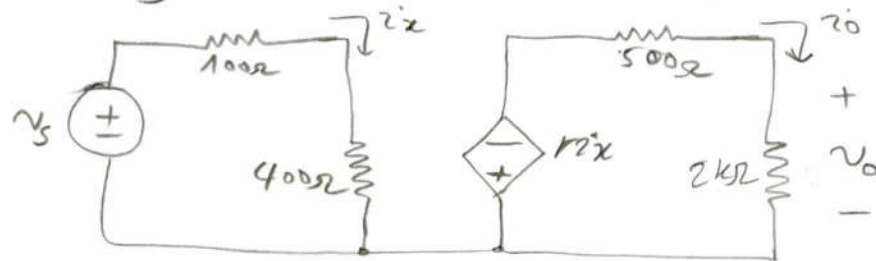


problem 4-1:

For the circuit shown in the following figure, we would like to find $\frac{v_o}{v_s}$ and $\frac{i_o}{i_x}$ where $r = 5 \text{ k}\Omega$.



First: For the portion of the circuit shown on the left hand side (LHS), and provided the two resistances are in series, we obtain:

$$i_x = \frac{v_s}{400 + 100} \Rightarrow \boxed{i_x = \frac{v_s}{500}} (*)$$

Second: For the portion of the circuit shown on the right hand side (RHS), and provided the two resistances are in series, we obtain:

$$i_o = \frac{-r i_x}{2500} \Rightarrow \frac{i_o}{i_x} = \frac{-r}{2500}$$

which then recalling $r = 5000 \Omega$, results the following:

$$\frac{i_o}{i_x} = -\frac{5000}{2500} \Rightarrow \boxed{\frac{i_o}{i_x} = -2} (*)$$

On the other hand, v_o can be also obtained by using voltage division technique which results the following:

$$v_o = -r i_x \left(\frac{2000}{2000 + 500} \right) \Rightarrow \boxed{v_o = -\frac{4r}{5} i_x}$$

which then by (*), we get:

$$v_o = -\frac{4r}{5} \left(\frac{v_s}{500} \right) \Rightarrow \boxed{v_o = -\frac{4r}{2500} v_s}$$

where then provided $r = 5000 \Omega$, we get:

$$v_o = -\frac{4 \times 5 \times 10^3}{2.5 \times 10^3} v_s \Rightarrow v_o = -8 v_s \Rightarrow \boxed{\frac{v_o}{v_s} = -8} (**)$$

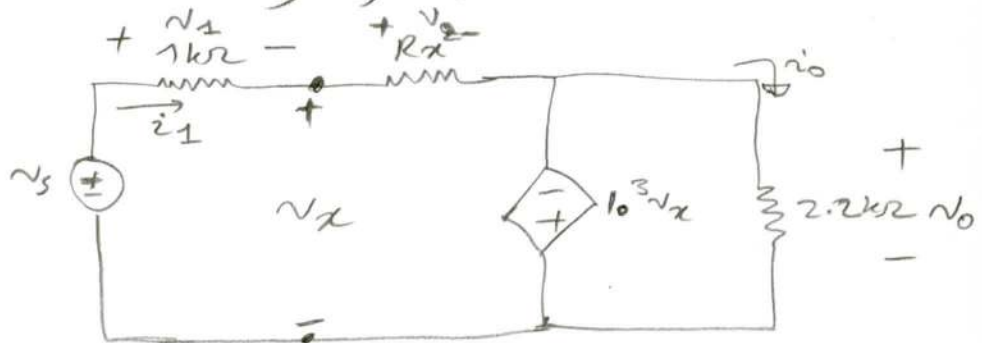
Therefore, having obtained (*) and (**), the solution to this problem is complete.

page 2



problem 4-8:

(a) Provided the circuit shown in the following figure, we would like to find $\frac{i_o}{v_s}$ in terms of R_x .



First: Referring to the circuit, we note that the value of the voltage drop, v_o , across $2.2k\Omega$ resistance is $v_o = -10^3 v_x$, where then the current i_o passing through this resistance can be also characterized as follows:

$$i_o = \frac{v_o}{2.2 \times 10^3} = -\frac{10^3 v_x}{2.2 \times 10^3} \Rightarrow \boxed{i_o = -\frac{v_x}{2.2}} \quad (*)$$

Second: Referring to the figure, the voltage drop across R_x , i.e., v_2 , can be obtained as follows:

$$v_2 = v_x + 10^3 v_x \Rightarrow \boxed{v_2 = 1001 v_x} \quad (**)$$

Moreover, in a similar way, the voltage drop across $1k\Omega$ resistance, i.e., v_1 , can be obtained as follows:

$$\boxed{v_1 = v_s - v_x} \quad (***)$$

where then, accordingly, the current i_1 which passes through $1k\Omega$ and R_x resistances can be developed in the following way:

$$i_1 = \frac{v_1}{1000} = \frac{v_2}{R_x} \xrightarrow[\text{ad(***)}]{\text{by(**)}} \boxed{\frac{v_s - v_x}{1000} = \frac{1001 v_x}{R_x}} \quad (*)$$

Therefore, according to eqn (*), we can find the following relationship b/w v_s and v_x :

$$\frac{v_s}{1000} = \frac{v_x}{1000} + \frac{1001}{R_x} v_x \Rightarrow v_s = 1000 \left(\frac{1}{1000} + \frac{1001}{R_x} \right) v_x$$

(page 5)

$$\Rightarrow V_s = \frac{R_x + 1001}{R_x} V_x \Rightarrow \boxed{\frac{V_x}{V_s} = \frac{R_x}{R_x + 1001000}} \quad (**)$$

Therefore, recalling eqn (*), we get:

$$i_o = -\frac{V_x}{2.2} \xrightarrow{By (**)} i_o = -\frac{1}{2.2} \left(\frac{R_x}{R_x + 1001000} \right) V_s$$

$$\Rightarrow \boxed{\frac{i_o}{V_s} = \frac{-R_x}{2.2R_x + 2202200}} \quad (+)$$

(b) Provided (+), obtained in the previous part of the problem, we shall find R_x such that $\frac{i_o}{V_s} = -0.227$ in the following way:

$$\frac{-R_x}{2.2R_x + 2202200} = -0.227 \Rightarrow R_x = 0.5R_x + 5 \times 10^5$$

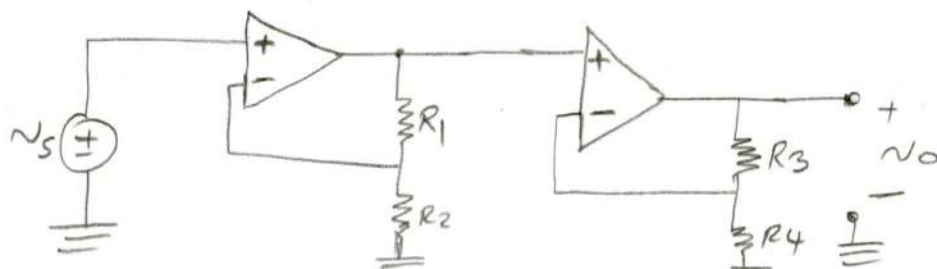
$$\Rightarrow \boxed{R_x = 1000 \text{ k}\Omega}$$

prob 4-24:

According to the problem statement, we would like to design OPAMP Circuits using standard 1% tolerance resistors maintaining selected values of voltage gains. In order to proceed, we shall categorize the voltage gains into positive and negative values and treat them in the following two parts, where K stands for the voltage gain.

→ Positive-valued voltage gains:

(*) $K = 200$: For this case, we have designed a circuit using two non-inverting OPAMP circuits as shown in the following figure:



therefore, recalling noninverting opamp gain and provided these 2 circuits are connected in series, the overall gain of this circuit would be obtained as follows:

(page 4)

$$K_{\text{tot}} = \left(\frac{R_1 + R_2}{R_2} \right) \times \left(\frac{R_3 + R_4}{R_4} \right)$$

more, on the other hand, we would like to maintain $K = 200$, therefore, we need to choose the resistances R_1, R_2, R_3 and R_4 such that the following holds:

$$\frac{R_1 + R_2}{R_2} \times \frac{R_3 + R_4}{R_4} = 200$$

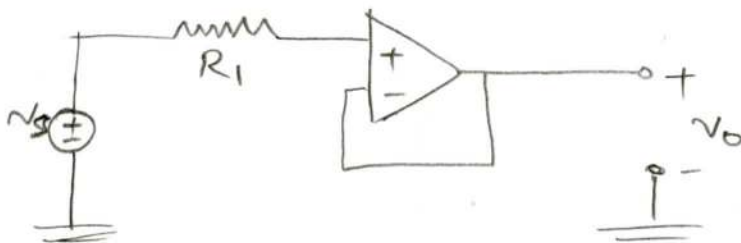
Hence, w/o loss of generality, we can consider the following:

$$\begin{cases} \frac{R_1 + R_2}{R_2} = 10 \Rightarrow R_1 = 9R_2 \quad (*) \\ \frac{R_3 + R_4}{R_4} = 20 \Rightarrow R_3 = 19R_4 \quad (**) \end{cases}$$

Provided (*) and (**) and recalling standard values for 10% resistance, we have considered the following values for chosen resistances:

$$\begin{aligned} R_2 &= 10\Omega & R_1 &= 90\Omega = 9 \times 10\Omega \text{ (9 resistances of } 10\Omega \text{ in series)} \\ R_4 &= 10\Omega & R_3 &= 190\Omega = 19 \times 10\Omega \text{ (19 resistances of } 10\Omega \text{ in series)} \end{aligned}$$

(*) $K = 1$: For this case, we have selected the buffer circuit, as shown in the following figure:



where recalling buffer gain, the following holds:

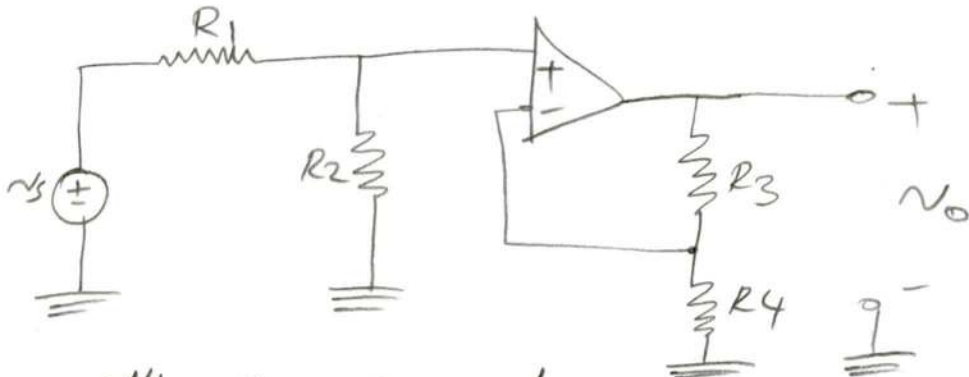
$$\frac{V_o}{V_s} = 1$$

which satisfies the design requirement. Resistance R_1 can be chosen to

be $R_1 = 10\text{ k}\Omega$ which is one of the 1% standard resistances.

(pages)

(*) $K = 0.5$: For this case, we have designed a circuit similar to Ex. 4-13 and as drawn in the following figure:



For which, recalling the analysis of Ex. 4-13, the following gain holds:

$$K_{tot} = \frac{R_2}{R_1 + R_2} \times \frac{R_3 + R_4}{R_4}$$

where we would like to ensure $K_{tot} = 0.5$, therefore the following needs to hold:

$$\frac{R_2}{R_1 + R_2} \times \frac{R_3 + R_4}{R_4} = 0.5$$

Then, in order to ensure the above and provided resistances are positive-valued quantities, we have considered:

$$\begin{cases} \frac{R_2}{R_1 + R_2} = 0.25 \Rightarrow \underline{R_1 = 3R_2} \\ \frac{R_3 + R_4}{R_4} = 2 \Rightarrow \underline{R_3 = R_4} \end{cases}$$

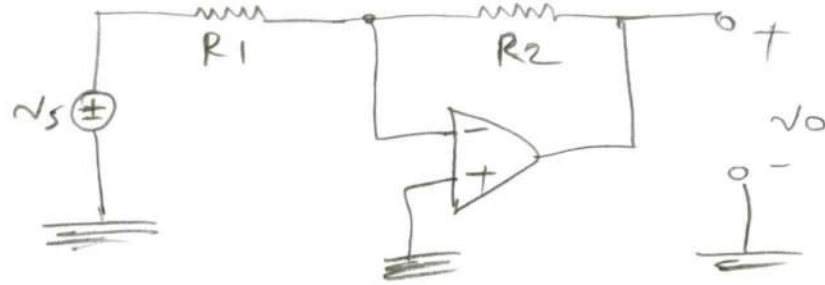
therefore, referring to the standard resistance values of 1% tolerance, we can pick:

$$\underline{R_2 = 10\text{ k}\Omega}, \underline{R_1 = 30\text{ k}\Omega = 3 \times 10\text{ k}\Omega} \text{ (3 resistances of } 10\text{ k}\Omega \text{ in series)}$$

$$\underline{R_3 = 10\text{ k}\Omega}, \underline{R_4 = 10\text{ k}\Omega}$$

→ Negative-valued voltage gain:

(*) $K = -10$: For this case, we have designed an inverting op Amp circuit as shown in the following figure:



where recalling inverting opamp gain, the following holds:

$$K_{tot} = -\frac{R_2}{R_1}$$

we also would like to ensure $K_{tot} = -100$, therefore we obtain:

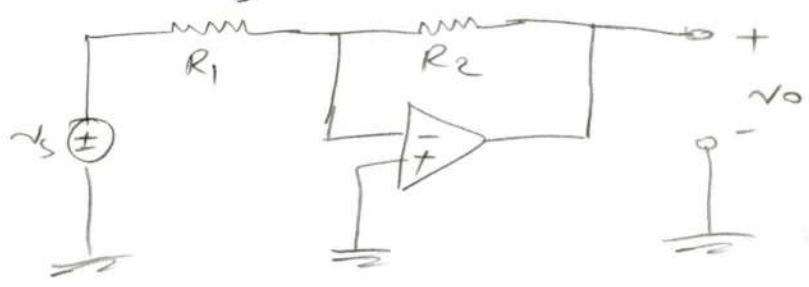
$$-\frac{R_2}{R_1} = -100 \Rightarrow \left[\frac{R_2}{R_1} = 100 \right]$$

In order to satisfy the above, we have chosen:

$$\begin{cases} R_2 = 10 \times 15\Omega = 10(68 + 82)\Omega = 10 \text{ series of } 68\Omega \text{ and } 82\Omega \text{ resistances} \\ R_1 = 15\Omega \end{cases}$$

we also note that 15, 68, and 82 Ω resistances are amongst the standard 10% values of resistances.

(*) $K = -0.5$: for this case, we again consider an inverting opamp circuit as shown in the following figure:



for which the following gain holds:

$$K_{tot} = -\frac{R_2}{R_1}$$

where we would like to ensure $K_{tot} = -0.5$, therefore:

$$-0.5 = -\frac{R_2}{R_1} \Rightarrow \left[\frac{R_2}{R_1} = 0.5 \right]$$

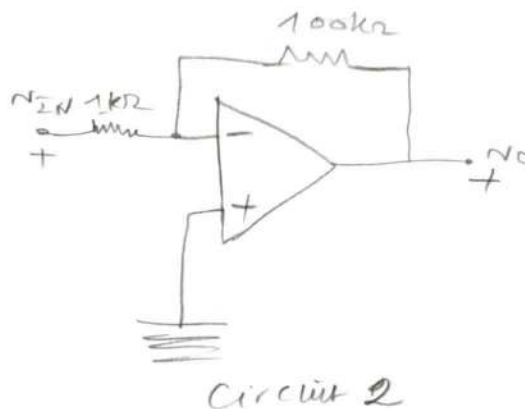
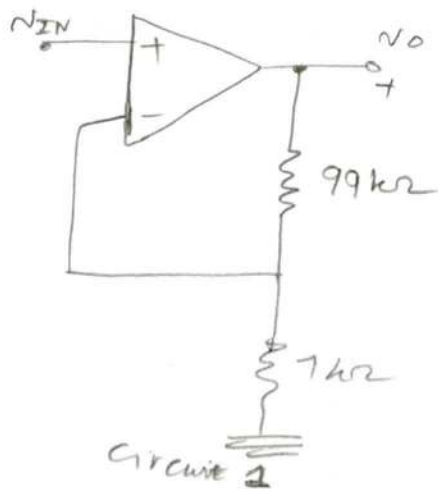
in order to satisfy the latter eq'n, we have considered the following values for resistances:

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$$\begin{cases} R_2 = 10\Omega \\ R_1 = 20\Omega = 2 \times 10\Omega = 2 \text{ resistances of } 10\Omega \text{ in series} \end{cases}$$

problem 4-25:

(a) For the two op-amp circuits shown below, we would like to show that either gain ± 60 is ensured.



Circuit 1: let first $R_1 = 99k\Omega$, $R_2 = 1k\Omega$, then we note that this circuit is indeed the non-inverting op-amp circuit for which the following holds:

$$K_1 = \frac{R_1 + R_2}{R_2}$$

therefore, for circuit 1 and provided $R_1 = 99k\Omega$, $R_2 = 1k\Omega$ the following holds:

$$K_1 = \frac{99k + 1k}{1k} \Rightarrow |K_1| = 100 (*)$$

Circuit 2: let first $R_1 = 1k\Omega$ and $R_2 = 100k\Omega$, then we note that this circuit is indeed the inverting op-amp circuit for which the following holds:

$$K_2 = -\frac{R_2}{R_1}$$

therefore, for circuit 2 and provided $R_1 = 1k\Omega$ and $R_2 = 100k\Omega$ the following holds:

$$K_2 = -\frac{100k}{1k} \Rightarrow |K_2| = 100 (**)$$

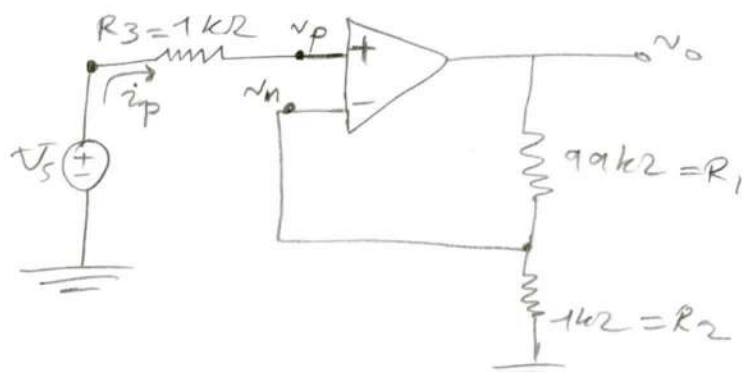
Having derived eqns (*) and (**), the claim is shown to be true.

(page 8)

That is to say these circuits produce gain of either ± 60 .

(b) Recalling Circuits 1 and 2 as discussed in the previous part of the problem, we consider in this part of the problem the connect of the circuits w/ a practical voltage source of V_S connected in series w/ $1k\Omega$ resistor. This is discussed below.

(Connect of Circuit 1): The obtained circuit is shown below:



we first note that provided it is an ideal opamp, therefore, $i_p = 0A$, therefore, referring to the circuit, there would be no voltage drop across $R_3 = 1k\Omega$ resistance, i.e., $v_p = V_S$. Having noted this fact, it is sufficient to recall what stated in the first part of this problem to conclude $|K_1| = 100$. (*)

(Connect of circuit 2): The obtained circuit is shown below:



In this case, R_1 and R_3 can be treated as 2 resistances connected in series with the equivalent resistance of $R_1 + R_3 = 2k\Omega$. Therefore, the new gain for this inverting opamp is: $K_2 = -\frac{R_2}{R_1 + R_3} = -\frac{100k}{2k} \Rightarrow |K_2| = -50$ (**)

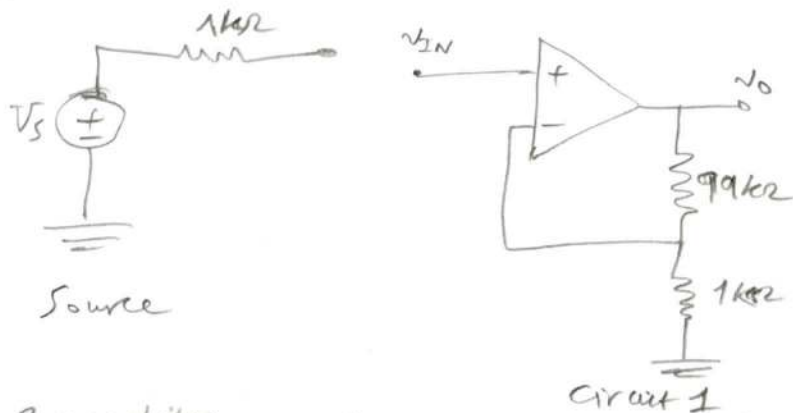
Having derived eqns (*) and (**), we now conclude that for the case of connected circuit, the original claim holds true only for circuit 1, thus not circuit 2.



problem 4-26:

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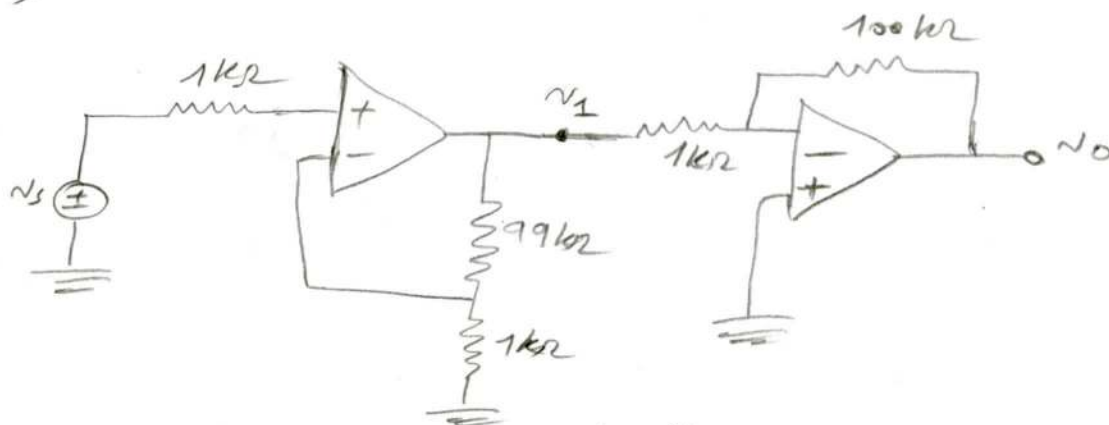
According to the problem statement, we would like to ensure the overall gain of $\frac{v_o}{v_s} = -10^4$ by properly connecting "Source", "Circuit 1", and "Circuit 2" which are shown below:



We note that 2 possibilities may be considered, i.e., [possibility #1: Source - Circuit 1 - Circuit 2] and [possibility #2: Source - Circuit 2 - Circuit 1]. In what follows we shall study these 2 possibilities addressing whether gain of $\frac{v_o}{v_s} = -10^4$ can be ensured.

(possibility #1: Source - circuit 1 - circuit 2):

The following circuit will be obtained:



First, we recall what we explained in problem 4-25:

$$K_1 = \frac{v_1}{v_s} = 100, \quad K_2 = \frac{v_o}{v_1} = -100$$

therefore, the overall gain is obtained as follows:

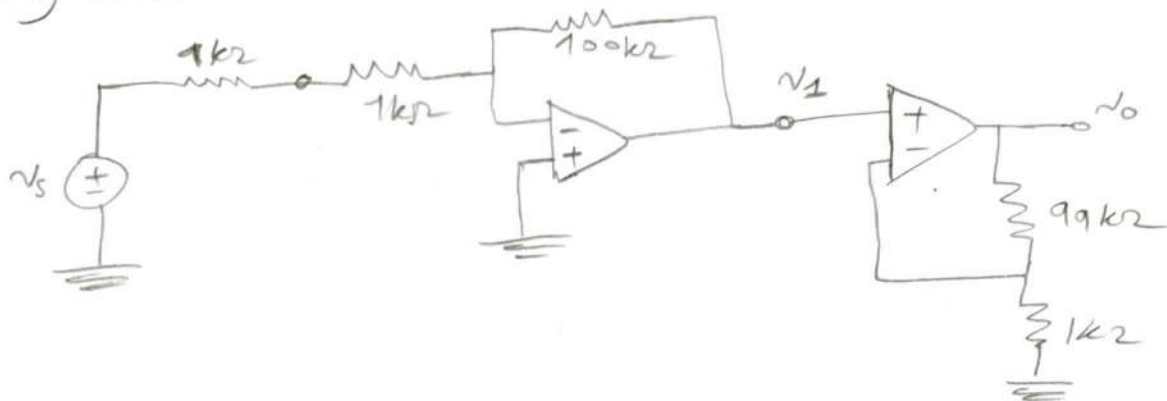
$$K_{tot} = \frac{v_o}{v_s} = \frac{v_o}{v_1} \times \frac{v_1}{v_s} = K_2 \times K_1 = (-100) \times (100) \\ \Rightarrow \boxed{K_{tot} = -10^4} (*)$$

let us now study possibility #2.

(possibility #2: Source - Circuit 2 - Circuit 1):

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The following circuit will be obtained:



First, we recall what we explained in problem 4-25, in particular, in part (b) for (connection of circuit 2):

$$k_1 = \frac{v_1}{v_s} = -50, \quad k_2 = \frac{v_o}{v_1} = 100$$

therefore, the overall gain is obtained as follows:

$$k_{tot} = \frac{v_o}{v_s} = \frac{v_o}{v_1} \times \frac{v_1}{v_s} = k_2 \times k_1 = (+100) \times (-50)$$

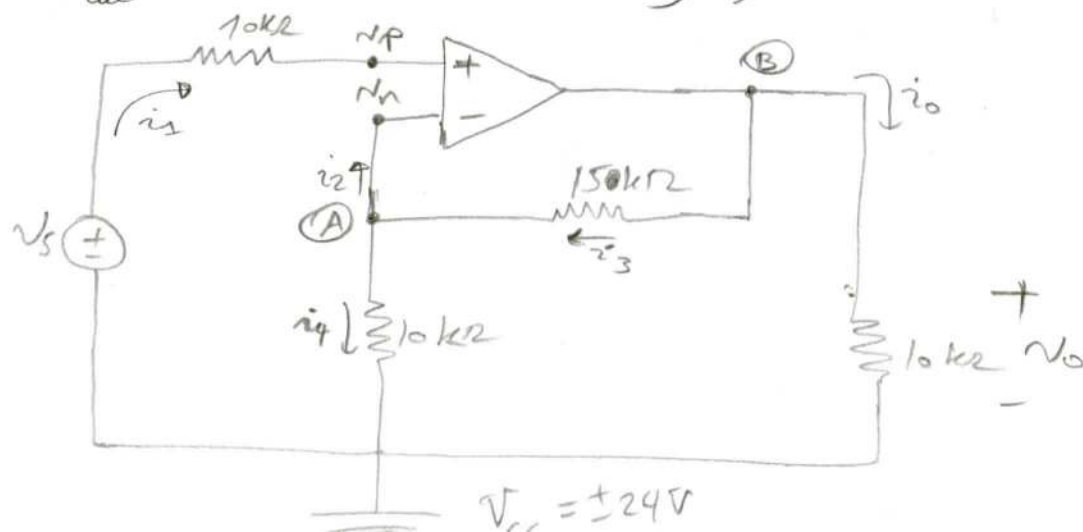
$$\Rightarrow \boxed{k_{tot} = -5000} \quad (**)$$

Hence, having derived (*) and (**), and provided the desired gain is -10^4 , therefore, we conclude that (possibility #1: Source - Circuit 1 - Circuit 2) has to be considered.



problem 4-30:

(a) For the circuit shown in the following figure:



we would like to find v_o in terms of v_s . In order to proceed, we shall Page 11
 first note that provided we are considering an ideal opamp, then $i_1 = i_2 = 0$, based on which we can derive the following:

$$\left. \begin{aligned} i_1 = 0 &\Rightarrow \text{no voltage drop across } 10k\Omega \text{ resistance connected to the positive terminal} \Rightarrow v_p = v_s \\ \text{provided we are considering an ideal op-amp} &\Rightarrow v_n = v_p \\ i_2 = 0 &\Rightarrow v_A = v_n \end{aligned} \right\} \Rightarrow \boxed{v_A = v_s} \quad (*)$$

Given eq'n (*), we can find the current, i_4 : $i_4 = \frac{v_A - 0}{10k} \stackrel{(*)}{=} \boxed{i_4 = \frac{v_s}{10} \text{ mA}} \quad (**)$

Then KCL at node (A), provided $i_2 = 0 \Rightarrow i_3 = i_4 \stackrel{(**)}{=} \boxed{i_3 = \frac{v_s}{10} \text{ mA}} \quad (***)$

We note that, on the other hand, i_3 can be obtained by developing voltage drop across $150k\Omega$ resistance $\Rightarrow \boxed{i_3 = \frac{v_B - v_A}{150} \text{ mA}} \quad (****)$

By (**), (****) and recalling (*), we get:

$$\frac{v_B - v_s}{150} = \frac{v_s}{10} \Rightarrow v_B - v_s = 15v_s \Rightarrow \boxed{v_B = 16v_s} \quad (*)$$

Also referring to the circuit, we note $v_o = v_B - 0 \stackrel{(*)}{=} \boxed{v_o = 16v_s} \quad (**)$

(b) In here, we would like to find i_o for $v_s = 1V$ and $v_s = 3V$. In order to proceed, we recall eq'n (**), where according to the circuit results in the following:

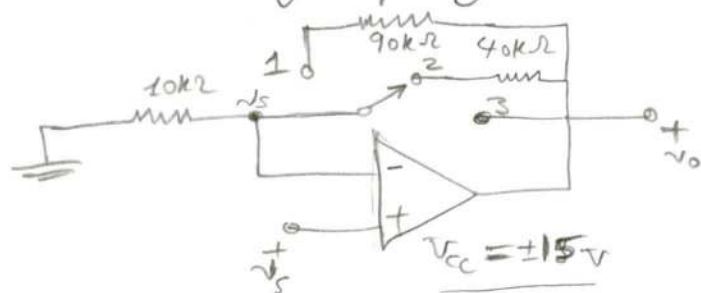
$$\boxed{i_o = \frac{v_o}{10k}}$$

Therefore, $\left\{ \begin{aligned} v_s = 1V &\stackrel{(**)}{\Rightarrow} v_o = 16 \times 1 = 16V \Rightarrow i_o = \frac{16}{10k} \Rightarrow \boxed{i_o = 1.6 \text{ mA}} \\ v_s = 3V &\stackrel{(**)}{\Rightarrow} v_o = 16 \times 3 = 48V, \text{ therefore opamp is saturated in this case, i.e., } v_o = v_{cc} = 24V \Rightarrow i_o = \frac{24}{10k} \Rightarrow \boxed{i_o = 2.4 \text{ mA}} \end{aligned} \right.$

proble-4-32:

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According to the problem statement, we would like to assess the circuit shown in the following figure, in terms of amplifying an input voltage of 2V, w/ gains of 1, 5, and 10.



Let first the resistance on the f/b line of the op-amp be R_k , then the gain provided by this op-amp circuit can be derived as follows:

$$\begin{aligned} \frac{V_S}{10k} &= \frac{V_O - V_S}{R_k} \Rightarrow \frac{R}{10} V_S = V_O - V_S \Rightarrow V_O = \left(\frac{R}{10} + 1\right) V_S \\ \Rightarrow k &= \frac{V_O}{V_S} = \left(\frac{R}{10} + 1\right) (*) \end{aligned}$$

This is provided for an ideal opamp the voltage of the positive and negative terminals are equal and that there is zero current on the negative terminal.

Hence, by (*) and for three values of resistances associated to three positions of the switch, we obtain the following values for the gain:

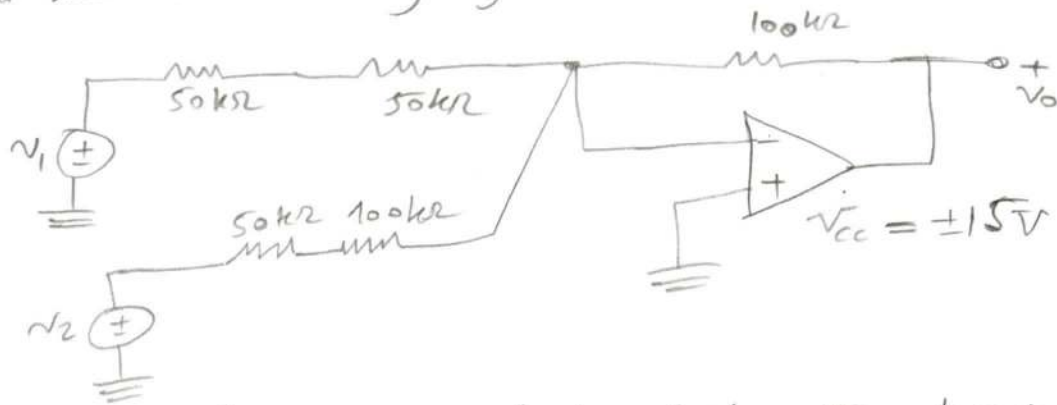
$$\begin{cases} R_1 = 90k\Omega \Rightarrow k_1 = 10 \\ R_2 = 40k\Omega \Rightarrow k_2 = 5 \\ R_3 = 0k\Omega \Rightarrow k_3 = 1 \end{cases}$$

Therefore, it seems that the three required gains have been fulfilled. However, the design is not correct b/c for $k_1 = 10$ saturation occurs, i.e., $V_O = k_1 V_S = 10 \times 2 = 20V$ which exceeds the saturation voltage of $V_{CC} = 15V$.

One way to fix the aforementioned issue of the design is to consider an opamp with higher value of V_{CC} . E.g., $V_{CC} = \pm 24V$ makes well for this design provided $V_O = 20V \leq 24V$.



(a) For the circuit shown in the following figure:



we would like to find V_0 in terms of V_1 and V_2 . In order to proceed, we first note that the considered circuit is an inverting summing amplifier, for which we can derive the following:

$$\left. \begin{aligned} \text{let } R_f &= 100k\Omega \\ R_1 &= 50k\Omega + 50k\Omega = 100k\Omega \\ R_2 &= 50k\Omega + 100k\Omega = 150k\Omega \end{aligned} \right\} \Rightarrow V_0 = k_1 V_1 + k_2 V_2 = \left(-\frac{R_f}{R_1}\right) V_1 + \left(-\frac{R_f}{R_2}\right) V_2$$

$$\Rightarrow V_0 = \left(-\frac{100k}{100k}\right) V_1 + \left(-\frac{100k}{150k}\right) V_2 \Rightarrow \boxed{V_0 = -V_1 - \frac{2}{3} V_2} \quad (*)$$

(b) Having derived (*), we would like to see that w/ $V_1 = 1V$, for which values of V_2 , we do not see saturation in op-amp. In order to proceed, we recall that $V_{cc} = \pm 15V$ for this op-amp, therefore, saturation does not occur if $-15 \leq V_0 \leq 15$, hence by (*) and $V_1 = 1V$, we get:

$$V_0 = -1 - \frac{2}{3} V_2 \Rightarrow -15 \leq -1 - \frac{2}{3} V_2 \leq 15 \Rightarrow -14 \leq -\frac{2}{3} V_2 \leq 16$$

$$\Rightarrow \left\{ \begin{aligned} \frac{2}{3} V_2 &\geq -16 \Rightarrow \underline{V_2 \geq -24V} \\ \frac{2}{3} V_2 &\leq 14 \Rightarrow \underline{V_2 \leq 21V} \end{aligned} \right\} \Rightarrow \boxed{-24 \leq V_2 \leq 21V} \quad (**)$$

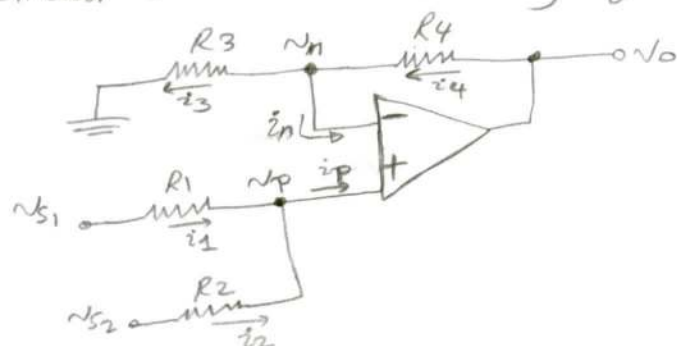
Therefore for the range of V_2 described in (**), saturation in op-amp would not occur.



problem 4-38:

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For the circuit shown in the following figure:



we would like to find v_o in terms of v_{s1} and v_{s2} . In order to proceed, we shall first note that provided it is an ideal op-amp, then $v_n = v_p$, and $i_n = i_p = 0$. The next computation would be as follows:

$$\left. \begin{array}{l} \text{KCL at } p: i_1 + i_2 = i_p = 0 \\ \text{voltage drop across } R_1: i_1 = \frac{v_{s1} - v_p}{R_1} \\ \text{voltage drop across } R_2: i_2 = \frac{v_{s2} - v_p}{R_2} \end{array} \right\} \Rightarrow \frac{v_{s1} - v_p}{R_1} + \frac{v_{s2} - v_p}{R_2} = 0$$

$$\Rightarrow v_p \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2} \Rightarrow v_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \left(\frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2} \right) \quad (*)$$

$$\left. \begin{array}{l} \text{KCL at } n: i_4 = i_n + i_3 \xrightarrow{i_n=0} i_4 = i_3 \\ \text{voltage drop across } R_3: i_3 = \frac{v_n - 0}{R_3} \\ \text{voltage drop across } R_4: i_4 = \frac{v_o - v_n}{R_4} \end{array} \right\} \Rightarrow \frac{v_o - v_n}{R_4} = \frac{v_n}{R_3}$$

$$\Rightarrow v_o = v_n \left(1 + \frac{R_4}{R_3} \right) \quad (**)$$

By (*), (**) and that $v_n = v_p$, we get:

$$v_o = \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \left(\frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2} \right) \right) \left(1 + \frac{R_4}{R_3} \right) = \frac{R_1 R_2}{R_1 + R_2} \left(\frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2} \right) \left(\frac{R_3 + R_4}{R_3} \right)$$

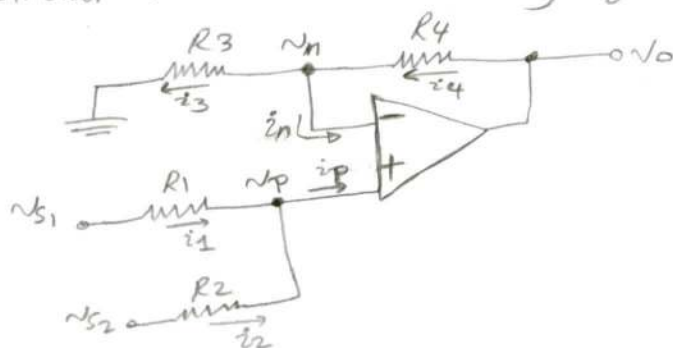
$$\Rightarrow v_o = \frac{R_1 R_2}{R_3} \left(\frac{R_3 + R_4}{R_1 + R_2} \right) \left(\frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2} \right)$$



problem 4-38:

page 14

For the circuit shown in the following figure:



we would like to find v_o in terms of v_{s1} and v_{s2} . In order to proceed, we shall first note that provided it is an ideal op-amp, then $v_n = v_p$, and $i_n = i_p = 0$. The next computation would be as follows:

$$\text{KCL at } p: i_1 + i_2 = i_p = 0$$

$$\text{voltage drop across } R_1: i_1 = \frac{v_{s1} - v_p}{R_1}$$

$$\text{voltage drop across } R_2: i_2 = \frac{v_{s2} - v_p}{R_2}$$

$$\Rightarrow \frac{v_{s1} - v_p}{R_1} + \frac{v_{s2} - v_p}{R_2} = 0$$

$$\Rightarrow v_p \left(\frac{1}{R_1} + \frac{1}{R_2} \right) = \frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2} \Rightarrow v_p = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \left(\frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2} \right) \quad (*)$$

$$\text{KCL at } n: i_4 = i_n + i_3 \xrightarrow{i_n=0} i_4 = i_3$$

$$\text{voltage drop across } R_3: i_3 = \frac{v_n - 0}{R_3}$$

$$\text{voltage drop across } R_4: i_4 = \frac{v_o - v_n}{R_4}$$

$$\Rightarrow \frac{v_o - v_n}{R_4} = \frac{v_n}{R_3}$$

$$\Rightarrow v_o = v_n \left(1 + \frac{R_4}{R_3} \right) \quad (**)$$

By (*), (**) and that $v_n = v_p$, we get:

$$v_o = \left(\frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \left(\frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2} \right) \right) \left(1 + \frac{R_4}{R_3} \right) = \frac{R_1 R_2}{R_1 + R_2} \left(\frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2} \right) \left(\frac{R_3 + R_4}{R_3} \right)$$

$$\Rightarrow v_o = \frac{R_1 R_2}{R_4} \left(\frac{R_3 + R_4}{R_1 + R_2} \right) \left(\frac{v_{s1}}{R_1} + \frac{v_{s2}}{R_2} \right)$$