HW#6
MAE140
Winter 2014

9-1

$$f(t) = 500[1 - e^{-100t}]u(t)$$
$$= 500u(t) - 500e^{-100t}u(t)$$

*using table 9-2 in page 458 for laplace transforms of basic functions, we have;

$$F(s) = \frac{500}{s} - \frac{500}{s+100} = \frac{50000}{s(s+100)}$$

$$\left.\begin{matrix} s=0 \\ s=-100 \end{matrix}\right\} \text{poles}$$

$\boxed{9-4}$

$$f(t) = 20\left[e^{-200t} - 2e^{-100t}\right]u(t)$$

$$= 20e^{-200t}u(t) - 40e^{-100t}u(t)$$

$$F(S) = \frac{20}{S+200} - \frac{40}{S+100} = \frac{-20(S+300)}{(S+200)(S+100)}$$

$S = -300$ zero

$\left.\begin{matrix} S = -200 \\ S = -100 \end{matrix}\right\}$ poles

$\boxed{9-6}$

$f(t) = 0.005[10 - 10\cos(1000t)]u(t)$

⊛Using table 9-2 in page 458:

$\Rightarrow F(s) = 0.005\left[\frac{10}{s} - 10\frac{s}{s^2+10^6}\right] = 0.05\frac{10^6}{s(s^2+10^6)}$

$\left.\begin{array}{l} s=0 \\ s^2+10^6=0 \Rightarrow s=\pm 1000j \end{array}\right\}$ poles

$\boxed{9-11}$

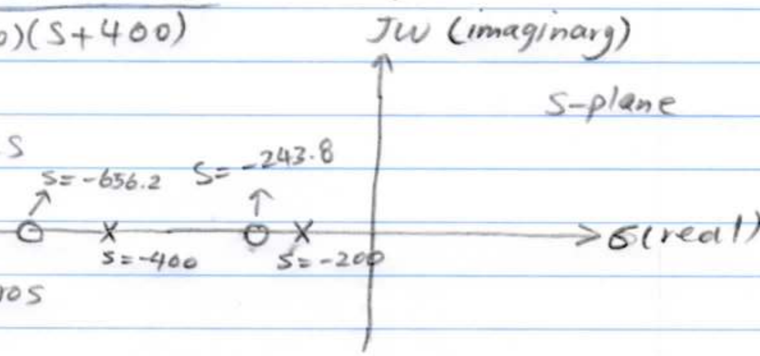
(a) $f_1(t) = 2\delta(t) + [200e^{-200t} + 400e^{-400t}]u(t)$

$$\Rightarrow F_1(S) = 2 + \frac{200}{S+200} + \frac{400}{S+400}$$

$$\Rightarrow F_1(S) = \frac{2S^2 + 1800S + 320000}{(S+200)(S+400)}$$

$\left.\begin{matrix} S=-200 \\ S=-400 \end{matrix}\right\}$ poles

$\left.\begin{matrix} S=-656.2 \\ S=-243.8 \end{matrix}\right\}$ zeros

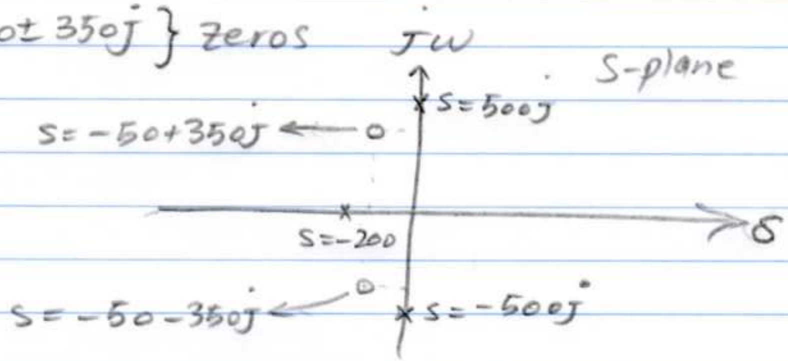


(b) $f_2(t) = [15e^{-200t} + 15\cos 500t]u(t)$

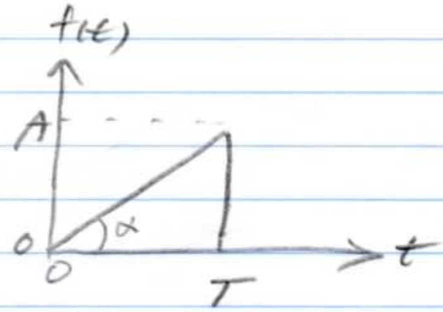
$$\Rightarrow F_2(S) = \frac{15}{S+200} + \frac{15S}{S^2+(500)^2} = \frac{15(2S^2+200S+250000)}{(S+200)(S^2+(500)^2)}$$

$\left.\begin{matrix} S=-200 \\ S=\pm 500j \end{matrix}\right\}$ poles

$S=-50\pm 350j\}$ zeros



$\boxed{9\text{-}16}$



ⓐ To write down the expression for $f(t)$, first we need the slope for ramp line. The slope can be obtained as,

$$\text{Slope} = \tan\alpha = \frac{A}{T}$$

$$\Rightarrow f(t) - 0 = \frac{A}{T}(t-0) \Rightarrow f(t) = \frac{A}{T}t$$

Note that $f(t) = 0$ for $t<0$ and $t>T$.

For $t<0$ & $f(t)=0$, we multiply $\frac{A}{T}t$ by $u(t)$.

$$\Rightarrow f(t) = \left(\frac{A}{T}t\right)u(t)$$

For $t>T$ & $f(t)=0$, we replace "$u(t)$" in above expression with "$u(t) - u(t-T)$". Therefore the final expression becomes;

$$\Rightarrow f(t) = \underbrace{\frac{A}{T}t}_{\text{ramp function}}\underbrace{\left[u(t) - u(t-T)\right]}_{\text{Step functions}}$$

ⓑ $f(t) = \frac{A}{T} t[u(t) - u(t-T)]$

$$= \frac{A}{T} t u(t) - \frac{A}{T} t u(t-T)$$

$$= \frac{A}{T} t u(t) - \frac{A}{T}(t - T + T) u(t-T)$$

$$= \frac{A}{T} t u(t) - \frac{A}{T}(t-T) u(t-T) - A u(t-T)$$

using t-Domain translation property:

$$\Rightarrow F(s) = \frac{A}{T}\frac{1}{s^2} - \frac{A}{T} e^{-sT}\frac{1}{s^2} - A e^{-sT}\frac{1}{s}$$

ⓒ $F(s) = \int_{0^-}^{\infty} f(t) e^{-st} dt$ equation (9-2) in page 449 for laplace transformation

$$\Rightarrow F(s) = \int_{0}^{\infty} \frac{A}{T} t[u(t) - u(t-T)] e^{-st} dt$$

$$= \int_{0^-}^{\infty} \frac{A}{T} t u(t) e^{-st} dt - \int_{0^-}^{\infty} \frac{A}{T} t u(t-T) e^{-st} dt$$

$$= \int_{0^-}^{\infty} \frac{A}{T} t e^{-st} dt - \int_{T}^{\infty} \frac{A}{T} t e^{-st} dt$$

$\left(\frac{A}{T}\frac{1}{s^2}\right) = \leftarrow$

$\Rightarrow$ becaus $u(t-T) = 0$ for $t < T$

Solving $\int_{T}^{\infty} \frac{A}{T} t e^{-st} dt$ using variable change as

$$t - T = \tau \Rightarrow t = \tau + T$$

$$\Rightarrow \int_T^\infty \frac{A}{T} t e^{-st}\, dt = \int_0^\infty \frac{A}{T}(\tau+T) e^{-s(\tau+T)} d\tau$$

$$= \frac{A}{T} e^{-sT}\left(\frac{1}{s^2} + \frac{T}{s}\right)$$

$$\Rightarrow F(s) = \frac{A}{T}\frac{1}{s^2} - \frac{A}{T} e^{-sT}\left(\frac{1}{s^2} + \frac{T}{s}\right)$$

$$\Rightarrow F(s) = \frac{A}{T}\frac{1}{s^2} - \frac{A}{T}\frac{e^{-sT}}{s^2} - \frac{Ae^{-sT}}{s}$$

this is the same as
answer to part b ✓

$\boxed{9-20}$

(a) $F_1(S) = \dfrac{50}{S(S+50)}$ * Finding partial fraction expansion as;

$$F_1(S) = \frac{K_1}{S} + \frac{K_2}{S+50}$$

$$K_1 = SF_1(S)\Big|_{S=0} = \lim_{S \to 0} SF_1(S) = 1$$

$$K_2 = (S+50)F_1(S)\Big|_{S=-50} = \lim_{S \to -50} (S+50)F_1(S) = -1$$

$$\Rightarrow F_1(S) = \frac{1}{S} - \frac{1}{S+50}$$

$$\Rightarrow \mathcal{L}^{-1}[F_1(S)] \underset{\downarrow}{=} f_1(t) = u(t) - e^{-50t}u(t) = [1 - e^{-50t}]u(t)$$

using table 9-2 in page 458 of book

(b) $F_2(S) = \dfrac{S+1}{(S+2)(S+3)}$

$$F_2(S) = \frac{K_1}{S+2} + \frac{K_2}{S+3}$$

$$K_1 = (S+2)F_2(S)\Big|_{S=-2} = -1$$

$$K_2 = (S+3)F_2(S)\Big|_{S=-3} = 2$$

$$\Rightarrow F_2(S) = \frac{-1}{S+2} + \frac{2}{S+3}$$

$$f_2(t) = \mathcal{L}^{-1}[f_2(S)] = -e^{-2t}u(t) + 2e^{-3t}u(t)$$
$$= (-e^{-2t} + 2e^{-3t})u(t)$$

9-22

(a) $F_1(S) = \dfrac{5000(S+1000)}{(S+500)(S+5000)}$

$\Rightarrow F_1(S) = \dfrac{k_1}{S+500} + \dfrac{k_2}{S+5000}$

$k_1 = (S+500)F_1(S)\Big|_{S=-500} = 555.6$

$k_2 = (S+5000)F_1(S)\Big|_{S=-5000} = 4444$

$\Rightarrow f_1(t) = \mathcal{L}^{-1}[F_1(S)] = [555.6e^{-500t} + 4444e^{-5000t}]u(t)$

(b) $F_2(S) = \dfrac{5S^2}{(S+100)(S+500)}$

$\Rightarrow F_2(S) = 5 - \dfrac{3000S + 250000}{(S+100)(S+500)}$

$= 5 + \dfrac{k_1}{S+100} + \dfrac{k_2}{S+500}$

$k_1 = (S+100)F_2(S)\Big|_{S=-100} = 125$

$k_2 = (S+500)F_2(S)\Big|_{S=-500} = -3125$

$\Rightarrow f_2(t) = \mathcal{L}^{-1}[f_2(S)] = 5\delta(t) + [125e^{-100t} - 3125e^{-500t}]u(t)$

9-24

(a) $F_1(S) = \frac{\beta(S+\beta)}{S(S^2+\beta^2)} = \frac{K_1}{S} + \frac{K_2}{S+\beta j} + \frac{K_3}{S-\beta j}$

$K_1 = S F_1(S)\big|_{S=0} = 1$

$K_2 = (S+\beta j)F_1(S)\big|_{S=-\beta j} = \frac{\sqrt{2}}{2} e^{j\frac{3}{4}\pi}$

$K_3 = (S-\beta j)F_1(S)\big|_{S=\beta j} = \frac{\sqrt{2}}{2} e^{-j\frac{3}{4}\pi} = K_2^*$

$\Rightarrow f_1(t) = \mathcal{L}^{-1}[F_1(S)] = \left[1 + \frac{\sqrt{2}}{2} e^{-\beta j t + j\frac{3}{4}\pi} + \frac{\sqrt{2}}{2} e^{\beta j t - j\frac{3}{4}\pi}\right] u(t)$

$= \left[1 + \sqrt{2}\cos(\beta t - \frac{3}{4}\pi)\right] u(t)$

$\left[\text{using } \cos\theta = \frac{e^{j\theta} + e^{-\theta j}}{2}\right]$

please See the sketch in next page.

(b) $F_2(S) = \frac{S(S+\beta)}{S^2+\beta^2} = 1 + \frac{\beta S - \beta^2}{S^2+\beta^2} = 1 + \frac{\beta S}{S^2+\beta^2} - \frac{\beta^2}{S^2+\beta^2}$

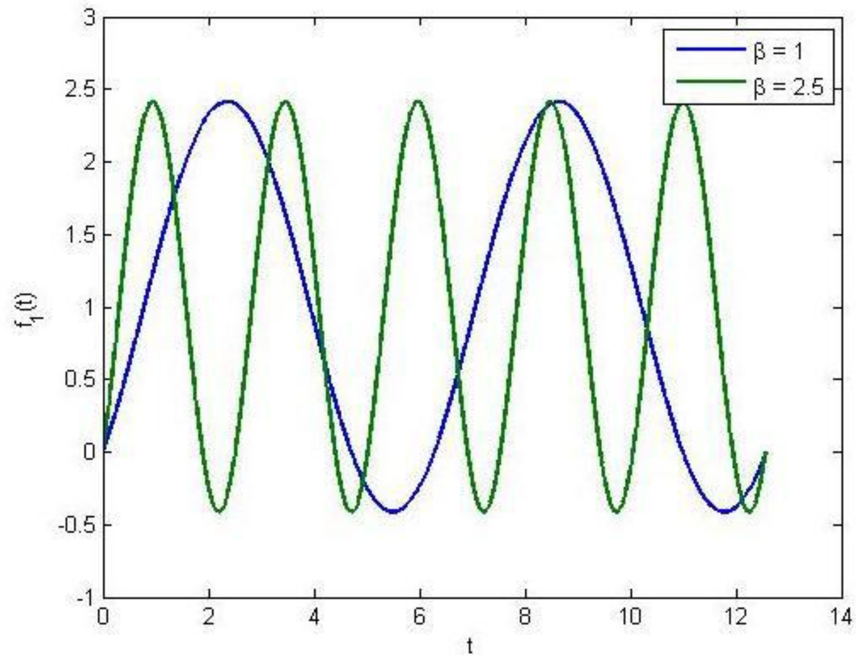
using table 9-2 in page 458;

$\Rightarrow f_2(t) = \mathcal{L}^{-1}[F_2(S)] = \delta(t) + \beta\cos(\beta t) - \beta\sin(\beta t)$

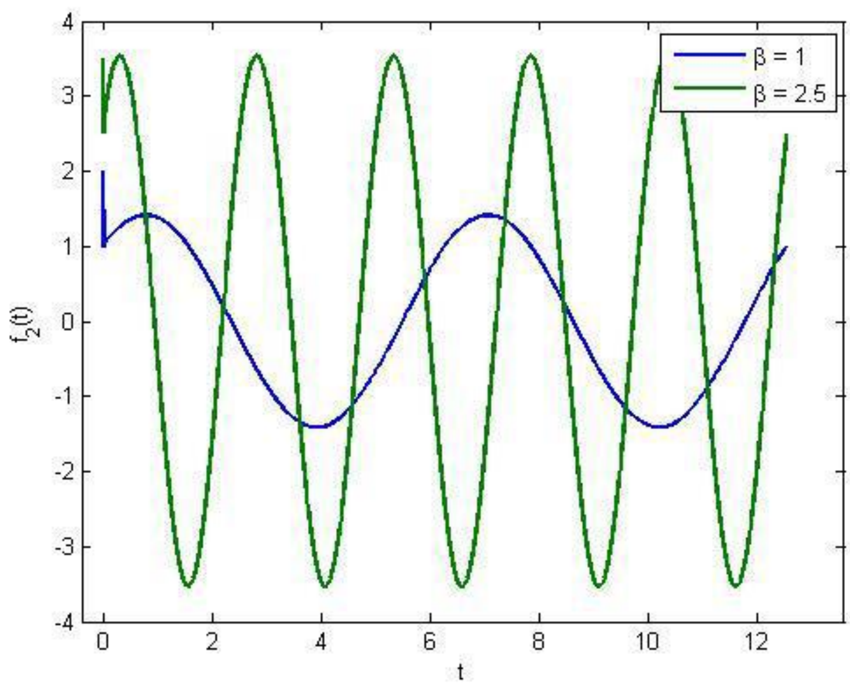
$= \delta(t) + \sqrt{2}\beta\cos(\beta t - \frac{\pi}{4})$

please See the sketch in next page.

(a)


β = 1
β = 2.5
$f_1(t)$
t
3
2.5
2
1.5
1
0.5
0
-0.5
-1
0
2
4
6
8
10
12
14

(b)


β = 1
β = 2.5
$f_2(t)$
t
4
3
2
1
0
-1
-2
-3
-4
0
2
4
6
8
10
12

$\boxed{9-25}$

(a) $F_1(s) = \dfrac{\alpha^2}{s^2(s+\alpha)}$

poles $\begin{cases} s=0 & \text{of order 2} \\ s=-\alpha \end{cases}$

$$F_1(s) = \frac{1}{s}\left[\frac{\alpha^2}{s(s+\alpha)}\right]$$

$$= \frac{1}{s}\left[\frac{K_1}{s} + \frac{K_2}{s+\alpha}\right] = \frac{K_1}{s^2} + \frac{K_2}{s(s+\alpha)}$$

$$= \frac{K_1}{s^2} + \frac{K_{21}}{s} + \frac{K_{22}}{s+\alpha}$$

$$\Rightarrow K_1 = \alpha,\ K_{21} = -1,\ K_{22} = 1$$

$$\Rightarrow f_1(t) = -u(t) + \alpha t u(t) + e^{-\alpha t} u(t)$$

(b) $F_2(s) = \dfrac{\alpha^2}{s(s+\alpha)^2}$

$$= \frac{1}{s+\alpha}\left[\frac{\alpha^2}{s(s+\alpha)}\right]$$

$$= \frac{1}{s+\alpha}\left[\frac{K_1}{s} + \frac{K_2}{s+\alpha}\right] = \frac{K_1}{s(s+\alpha)} + \frac{K_2}{(s+\alpha)^2}$$

$$= \frac{K_{11}}{s} + \frac{K_{12}}{s+\alpha} + \frac{K_2}{(s+\alpha)^2}$$

$$\Rightarrow K_{11} = 1,\ K_{12} = -1,\ K_2 = -\alpha$$

$$\Rightarrow f_2(t) = u(t) - e^{-\alpha t} u(t) - \alpha t e^{-\alpha t} u(t)$$

$\boxed{9-34}$

(a) $F_1(S) = \dfrac{S^2}{(S+5)}$

$$= \frac{S^2-25+25}{S+5} = \frac{(S-5)(S+5)+25}{S+5} = S-5+\frac{25}{S+5}$$

$$\Rightarrow f_1(t) = \frac{d\delta(t)}{dt} - 5\delta(t) + \left[25e^{-5t}\right]u(t)$$

(b) $F_2(S) = \dfrac{(S+1000)^2}{(S+2000)^2} = \dfrac{(S+2000)^2 - 2000S - 3000000}{(S+2000)^2}$

$$= 1 - \frac{2000S+3000000}{(S+2000)^2}$$

$$= 1 - \frac{2000}{S+2000} + \frac{1000000}{(S+2000)^2}$$

$$\Rightarrow f_2(t) = \delta(t) + \left[-2000e^{-2000t} + 1000000\,te^{-2000t}\right]u(t)$$