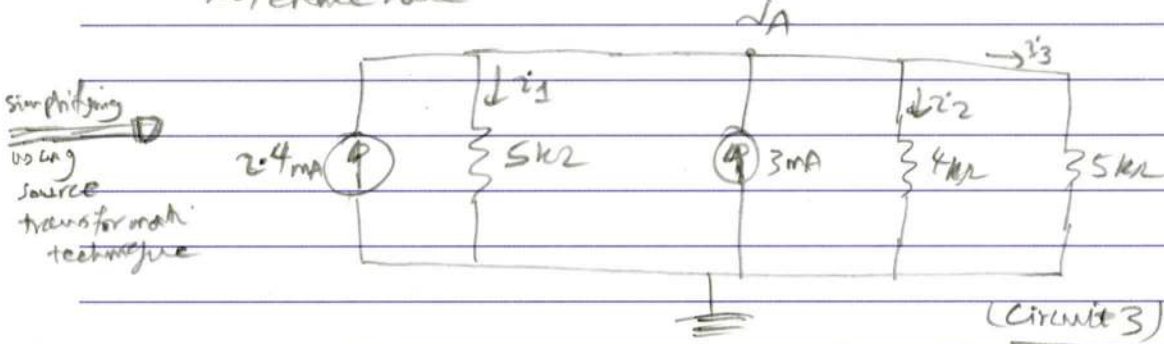
$$v_x = v_B - v_A = v_B - v_S = 17 - 25 \Rightarrow \boxed{v_x = -8\,V}$$

$$i_x = i_3 = \frac{v_A - v_C}{R_3} = \frac{25-26}{10\times10^{3}} \Rightarrow \boxed{i_x = -0.1\,mA}$$

problem 3-15:

For the following circuit, we shall first find v_A by formulating node-voltage eq'ns as discussed below, where we use source transformation technique to transform the voltage to current source as shown in the following figure.





Referring to circuit 3, we write KCL at node (A):

KCL at node (A): $2.4\times10^{-3} + 3\times10^{-3} = i_1 + i_2 + i_3$

$$\Rightarrow 5.4\times10^{-3} = \frac{v_A}{5}\times10^{-3} + \frac{v_A}{4}\times10^{-3} + \frac{v_A}{5}\times10^{-3} \Rightarrow 5.4 = v_A(0.2+0.25+0.2)$$

$$\Rightarrow v_A = \frac{5.4}{0.65} \Rightarrow \boxed{v_A = 8.308\,V}$$

Having found v_A, we then refer to circuit 2 to find i_A, i_x, and i_B, where we note $v_C = 12V$:

$$i_A = \frac{v_C - v_A}{5\times10^3} = \frac{12 - 8.308}{5\times10^3} \Rightarrow \boxed{i_A = 0.738\,mA} \; ; \; i_B = \frac{v_A}{5\times10^3} = \frac{8.308}{5}\times10^{-3}$$

$$\Rightarrow \boxed{i_B = 1.66\,mA}$$

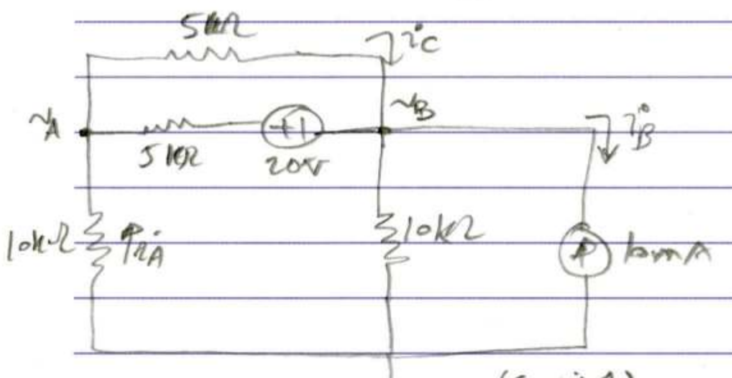
Then, KCL at (A) on circuit 2 : $i_x = i_B - i_A = 1.66 - 0.738 \Rightarrow \boxed{i_x = 0.922 \text{ mA}}$

At last, we shall find v_x, referring to circuit 1:

$$v_x = 4\times10^3 \times i_B = 4\times10^3 \times 1.66\times10^{-3} \Rightarrow \boxed{v_x = 6.64\ v}$$

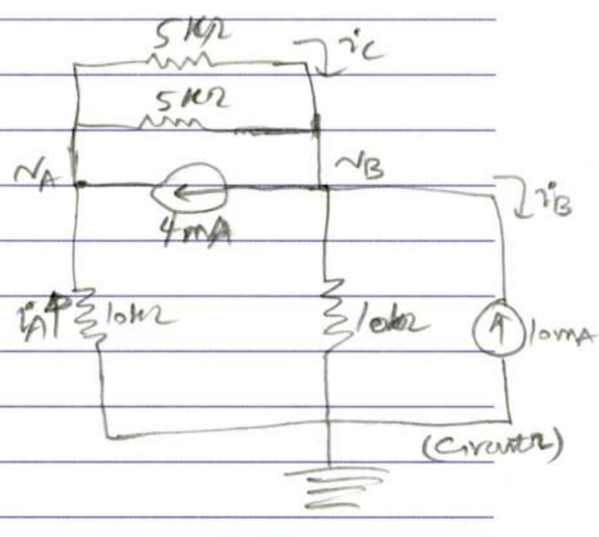
problem 3-26:

(b) For the following circuit, we shall develop node-voltage eq'ns, where we have used the source transformation technique to transform the voltage source to a current source as shown in the following figure:



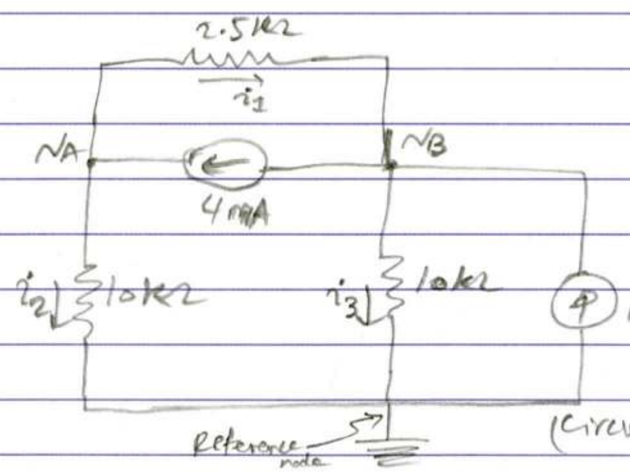
(Circuit 1)

applying source transformation technique



(Circuit 2)

simplifying by equivalent resistance technique



(Circuit 3)

Referring to circuit 3 : we note that there are 2 nodes in the circuit, A and B, therefore, we employ the method by inspection in order to obtain the final result. On this way, we need to develop matrix G as in $Gv = i$. The properties of this matrix has been discussed in proble 3-2

$$Gv = i \Rightarrow \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 4\times10^{-3} \\ (10-4)\times10^{-3} \end{bmatrix}$$

provided current source of 4 mA is approaching node A. And that total current of (10−4) mA is approaching B

inspecting the circuits:

$$G_{11} = \frac{1}{2.5k} + \frac{1}{10k} = 0.5\times10^{-3}$$

$$G_{12} = \frac{1}{2.5k} = 0.4\times10^{-3}$$

$$G_{21} = \frac{1}{2.5k} = 0.4\times10^{-3}$$

$$G_{22} = \frac{1}{2.5k} + \frac{1}{10k} = 0.5\times10^{-3}$$

which then in matrix form:

$$\boxed{\underbrace{\begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix}}_{A} \underbrace{\begin{pmatrix} v_A \\ v_B \end{pmatrix}}_{x} = \underbrace{\begin{pmatrix} 40 \\ +60 \end{pmatrix}}_{b}} \quad (*)$$

(d) We then solve (*) for v_A, v_B, by using Cramer's rule to obtain v_A and v_B:

$$A = \begin{pmatrix} 5 & -4 \\ -4 & 5 \end{pmatrix} \Rightarrow \boxed{\det(A) = 9} \qquad A_B = \begin{pmatrix} 5 & 40 \\ -4 & 60 \end{pmatrix} \Rightarrow \boxed{\det(A_B) = 460}$$

$$A_A = \begin{pmatrix} 40 & -4 \\ +60 & 5 \end{pmatrix} \Rightarrow \boxed{\det(A_A) = 440}$$

$$\Rightarrow v_A = \frac{\det(A_A)}{\det(A)} \Rightarrow \boxed{v_A = \frac{440}{9}\,\text{v}} \text{ and } v_B = \frac{\det(A_B)}{\det(A)} \Rightarrow \boxed{v_B = \frac{460}{9}\,\text{v}}$$

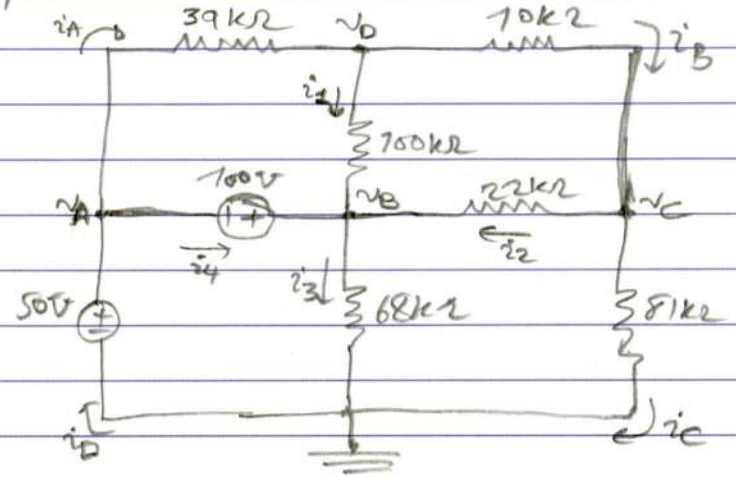
At last, returning to circuit 2:

$$\boxed{i_B = -10\text{mA}} \qquad i_C = \frac{v_A - v_B}{5\times10^3} = \frac{440/9 - 460/9}{5\times10^3} \Rightarrow \boxed{i_C = -0.44\text{mA}}$$

$$i_A = \frac{0 - v_A}{10\times10^3} = \frac{-440/9}{10\times10^3} \Rightarrow \boxed{i_A = -4.89\text{ mA}}$$

problem 3–27 :

(c) For the circuit shown in the following figure, we shall formulate node-voltage eqns:



On this way we use the first method where we know $\boxed{v_A = 50V}$ and that $v_B - v_A = 100 \Rightarrow \boxed{v_B = 150\ V}$.

First: We derive $GV = i$ eqn for nodes A, B, C, and D, considering i_D and i_4 as current sources approaching/leaving nodes A and B. Therefore, one obtains:

$$\begin{bmatrix} G_{11} & G_{12} & G_{13} & G_{14} \\ G_{21} & G_{22} & G_{23} & G_{24} \\ G_{31} & G_{32} & G_{33} & G_{34} \\ G_{41} & G_{42} & G_{43} & G_{44} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_C \\ v_D \end{bmatrix} = \begin{bmatrix} i_D - i_4 \\ i_4 \\ 0 \\ 0 \end{bmatrix}$$

$G_{11} = \frac{1}{39k}$; $G_{12} = 0$, $G_{13} = 0$; $G_{14} = \frac{-1}{39k}$;
$G_{21} = 0$; $G_{22} = \frac{1}{100k} + \frac{1}{22k} + \frac{1}{68k}$;
$G_{23} = \frac{-1}{22k}$; $G_{24} = \frac{-1}{100k}$;
$G_{31} = 0$; $G_{32} = \frac{-1}{22k}$; $G_{33} = \frac{1}{22k} + \frac{1}{10k} + \frac{1}{81k}$
$G_{34} = \frac{-1}{10k}$; $G_{41} = \frac{-1}{39k}$; $G_{42} = \frac{-1}{100k}$;
$G_{43} = \frac{-1}{10k}$; $G_{44} = \frac{1}{100k} + \frac{1}{10k} + \frac{1}{39k}$
and $v_A = 50$; $v_B = 150$

Accordingly, we shall obtain the following eqns:

$$\begin{cases} \frac{1}{39k} 50 - \frac{1}{39k} v_D = i_D - i_4 \\ \left(\frac{1}{100k} + \frac{1}{22k} + \frac{1}{68k}\right)150 - \frac{1}{22k} v_C - \frac{1}{100k} v_D = i_4 \\ -\left(\frac{1}{22k}\right)150 + \left(\frac{1}{22k} + \frac{1}{10k} + \frac{1}{81k}\right) v_C - \frac{1}{10k} v_D = 0 \\ -\frac{1}{39k} 50 - \frac{1}{100k} 150 - \frac{1}{10k} v_C + \left(\frac{1}{100k} + \frac{1}{10k} + \frac{1}{39k}\right) v_D = 0 \end{cases}$$

where by some rearrangement in terms yields:

$$\begin{cases} \frac{1}{39k}50 - \frac{1}{39k}v_D = i_D - i_4 & (*) \\ \left(\frac{1}{100k}+\frac{1}{22k}+\frac{1}{68k}\right)150 - \frac{1}{22k}v_C - \frac{1}{100k}v_D = i_4 & (**) \\ \frac{-1}{22k}150 + \left(\frac{1}{22k}+\frac{1}{10k}+\frac{1}{81k}\right)v_C - \frac{1}{10k}v_D = 0 & (***) \\ \frac{-1}{39k}50 - \frac{1}{100k}150 - \frac{1}{10k}v_C + \left(\frac{1}{100k}+\frac{1}{10k}+\frac{1}{39k}\right)v_D = 0 & (****) \end{cases}$$

Eq'ns (***) and (****) can be written in a matrix form which results the following.

$$\begin{pmatrix} \frac{1}{22}+\frac{1}{10}+\frac{1}{81} & \frac{-1}{10} \\ \frac{-1}{10} & \frac{1}{100}+\frac{1}{10}+\frac{1}{39} \end{pmatrix}\begin{pmatrix} v_C \\ v_D \end{pmatrix} = \begin{pmatrix} \frac{+150}{22} \\ \frac{+150}{100}+\frac{50}{39} \end{pmatrix}$$

(d) we shall then solve the latest result of the previous section, using Cramer's rule:

$$A = \begin{pmatrix} \frac{1}{22}+\frac{1}{10}+\frac{1}{81} & \frac{-1}{10} \\ \frac{-1}{10} & \frac{1}{100}+\frac{1}{10}+\frac{1}{39} \end{pmatrix} \Rightarrow \boxed{\det(A) = 0.0114}$$

$$A_D = \begin{pmatrix} \frac{150}{22} & \frac{-1}{10} \\ \frac{150}{100}+\frac{50}{39} & \frac{1}{100}+\frac{1}{10}+\frac{1}{39} \end{pmatrix} \Rightarrow \boxed{\det(A_D) = 1.1208}$$

$$A_C = \begin{pmatrix} \frac{1}{22}+\frac{1}{10}+\frac{1}{81} & \frac{150}{22} \\ \frac{-1}{10} & \frac{150}{100}+\frac{50}{39} \end{pmatrix} \Rightarrow \boxed{\det(A_C) = 1.2028}$$

$$\Rightarrow v_D = \frac{\det(A_D)}{\det(A)} = \frac{1.1208}{0.0114} \Rightarrow \boxed{v_D = 98.32\,v}$$

and,

$$v_C = \frac{\det(A_C)}{\det(A)} = \frac{1.2028}{0.0114} \Rightarrow \boxed{v_C = 105.51\,v}$$

which when recalling $v_A = 50v$, $v_B = 150v$, by referring to the circuit, we obtain:

$$i_A = \frac{v_A - v_D}{39 \times 10^3} = \frac{50 - 98.32}{39 \times 10^3} \Rightarrow i_A = \frac{-48.32}{39 \times 10^3} \Rightarrow \boxed{i_A = -1.24 mA}$$

$$i_B = \frac{v_D - v_C}{10 \times 10^3} = \frac{98.32 - 105.51}{10 \times 10^3} \Rightarrow \boxed{i_B = -0.72 mA}$$

$$i_C = \frac{v_C - 0}{81 \times 10^3} = \frac{105.51}{81 \times 10^3} \Rightarrow \boxed{i_C = 1.30 mA}$$

$$i_3 = \frac{v_B - 0}{68 \times 10^3} = \frac{150}{68 \times 10^3} \Rightarrow \boxed{i_3 = 2.20 mA}$$

$$i_1 = \frac{v_D - 150}{10^5} = \frac{98.32 - 150}{10^5} \Rightarrow \boxed{i_1 = -0.52 mA}$$

$$i_2 = \frac{-150 + v_C}{22 \times 10^3} = \frac{-150 + 105.51}{22 \times 10^3} \Rightarrow \boxed{i_2 = -2.02 mA}$$

Then, recalling eqns (*) and (**) from previous part, we can obtain i_D and i_4:

$$\begin{cases} \frac{1}{39k} 50 - \frac{1}{39k}(98.32) = i_D - i_4 \Rightarrow \boxed{i_D = 3.5060 mA} \\ \left(\frac{1}{100k} + \frac{1}{22k} + \frac{1}{68k}\right)150 - \frac{1}{22k}(105.51) - \frac{1}{100k}(98.32) = i_4 \Rightarrow \boxed{i_4 = 4.7450 mA} \end{cases}$$

END of HW#3 Solns