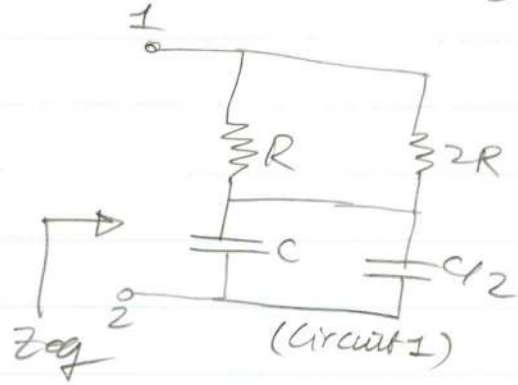
MAE 140, Winter 2014, HW#7 Sol'ns

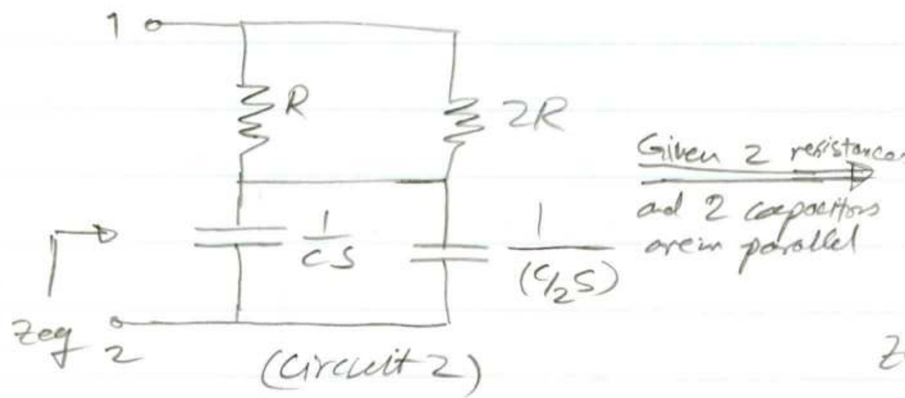
problem 10-4:

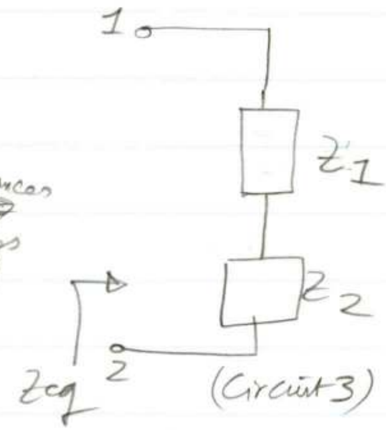
(a) we would like to express $Z_{eq}(s)$ and locate its poles and zeros, for the circuit shown in the following figure:



(Circuit 1)

In order to do so, we shall first transform the circuit into s-domain, where we recall the impedance of a capacitor is $Z_C = \frac{1}{Cs}$. This is performed under zero-state condition, i.e., no current/voltage source will be considered due to initial cond'ns of the capacitor. We obtain then the following circuit in s-domain:





w/ Z_1 and Z_2:

$$Z_1 = R // 2R = \frac{(R)(2R)}{R+2R} \Rightarrow \boxed{Z_1 = \frac{2}{3}R} \quad (*)$$

$$Z_2 = \left(\frac{1}{Cs}\right) // \left(\frac{2}{Cs}\right) = \frac{(1/Cs)(2/Cs)}{\frac{1}{Cs}+\frac{2}{Cs}} = \frac{\frac{2}{C^2s^2}}{\frac{3}{Cs}} \Rightarrow \boxed{Z_2 = \frac{2}{3}\frac{1}{Cs}} \quad (**)$$

Therefore, referring to (circuit 3), we note that z_1 and z_2 are in series, z_{eq} can be then obtained as follows:

$$z_{eq} = z_1 + z_2 \xrightarrow[(**)]{\text{By}(*)} \frac{2}{3}R + \frac{2}{3}\frac{1}{CS} \Rightarrow \boxed{z_{eq} = \frac{2}{3}\left(\frac{RCS+1}{CS}\right)} \quad (*\!*\!*)$$

Hence, by $(***)$, we can locate zero and pole of the equivalent impedance as follows:

$$\begin{cases} \underline{P = 0} & (\star) \\ \underline{Z = \frac{-1}{RC}} & (\star\star) \end{cases}$$

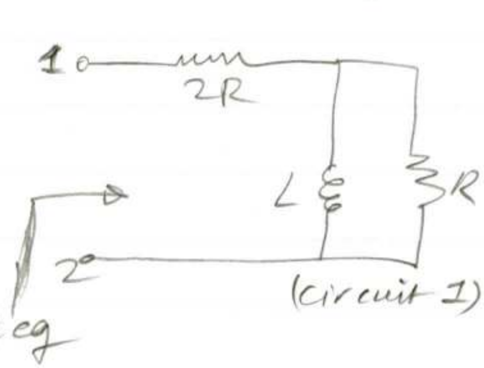
(b) we would like to select values of R and C to locate a zero at $-33 \frac{\text{krad}}{\text{sec}}$. Therefore, we recall our analysis in the previous part of this problem, i.e., eqn $(\star\star)$ which characterizes the value of zero whereby we get:

$$z = \frac{-1}{RC} = -33\times10^{3}\ \frac{\text{rad}}{\text{sec}} \Rightarrow RC = \frac{10^{-3}}{33} = 30.3\times10^{-6}$$

Accordingly, let $\boxed{R = 1k\Omega}$ then we get $\boxed{C = 30.3nF}$.

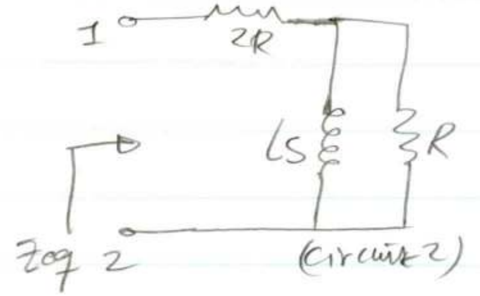
problem 10-6:

(a) we would like to express $z_{eq}(s)$ and locate its poles and zeros, for the circuit shown in the following figure:



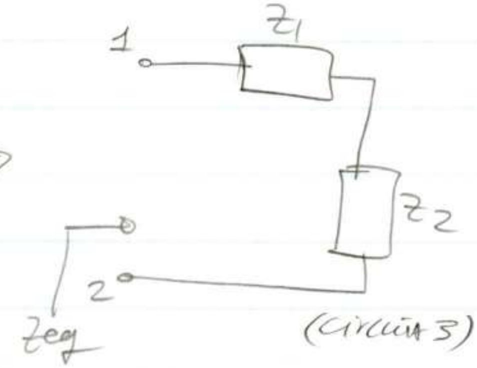
(circuit 1)

In order to do so, we shall first transform the circuit into s-domain, where we recall the impedance of an inductor is $Z_L = LS$. Along the same lines as in the previous problem, we perform the analysis under zero-state assumption. Hence, we obtain the following circuit in the s-domain:



(circuit 2)

Given the resistance and inductor are in parallel



(circuit 3)

w/ Z_1 and Z_2:

$$\boxed{Z_1 = 2R} \quad (*)$$

$$\boxed{Z_2 = R \parallel LS = \frac{RLS}{R+LS}} \quad (**)$$

then, according to (circuit 3) and that impedances Z_1 and Z_2 are in series, we get:

$$Z_{eq} = Z_1 + Z_2 \xrightarrow[(**)]{\text{by } (*)} Z_{eq} = 2R + \frac{RLS}{R+LS} \Rightarrow Z_{eq} = \frac{2R^2 + 2RLS + RLS}{R+LS}$$

$$\Rightarrow \boxed{Z_{eq} = \frac{3RLS + 2R^2}{LS + R}} \quad (***)$$

Hence, by $(***)$, we can locate zero and pole of the equivalent impedance as follows:

$$\begin{cases} \boxed{P = -R/L} & (\star) \\ \boxed{Z = \dfrac{-2R}{3L}} & (\star\star) \end{cases}$$

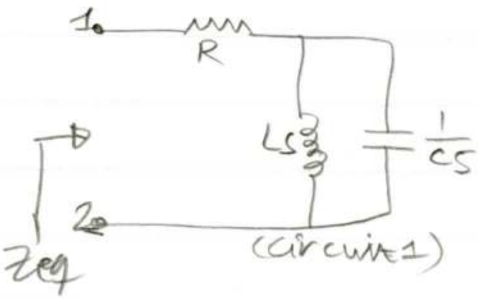
(b) we would like to select values of R and L to locate a pole at $-150 \frac{rad}{sec}$. Therefore we recall our analysis in the previous part of this problem, i.e., eq'n (*) which characterizes the value of the pole, whereby we get:

$$p = -\frac{R}{L} = -150 \Rightarrow R = 150L$$

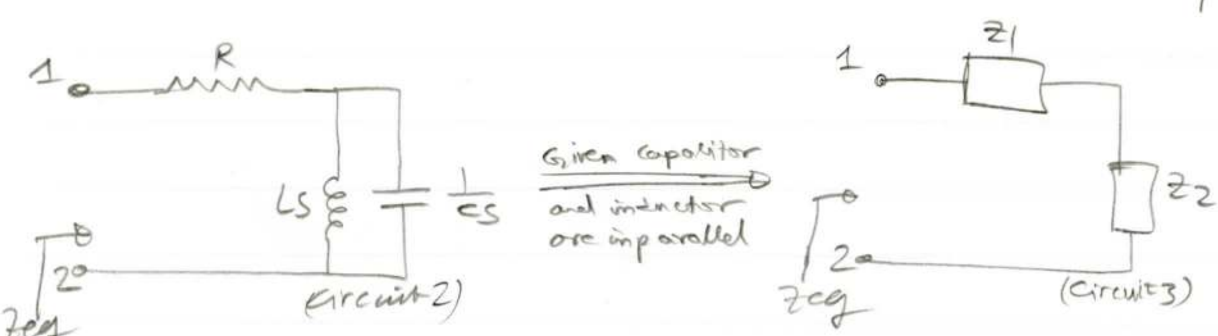
Accordingly, let $\boxed{L = 250\,mH}$, we then get $\boxed{R = 37.5\ \Omega}$. Therefore, the resulting zero, by (**) is at: $z = -\frac{2R}{3L} = \frac{-2\times 37.5}{3\times 250\times 10^{-3}} \Rightarrow \boxed{z = 100\ rad/sec}$

problem 10-8:

(a) we would like to express $Z_{eq}(s)$ and locate its poles and zeros, for the circuit shown in the following figure:



We first note that the given circuit is already in the s-domain, thus there is no need to transform it into s-domain. We shall then proceed as follows:



w/ Z_1 and Z_2:

$\boxed{Z_1 = R}$ (*)

$$Z_2 = (Ls)\,//\,(\tfrac{1}{Cs}) = \frac{(Ls)(\frac{1}{Cs})}{Ls + \frac{1}{Cs}} = \frac{\frac{L}{C}}{\frac{CLs^2+1}{Cs}} = \frac{LCs}{C(CLs^2+1)} \Rightarrow \boxed{Z_2 = \frac{Ls}{CLs^2+1}} \quad (**)$$

Therefore, referring to (circuits3), we note that z_1 and z_2 are in series, Zeq can be then developed as follows:

$$Z_{eq} = Z_1 + Z_2 \xrightarrow[(**)]{\text{By}(*)} R + \frac{LS}{CLS^2+1} = \frac{RLCS^2 + R + LS}{CLS^2+1}$$

$$\Rightarrow \boxed{Z_{eq} = \frac{RS^2 + \frac{1}{C}S + \frac{R}{LC}}{S^2 + \frac{1}{LC}}} \quad (***)$$

Hence, by $(***)$, we can locate zeros and poles of the equivalent impedance as follows:

poles: roots of $S^2 + \frac{1}{LC}$ polynomial $\Rightarrow \boxed{P_{1,2} = \pm\sqrt{\frac{1}{LC}}\, j}$ $(\star)$

zeros: roots of $RS^2 + \frac{1}{C}S + \frac{R}{LC}$ polynomial $\Rightarrow Z_{1,2} = \dfrac{-\frac{1}{C} \pm \sqrt{\left(\frac{1}{C}\right)^2 - 4(R)\left(\frac{R}{LC}\right)}}{2R}$

$$\Rightarrow Z_{1,2} = \frac{-1/C \pm \sqrt{(L-4R^2C)/LC^2}}{2R} = \frac{-1 \pm \sqrt{(L-4R^2C)/L}}{2RC}$$

$$\Rightarrow \boxed{Z_{1,2} = \frac{-L \pm \sqrt{L^2 - 4R^2LC}}{2RLC}} \quad (\star\star)$$

(b) for $R = 10\,k\Omega$, we would like to select values of L and C to locate poles at $\pm j200 \frac{krad}{sec}$. Therefore we recall our analysis in the previous part of this problem, i.e., eq'n $(\star)$ which characterizes the values of poles whereby we get:

$$P_{1,2} = \pm\sqrt{\frac{1}{LC}}\, j = \pm j200\times10^3 \Rightarrow \sqrt{LC} = \frac{1}{2\times10^5} \Rightarrow \boxed{LC = \frac{10^{-10}}{4} = \left(\frac{1}{4}10^{-6}\right)(10^{-1}\times10^{-3})}$$

thus, given this latter eq'n and w/o loss of generality, the following values can be chosen:

$$C = \frac{1}{4}10^{-6} \Rightarrow \boxed{C = 0.25\,\mu F} \quad (+)$$

$$L = 10^{-1}\times10^{-3} \Rightarrow \boxed{L = 0.1\,mH} \quad (++)$$

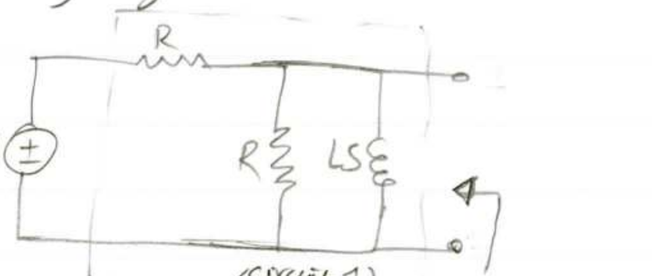
provided (+) and (++) are proper values for C and L, the resulting zeros would be at the following locations:

$$z_{1,2} = \frac{-10^{-4} \pm \sqrt{10^{-8} - 4(10^{8})(10^{-4})(0.25\times10^{-6})}}{2(10^{4})(10^{-4})(0.25\times10^{-6})} = \frac{-10^{-4} \pm 10^{-1}\sqrt{10^{-6}-1}}{0.5\times10^{-6}}$$

$$\Rightarrow z_{1,2} = -2\times10^{2} \pm 2\times10^{5}\sqrt{10^{-6}-1} \Rightarrow \boxed{z_{1,2} = -2\times10^{2} \pm 2\times10^{5} j}$$

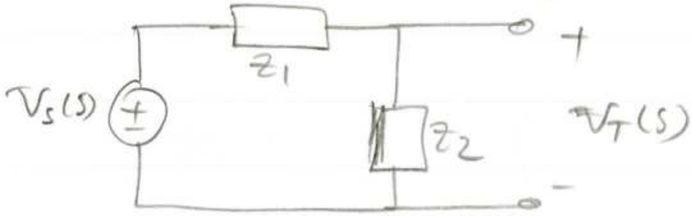
problem 10-17:

First: we would like to find the Thévenin equivalent for the circuit shown in the following figure:



(circuit 1)

we note that $V_T(s)$ is the voltage drop across resistance R and inductance LS, on the other hand, we note that these two impedances are in parallel; therefore we can simplify (circuit 1) as follows:



(circuit 2)

where,

$$z_1 = R \quad (*)$$

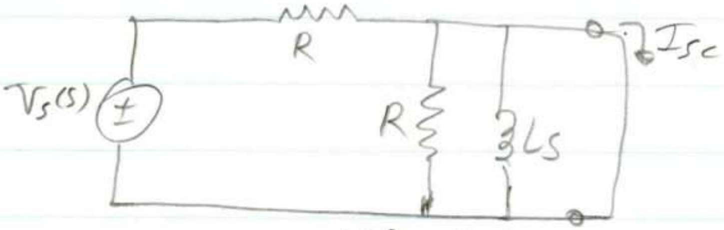
$$z_2 = R \parallel LS = \frac{RLS}{R+LS} \quad (**)$$

Hence, given eqns. (*), (**), and according to (circuit 2), by recalling voltage division technique, we get the following:

$$V_T(S) = \frac{Z_2(S)}{Z_1(S)+Z_2(S)} V_s(S) = \frac{\frac{RLS}{R+LS}}{R+\frac{RLS}{R+LS}} V_{s(S)} = \frac{\frac{RLS}{R+LS}}{\frac{R^2+RLS+RLS}{R+LS}} V_s(S)$$

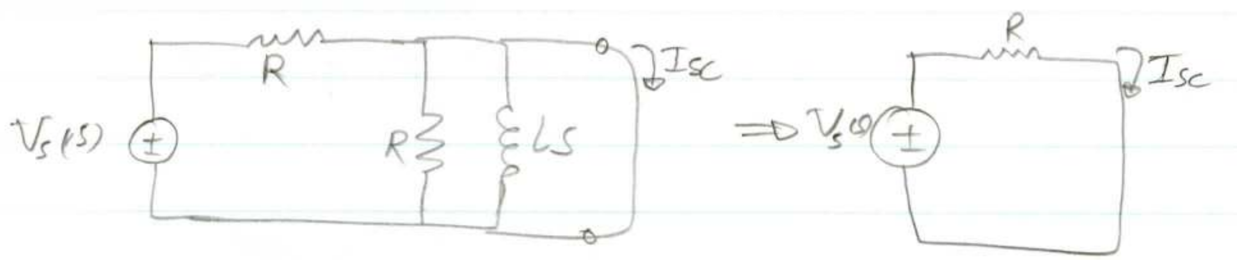
$$\Rightarrow \boxed{V_T(S) = \frac{S}{2S+\frac{R}{L}} V_s(S)} \quad (\star)$$

Having found $V_T(S)$ in the above eq'n, we shall find $Z_T(S)$ in the following. In order to do so, we refer to (circuit 1) where we short circuit the designated ports to obtain I_{sc} as shown in the next figure:



(circuit 3)

Then, referring to (circuit 3) and given that the short circuit port is in parallel w/ the impedances R and LS, hence, we can compute it as follows:



$$\boxed{I_{sc}(S) = \frac{V_s(S)}{R}} \quad (\star\star)$$

Therefore, by ($\star$) and ($\star\star$), we can obtain $Z_T(S)$ as performed in the following:

$$Z_T(S) = \frac{V_T(S)}{I_{sc}(S)} = \frac{\frac{S}{2S+R/L} V_s(S)}{\frac{1}{R} V_s(S)} \Rightarrow Z_T(S) = \frac{RS}{2S+R/L} \Rightarrow \boxed{Z_T(S) = \frac{RLS}{2LS+R}} \quad (\star\star\star)$$

By (*) and (***), we have characterized $V_T(S)$ and $Z_T(S)$ and thus the Thévenin equivalent circuit is obtained.

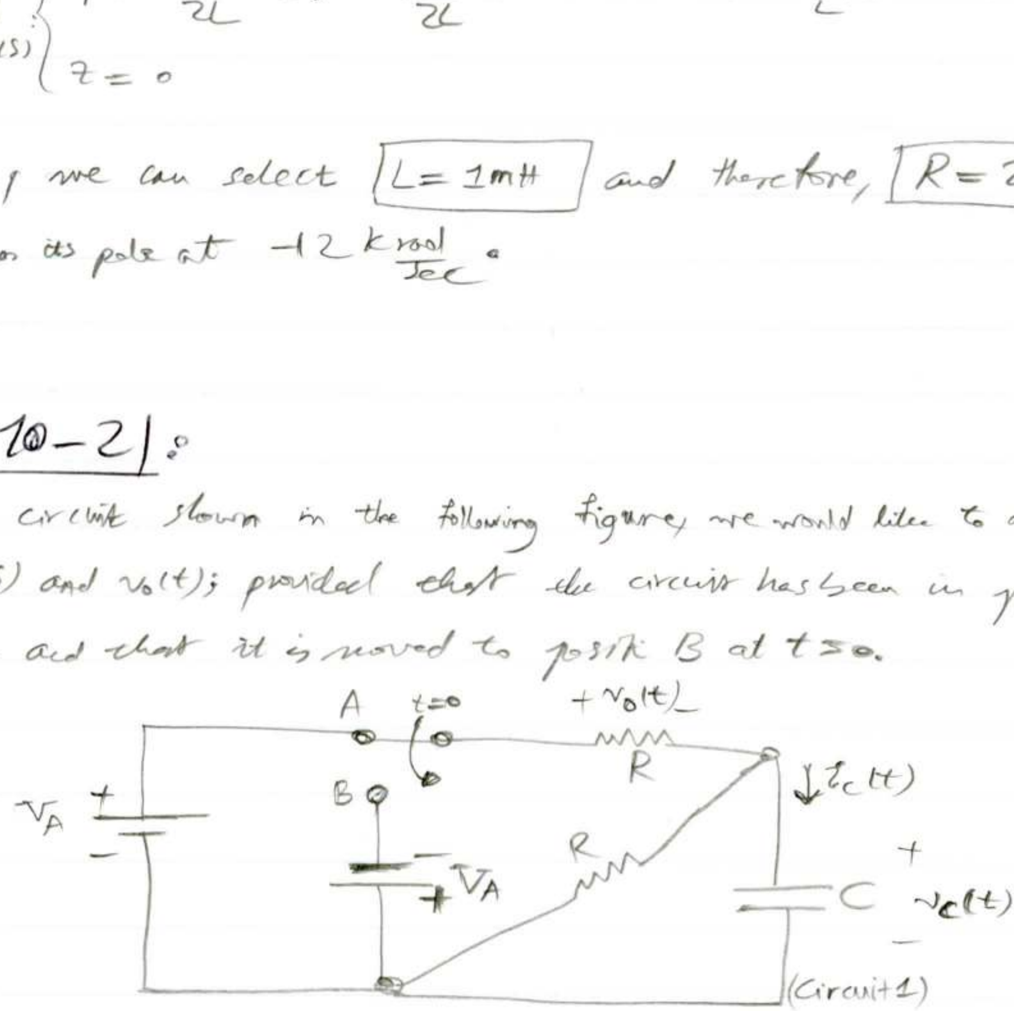
Second: we would like to select values for R and L so that $V_T(S)$ has a pole at -12 krad/sec. In order to do so, we recall eq'n (*) for $V_T(S)$ and we locate its pole:

pole and zero of $V_T(S)$: $\begin{cases} p = \frac{-R}{2L} \Rightarrow -\frac{R}{2L} = -12\times10^3 \Rightarrow \frac{R}{L} = 24\times10^3 \Rightarrow \boxed{R = 24\times10^3 L} \\ z = 0 \end{cases}$

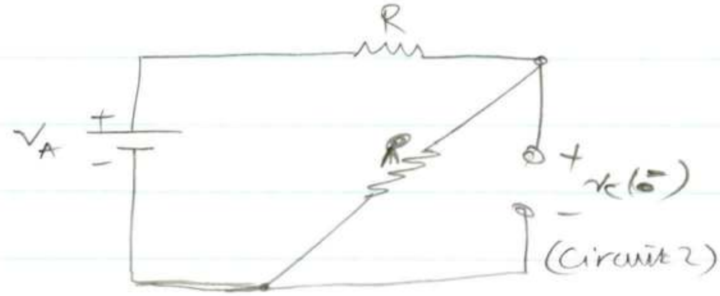
Hence, we can select $\boxed{L = 1mH}$ and therefore, $\boxed{R = 24\,\Omega}$, where then $V_T(S)$ has its pole at $-12\ \frac{\text{krad}}{\text{sec}}$.

problem 10-21:

For the circuit shown in the following figure, we would like to derive $I_C(S)$, $i_C(t)$, $V_O(S)$ and $v_O(t)$; provided that the circuit has been in position A for a long time and that it is moved to position B at $t=0$.



First: Given the circuit has been at position A for a long time, we can then find $v_C(0^-)$ that is the initial cond'n of the voltage across the capacitor just before switching to position B. In order to do so, and, again, provided the circuit has been in position A for a long time, we consider the capacitor as an open circuit. The circuit then looks like:

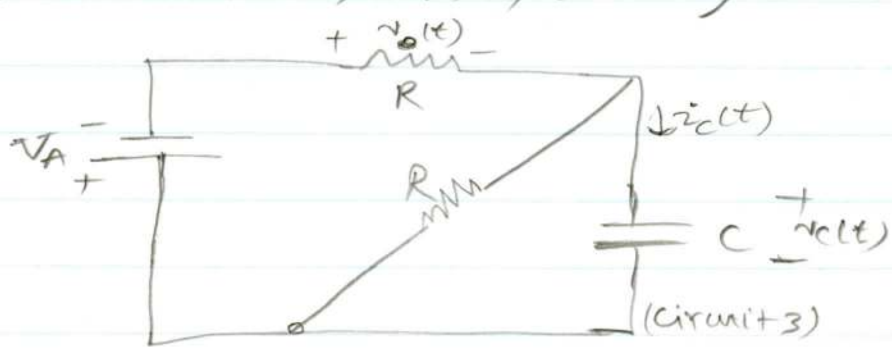


According to (Circuit 2), $v_c(0^-)$ is the voltage drop across the R-resistor and therefore, it can be obtained by voltage division eqn:

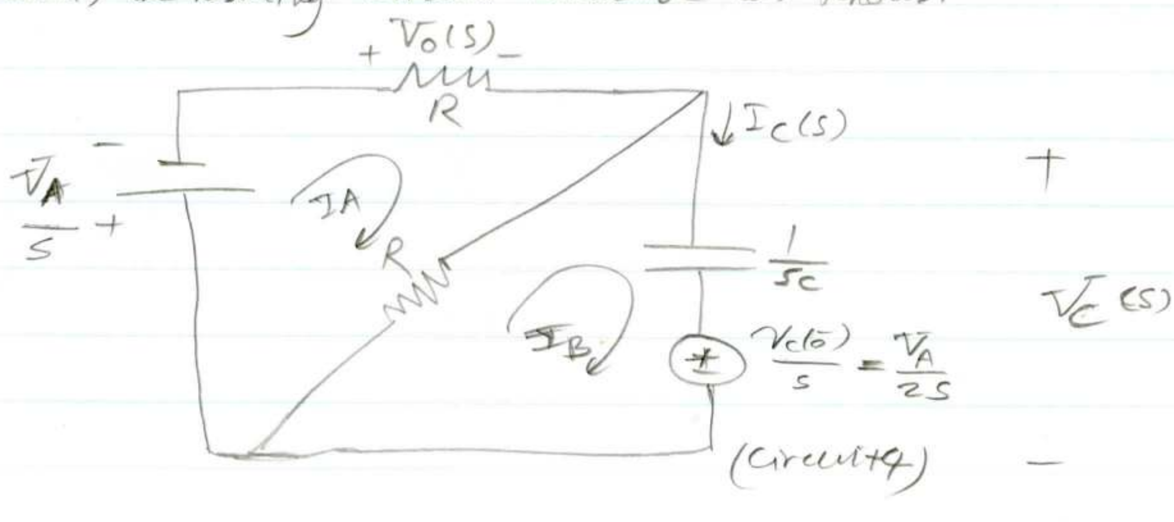
$$v_c(0^-) = \frac{R}{R+R} V_A \Rightarrow \boxed{v_c(0^-) = \frac{1}{2} V_A} \quad (*)$$

Having obtained (*), we can then analyze the circuit when the switch is at position B.

<u>Second:</u> Referring to (Circuit 1), we then consider that the switch to position B has occured at $t=0$, therefore, the resulting circuit would be as follows:



Then, recalling eqn (*), we can then consider transforming (Circuit 3) into s-domain, the resulting circuit would be as follows:



We then perform mesh analysis in order to analyze (circuit 4). On this way, we consider mesh currents I_A and I_B, we then obtain:

$$\begin{bmatrix} R+R & -R \\ -R & R+\frac{1}{sC} \end{bmatrix} \begin{bmatrix} I_A(s) \\ I_B(s) \end{bmatrix} = \begin{bmatrix} -\frac{V_A}{s} \\ -\frac{V_A}{2s} \end{bmatrix} \quad (**)$$

In order to solve for $I_A(s)$ and $I_B(s)$, given $(**)$, we use Cramer's rule:

$$\Delta(s) = \det\left(\begin{bmatrix} 2R & -R \\ -R & R+\frac{1}{sC} \end{bmatrix} \right) = 2R\left(R+\frac{1}{sC}\right) - (-R)(-R) = 2R^2 + \frac{2R}{sC} - R^2$$

$$\Rightarrow \boxed{\Delta(s) = R^2 + \frac{2R}{sC}}$$

$$D_A(s) = \det\left(\begin{bmatrix} -\frac{V_A}{s} & -R \\ -\frac{V_A}{2s} & R+\frac{1}{sC} \end{bmatrix} \right) = -\frac{V_A}{s}\left(R+\frac{1}{sC}\right) - (-R)\left(-\frac{V_A}{2s}\right)$$

$$\Rightarrow \boxed{D_A(s) = -V_A\left(\frac{3R}{2s} + \frac{1}{Cs^2}\right)}$$

$$D_B(s) = \det\left(\begin{bmatrix} 2R & -\frac{V_A}{s} \\ -R & -\frac{V_A}{2s} \end{bmatrix} \right) = -\frac{V_A}{2s}(2R) - \left(-\frac{V_A}{s}\right)(-R)$$

$$\Rightarrow \boxed{D_B(s) = -V_A\left(\frac{2R}{s}\right)}$$

by which $I_A(s)$ and $I_B(s)$ can be obtained as follows:

$$I_A(s) = \frac{D_A(s)}{D(s)} \Rightarrow I_A(s) = \frac{\frac{1}{s}\left(\frac{3R}{2} + \frac{1}{Cs}\right)}{\frac{2R}{Cs} + R^2}(-V_A)$$

$$\Rightarrow \boxed{I_A(s) = \frac{3RCs+2}{(2R^2Cs+4R)s}(-V_A)} \quad (\star)$$

$$I_B(s) = \frac{\Delta_B(s)}{\Delta(s)} \Rightarrow I_B(s) = \frac{\frac{2R}{s}}{R^2 + \frac{2R}{cs}}(-V_A) \Rightarrow \boxed{I_B(s) = \frac{2C}{RCs+2}(-V_A)} \quad (**)$$

Having obtained (*) and (**), we then refer to (circuit 4), where we get:

$$\boxed{I_C(s) = I_B(s) = \frac{2C}{RCs+2}(-V_A)} \quad (+)$$

$$V_0(s) = R\, I_A(s) \Rightarrow \boxed{V_0(s) = \frac{3RCs+2}{(2RCs+4)s}(-V_A)} \quad (++)$$

In order to finalize the solution, we need to find $i_c(t)$ and $v_o(t)$. This can be performed by using eqns (+), (++), and laplace inverse technique:

$$I_C(s) = \frac{2C}{RCs+2}(-V_A) = \frac{\frac{2}{R}}{s+\frac{2}{RC}}(-V_A) = \frac{2}{R}\left(\frac{1}{s+\frac{2}{RC}}\right)(-V_A)$$

So, we get:

$$i_c(t) = \mathcal{L}^{-1}\{I_C(s)\} = \mathcal{L}^{-1}\left\{\frac{2}{R}\left(\frac{1}{s+\frac{2}{RC}}\right)(-V_A)\right\} = \frac{-2V_A}{R}\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s+2/RC}\right\}}_{= e^{-2/RC\, t}}$$

$$\Rightarrow \boxed{i_c(t) = \frac{-2V_A}{R} e^{-\frac{2}{RC}t} u(t)} \quad (+++)$$

$$v_o(t) = \mathcal{L}^{-1}\{V_0(s)\} = \mathcal{L}^{-1}\left\{\frac{3RCs+2}{(2RCs+4)s}(-V_A)\right\} = \mathcal{L}^{-1}\left\{\frac{3RC}{2RCs+4}(-V_A)\right\} + \mathcal{L}^{-1}\left\{\frac{2}{(2RCs+4)s}(-V_A)\right\}$$

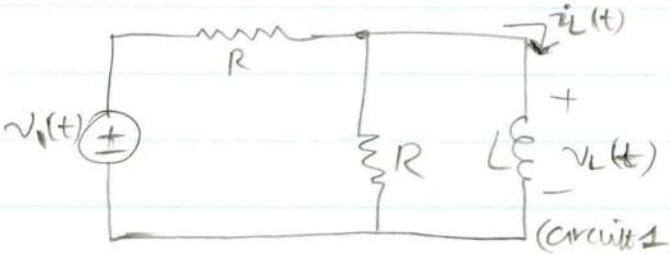
$$\Rightarrow v_o(t) = \underbrace{\mathcal{L}^{-1}\left\{\frac{\frac{3}{2}}{s+\frac{2}{RC}}(-V_A)\right\}}_{= \frac{3}{2}\left(e^{-2/RC\, t}(-V_A)u(t)\right)} + \underbrace{\mathcal{L}^{-1}\left\{\left(\frac{1}{2s} - \frac{RC}{2RCs+4}\right)(-V_A)\right\}}_{= \left(\frac{1}{2}u(t) - \frac{1}{2}e^{\frac{-2}{RC}t}(-V_A)u(t)\right)}$$

$$\Rightarrow \boxed{v_o(t) = -V_A\left(e^{\frac{-2}{RC}t} + \frac{1}{2}\right)u(t)} \quad (++++)$$

Hence, having obtained (+), (++), (+++), and (++++), the soln is Complete.

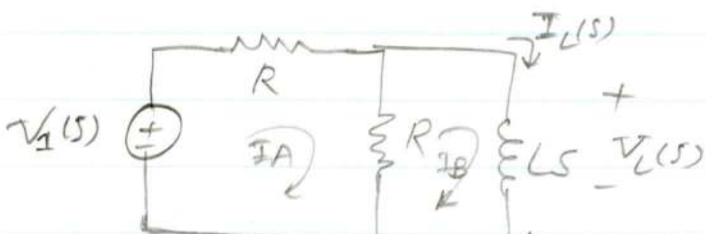
problem 10-24:

For the circuit shown in the following figure, we would like to find $I_L(s)$ and $i_L(t)$, where $v_1(t) = V_A e^{-1000t} u(t)$, $R = 200\Omega$, $L = 100mH$, and $i_L(0) = 0A$.



(circuit 1)

we shall first transform (circuit 1) into s-domain, where we note $i_L(0)=0A$; the resulting circuit is shown next:



(circuit 2)

we then analyze (circuit 2) using mesh analysis as follows:

$$\begin{bmatrix} R+R & -R \\ -R & R+LS \end{bmatrix}\begin{bmatrix} I_A \\ I_B \end{bmatrix} = \begin{bmatrix} V_1(S) \\ 0 \end{bmatrix}, \quad (*)$$

where then, (*) can be solved using Cramer's rule:

$$D = \det\left(\begin{bmatrix} 2R & -R \\ -R & R+LS \end{bmatrix}\right) = 2R^2 + 2RLS - R^2 \Rightarrow \boxed{D = R^2 + 2RLS}$$

$$D_A = \det\left(\begin{bmatrix} V_1(S) & -R \\ 0 & R+LS \end{bmatrix}\right) \Rightarrow \boxed{D_A = V_1(S)\,(R+LS)}$$

$$D_B = \det\left(\begin{bmatrix} 2R & V_1(S) \\ -R & 0 \end{bmatrix}\right) \Rightarrow \boxed{D_B = V_1(S)\,R}$$

$$\Rightarrow \begin{cases} I_A(S) = \dfrac{D_A(S)}{D(S)} = \dfrac{V_1(S)\,(R+LS)}{R^2+2RLS} \Rightarrow \boxed{I_A(S) = \dfrac{R+LS}{R^2+2RLS}\,V_1(S)} \quad (*) \\[2ex] I_B(S) = \dfrac{D_B(S)}{D(S)} = \dfrac{V_1(S)\,R}{R^2+2RLS} \Rightarrow \boxed{I_B(S) = \dfrac{1}{R+2LS}\,V_1(S)} \quad (**) \end{cases}$$

Then, referring to (circuit 2), we note that:

$$\boxed{I_L(S) = I_B(S) = \frac{1}{R+2LS}\,V_1(S)} \quad (+)$$

Moreover we recall that $v_1(t) = V_A\,e^{-1000t}\,u(t)$, hence by Laplace transform, we get:

$$\boxed{V_1(S) = \mathcal{L}\{v_1(t)\} = \mathcal{L}\left\{V_A\,e^{-1000t}\,u(t)\right\} = \frac{V_A}{S+1000}} \quad (++)$$

By (+) and (++), we get the following, where we recall $R = 200\Omega$, $L = 100mH$

$$I_L(S) = \left(\frac{1}{200+0.2S}\right)\left(\frac{1}{S+1000}\right)V_A \Rightarrow \boxed{I_L(S) = \frac{5}{(S+1000)^2}\,V_A} \quad (+++)$$

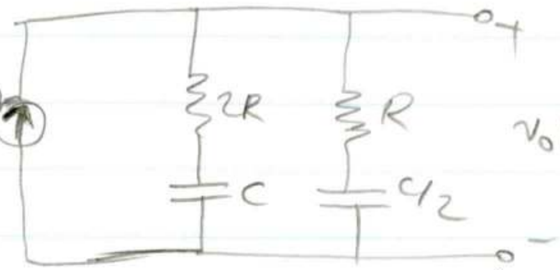
Hence, by (+++), we can ~~obtain~~ obtain $i_L(t)$, by applying Laplace inverse technique:

$$i_L(t) = \mathcal{L}^{-1}\{I_L(S)\} = \mathcal{L}^{-1}\left\{\frac{5}{(S+1000)^2}V_A\right\} \Rightarrow \boxed{i_L(t) = \left(t\,e^{-1000t}\,u(t)\right)5V_A} \quad (++++)$$

By (+++) and (++++), the solution is complete. ∎

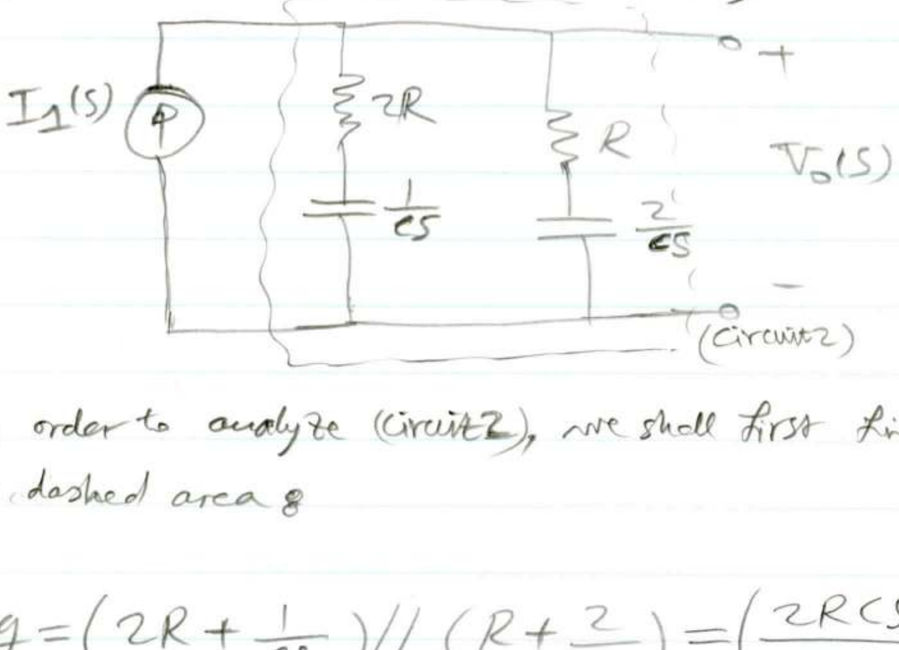
prob. 10-28 :

For the circuit shown in the following figure, we would like to find the s-domain relationship b/w the input $I_1(s)$ and the output $V_0(s)$. This is assumed when the circuit is in zero-state.



(circuit 1)

First, we shall transform (circuit 1) into the s-domain where provided the circuit is assumed to be at zero-state, i.e. the initial cond'n for the voltage across the capacitors are zero, we get the following circuit:



(Circuit 2)

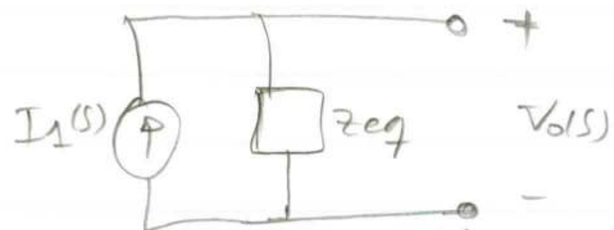
In order to analyze (Circuit 2), we shall first find the equivalent impedance of the dashed area :

$$Z_{eq} = \left(2R + \frac{1}{Cs}\right) // \left(R + \frac{2}{Cs}\right) = \left(\frac{2RCs+1}{Cs}\right) // \left(\frac{RCs+2}{Cs}\right)$$

$$\Rightarrow Z_{eq} = \frac{\left(\frac{2RCs+1}{Cs}\right)\left(\frac{RCs+2}{Cs}\right)}{\frac{2RCs+1}{Cs} + \frac{RCs+2}{Cs}} = \frac{\frac{(2RCs+1)(RCs+2)}{C^2s^2}}{\frac{3RCs+3}{Cs}}$$

$$\Rightarrow Z_{eq} = \frac{(RCs+2)(2RCs+1)(Cs)}{(3RCs+3)(C^2s^2)} \Rightarrow \boxed{Z_{eq} = \frac{(RCs+2)(2RCs+1)}{(3RCs+3)(Cs)}} \quad (*)$$

Having found Zeq, we can then consider the following circuit that is equivalent to (circuit 2):



(Circuit 3)

for which the following holds:

$$Zeq = \frac{V_o(s)}{I_1(s)} \xRightarrow{\text{By }(*)} \frac{V_o(s)}{I_1(s)} = \frac{(RCs+2)(2RCs+1)}{(3RCs+3)(Cs)}$$

$$\Rightarrow \boxed{\frac{V_o(s)}{I_1(s)} = \frac{2R^2C^2s^2 + 5RCs + 2}{(3RC^2s + 3C)s}} \quad (**)$$

Having found (**), we are then left to compute its zeros and poles:

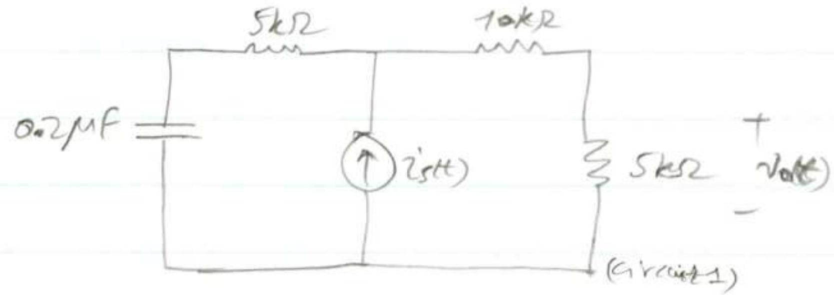
$$(***)\begin{cases} \text{Poles: } \boxed{P_1 = 0} \quad \boxed{P_2 = \frac{-1}{RC}} \\ \text{Zeros: } \boxed{z_1 = \frac{-1}{2RC}} \quad \boxed{z_2 = \frac{-2}{RC}} \end{cases}$$

By (**) and (***), the sol'n is complete.

problem 10-33:

For the circuit shown in the following figure, we would like to find $v_o(t)$ by applying s-domain transformation and using current division technique. We also given that no energy is stored in the capacitor, at $t=0$, and that $i_s(t) = 2.5\cos(2000t)u(t)$ mA.

The circuit is shown on the next page.

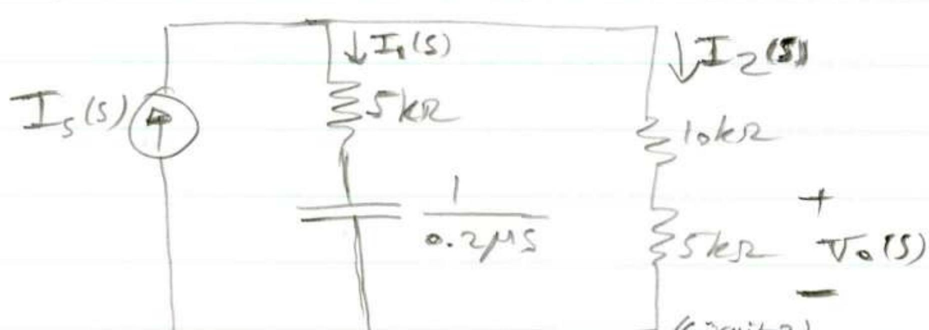


(Circuit 1)

First and in order to be consistent w/ current division technique circuitry, we rearrange the elements in (Circuit 1), where we obtain the next circuit:

(Circuit 2)

We then transform the above circuit into the s-domain, where we note that given at $t=0$ there is no energy stored in the capacitor, $v_c(0^-)=0$. We get the following circuit:



(Circuit 3)

In order to find $V_o(s)$, we first find $I_1(s)$ and $I_2(s)$ by using current division technique. In order to do so, we first find the admittance of each branch and the equivalent admittance of the circuit:

equivalent impedance of the first branch: $Z_1 = 5k + \frac{1}{0.2\mu s} \Rightarrow \boxed{Z_1 = \frac{5ms+5}{1\mu s}}$ (*)

$$\Rightarrow Y_1 = \frac{1}{Z_1} = \frac{1\mu s}{5ms+5} \Rightarrow \boxed{Y_1 = \frac{0.001s}{5s+5000}} \ (**)$$

equivalent impedance of the second branch: $\boxed{Z_2 = 10k + 5k = 15k}$ $(\bigstar)$ $\Rightarrow$ $\boxed{Y_2 = \frac{1}{15000}}$ $(\bigstar\bigstar)$

By $(**)$ and $(\bigstar\bigstar)$ $\Rightarrow$ $Y_{eq} = Y_1 + Y_2 = \frac{0.001S}{5S+5000} + \frac{1}{15000}$ $\Rightarrow$ $\boxed{Y_{eq} = \frac{0.004S+1}{15S+15000}}$ $(+)$

Therefore, having derived $(**)$, $(\bigstar\bigstar)$, and $(+)$, applying current division, we get the following:

$$I_1(S) = \frac{Y_1}{Y_{eq}} I_s(S) = \frac{\frac{0.001S}{5S+5000}}{\frac{0.004S+1}{15S+15000}} I_s(S) \Rightarrow \boxed{I_1(S) = \frac{0.003S}{0.004S+1} I_s(S)} \quad (++)$$

$$I_2(S) = \frac{Y_2}{Y_{eq}} I_s(S) = \frac{\frac{1}{15000}}{\frac{0.004S+1}{15S+15000}} I_s(S) \Rightarrow \boxed{I_2(S) = \frac{S+1000}{4S+1000} I_s(S)} \quad (+++)$$

By $(+++)$ and referring to (circuit 3), we can find $V_o(S)$:

$$V_o(S) = 5k \times I_2(S) \Rightarrow \boxed{V_o(S) = \frac{S+1000}{4S+1000}\left(5000 I_s(S)\right)} \quad (\diamond)$$

In order to complete the analysis, we need to compute $I_s(S)$, which provided $i_s(t) = 2.5\cos(2000t)u(t)$ mA, can be computed as follows:

$$i_s(t) = 2.5\cos(2000t) \times 10^{-3} u(t) \Rightarrow \boxed{I_s(S) = 2.5 \times 10^{-3}\left(\frac{S}{S^2 + (2000)^2}\right)} \quad (\diamond\diamond)$$

By $(\diamond)$ and $(\diamond\diamond)$, we get:

$$V_o(S) = \left(\frac{S+1000}{4S+1000}\right)\left(5000 \times 2.5 \times 10^{-3}\left(\frac{S}{S^2+(2000)^2}\right)\right)$$

$$\Rightarrow \boxed{V_o(S) = \frac{12.5S(S+1000)}{(4S+1000)(S^2+(2000)^2)}} \quad (\diamond\diamond\diamond)$$

In order to find $v_o(t)$, we need to apply laplace inverse on $V_o(s)$, characterized in (◇◇◇). This is performed as follows: (the details of this computation have been provided on page 22)

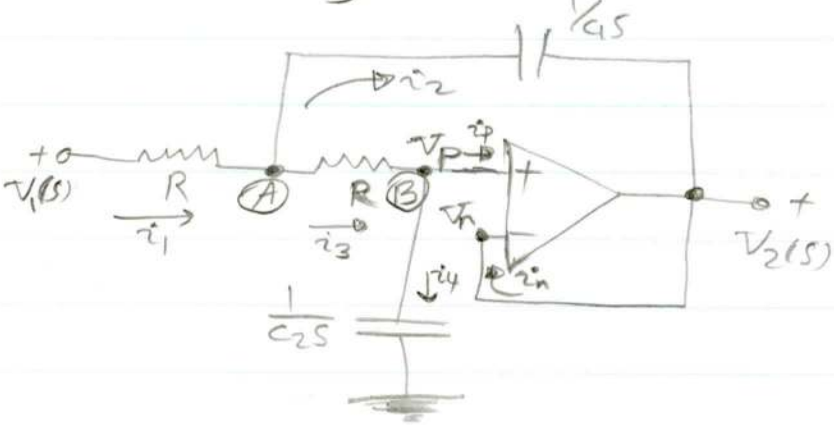
$$\boxed{v_o(t)=\mathcal{L}^{-1}\{V_o(s)\}=\frac{85}{26}\cos(2000t)+\frac{15}{13}\sin(2000t)-\frac{15}{104}e^{-250t}}$$

Having obtained $v_o(t)$, it is enough to characterize the forced and natural poles of $V_o(s)$ to finalize the solution. We then refer to eq in (◇◇◇), where we note:

poles of $V_o(s)$: $\begin{cases} \boxed{P_{1,2}=\pm 2000j} \text{ : forced poles which come from current source transformation to s-domain} \\ \boxed{P_3=-250} \text{ : natural pole which comes from circuit impedances} \end{cases}$

problem 10-53:

For the circuit shown in the following figure, we would like to find the circuit determinant, assuming it is in zero-state, and by using node-voltage eq'ns.



We first note that it is an ideal op-amp, therefore $i_n=i_p=0$ and $V_n=V_p$. Also, it is easy to verify that $V_n(s)=V_2(s)$, hence:

$$\boxed{V_p(s)=V_n(s)=V_2(s)}\ (*)$$

Moreover, the node-voltage eq'ns can be written as follows:

node A: $\dfrac{V_1-V_A}{R}=\dfrac{V_A-V_B}{\frac{1}{C_1 s}}+\dfrac{V_A-V_p}{R}\xrightarrow[V_B=V_p]{\text{By}(*)\ \&}\boxed{\dfrac{V_1-V_A}{R}=C_1 s\,(V_A-V_2)+\dfrac{1}{R}(V_A-V_2)}$ (**)

node B: $\frac{V_A - V_B}{R} = \frac{V_B - 0}{\frac{1}{C_2 S}} \xrightarrow[V_B = V_P]{By(*)\,\&} \boxed{\frac{V_A - V_2}{R} = C_2 S (V_2)}$ (***)

By (**) and (***), we can develop the following:

$$\begin{cases} (**) \Rightarrow \left(\frac{2}{R} + C_1 S\right) V_A = \frac{V_1}{R} + C_1 S V_2 + \frac{1}{R} V_2 \\ (***) \Rightarrow \left(\frac{1}{R}\right) V_A = \left(C_2 S + \frac{1}{R}\right) V_2 \end{cases}$$

from the above pair of eqns, we can find relation b/w V_2 and V_1 by eliminating V_A, where we obtain:

$$\left(\frac{2}{R} + C_1 S\right)(R C_2 S + 1) V_2 = \frac{V_1}{R} + C_1 S V_2 + \frac{1}{R} V_2$$

$$\Rightarrow (2 + R C_1 S)(1 + R C_2 S) V_2 = V_1 + (R C_1 S + 1) V_2$$

$$\Rightarrow \Big[\underbrace{(2 + R C_1 S)(1 + R C_2 S) - (R C_1 S + 1)}_{1 + 2R C_2 S + R^2 C_1 C_2 S^2}\Big] V_2 = V_1$$

$$\Rightarrow \boxed{\frac{V_2(S)}{V_1(S)} = \frac{1}{(R^2 C_1 C_2) S^2 + 2 R C_2 S + 1}} \quad (\star)$$

Thus, by ($\star$), the determinant of the circuit can be obtained as follows:

$$\boxed{D(S) \equiv (R^2 C_1 C_2) S^2 + 2 R C_2 S + 1} \quad (\star\star)$$

we would also like to select values of R, C_1 and C_2 to have $\zeta = 1$, $\omega_0 = 10 \frac{krad}{sec}$. In order to do so, we rewrite ($\star\star$) in standard form: $S^2 + 2\zeta\omega_0 S + \omega_0^2$, we obtain:

$$D(S) = S^2 + \frac{2 R C_2}{R^2 C_1 C_2} S + \frac{1}{R^2 C_1 C_2} \equiv S^2 + 2\zeta\omega_0 S + \omega_0^2 \quad (\star\star\star)$$

By (★★★):

$$\begin{cases} \omega_0^2 = \dfrac{1}{R^2 C_1 C_2} \Rightarrow \boxed{\omega_0 = \sqrt{\dfrac{1}{R^2 C_1 C_2}}} \quad (+) \\ 2\zeta\omega_0 = \dfrac{2}{R C_1} \Rightarrow \boxed{\zeta = \sqrt{\dfrac{C_2}{C_1}}} \quad (++) \end{cases}$$

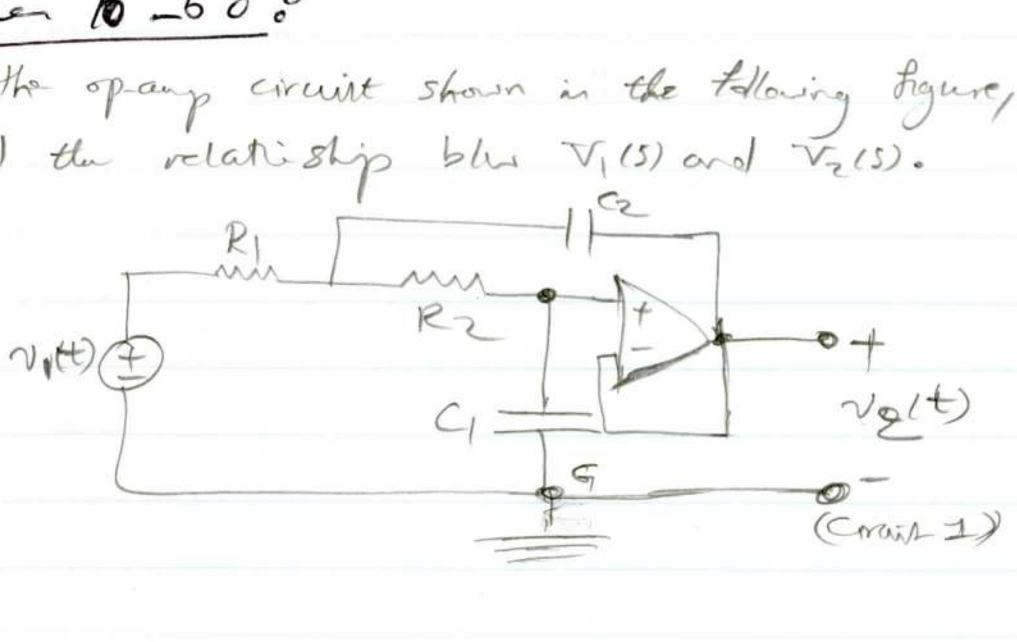
Provided we would like to maintain $\zeta = 1$, $\omega_0 = 10^4 \frac{rad}{sec}$, the following needs to be true:

$$\begin{cases} 10^4 = \sqrt{\dfrac{1}{R^2 C_1 C_2}} \Rightarrow \underline{\sqrt{R^2 C_1 C_2} = 10^{-4}} \\ 1 = \sqrt{\dfrac{C_2}{C_1}} \Rightarrow \underline{C_1 = C_2} \end{cases}$$

We have then selected $\boxed{R = 1\ k\Omega}$ and $\boxed{C_1 = C_2 = 0.1\mu F}$ which satisfy both above eq's.

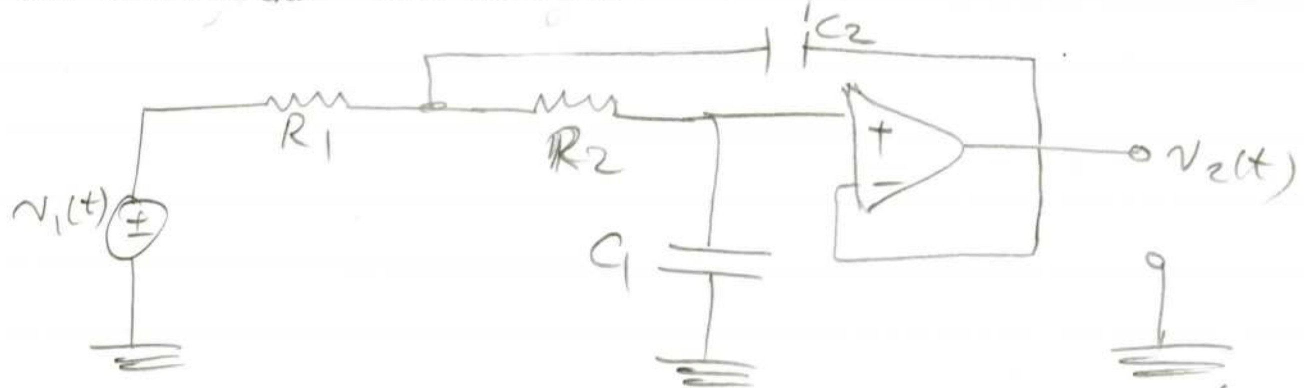
problem 10-68:

For the op-amp circuit shown in the following figure, we would like to find the relationship b/w $V_1(s)$ and $V_2(s)$.



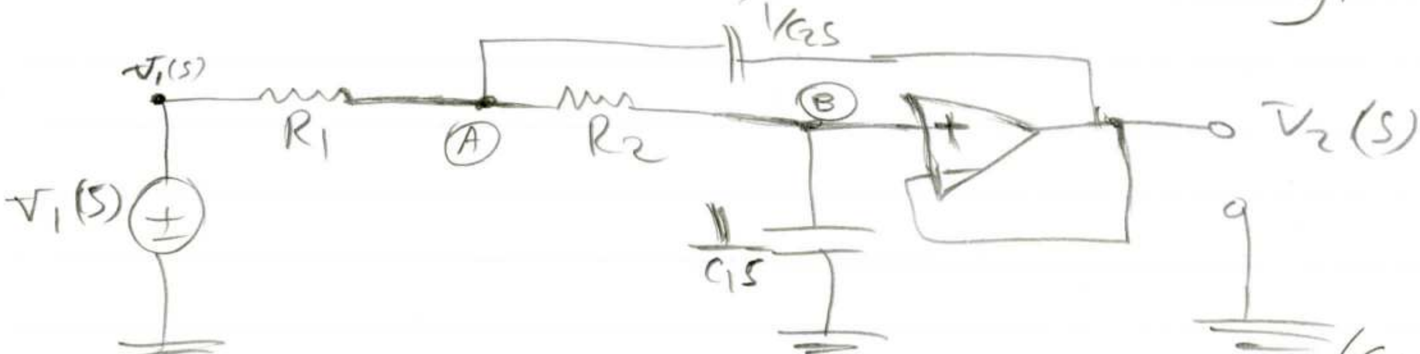
(Circuit 1)

Refering to (Circuit 1), we first note that the node G is grounded, therefore we can transform that circuit as follows:



(Circuit 2)

we then transform (Circuit 2) into the s-domain, where provided that the circuit is at zero-state, we obtain the following:



(Circuit 3)

At this stage, it is easy to note that the resulting (Circuit 3) is equivalent to the circuit we have already studied in problem 10-53, w/ the only difference that the two capacitors, $\frac{1}{C_1S}$ and $\frac{1}{C_2S}$ have been swapped in their labeling, and that the two resistors, R_1 and R_2, are no longer equal. Therefore, we can perform similar computations as follows:

node (A): $\frac{V_1-V_A}{R_1} = \frac{V_A-V_2}{\frac{1}{C_2S}} + \frac{V_A-V_2}{R_2} \Rightarrow \boxed{\frac{V_1-V_A}{R_1} = C_2S(V_A-V_2) + \frac{V_A-V_2}{R_2}}$ (* *)

node (B): $\frac{V_A-V_B}{R_2} = \frac{V_B-0}{\frac{1}{C_1S}} \Rightarrow \boxed{\frac{V_A-V_2}{R_2} = C_1S(V_2)}$ (* * *)

$$\Rightarrow \begin{cases} \frac{1}{R_1}V_1 + C_2SV_2 + \frac{1}{R_2}V_2 = \left(\frac{1}{R_1}+C_2S+\frac{1}{R_2}\right)V_A \\ \left(C_1S+\frac{1}{R_2}\right)V_2 = \frac{1}{R_2}V_A \end{cases}$$

Eliminating V_1 in the above eqns, we obtain the following relati btw V_2 and V_1:

$$\left(\frac{1}{\frac{1}{R_1}+C_2 s+\frac{1}{R_2}}\right)\left(\frac{1}{R_1}V_1 + C_2 s V_2 + \frac{1}{R_2}V_2\right) = R_2\left(C_1 s + \frac{1}{R_2}\right)V_2$$

$$\Rightarrow \left(\frac{R_1 R_2}{R_1 R_2 C_2 s + R_1 + R_2}\right)\left(\frac{1}{R_1}V_1 + C_2 s V_2 + \frac{1}{R_2}V_2\right) = (R_2 C_1 s + 1)V_2$$

$$\Rightarrow \left(\frac{R_2}{R_1 R_2 C_2 s + R_1 + R_2}\right)V_1 = \left[\frac{(R_2 C_1 s + 1)(R_1 + R_2 + R_1 R_2 C_2 s) - R_1 R_2 C_2 s - R_1}{R_1 R_2 C_2 s + R_1 + R_2}\right]V_2$$

$$\Rightarrow \frac{V_2(s)}{V_1(s)} = \frac{R_2}{(R_1 R_2 C_1 + R_2^2 C_1)s + (R_1 R_2^2 C_1 C_2)s^2 + R_2}$$

$$\Rightarrow \boxed{\frac{V_2(s)}{V_1(s)} = \frac{1}{(R_1 R_2 C_1 C_2)s^2 + (R_1 C_1 + R_2 C_1)s + 1}}$$

END of Solns to HW#7

ADDENDUM

Computations related to eq'n (◇▽◇) stated on page 17:

$$\boxed{V_o(s) = \frac{12.5}{4}\left(\frac{s(s+1000)}{(s+250)(s^2+(2000)^2)}\right) = \frac{12.5}{4}\left(\frac{s(s+1000)}{(s+250)(s+2000j)(s-2000j)}\right) = \frac{A_1}{s+250}+\frac{A_2}{s+2000j}+\frac{A_3}{s-2000j}} \quad (i)$$

where

$$A_1 = \frac{12.5}{4}\left(\frac{-250(-250+1000)}{(-250)^2+(2000)^2}\right) = \frac{12.5}{4}\left(\frac{(-250)(-250+1000)}{(-250)^2+(2000)^2}\right)$$

$$\Rightarrow \boxed{A_1 = \frac{-15}{104}} \quad (ii)$$

$$A_2 = \frac{12.5}{4}\left(\frac{(-2000j)(-2000j+1000)}{(-2000j+250)(-4000j)}\right) = \frac{12.5}{4}\left(\frac{-4\times 10^6 - 2\times 10^6 j}{-8\times 10^6 + 10^6 j}\right)$$

$$\Rightarrow A_2 = \frac{12.5}{4}\left(\frac{-4-2j}{-8+j}\right) = \frac{12.5}{4}\left(\frac{(-4-2j)(-8-j)}{65}\right) \Rightarrow \boxed{A_2 = \frac{12.5}{26}(3+2j)} \quad (iii)$$

Also, by partial fraction expansion technique, $A_3 = A_2^* \Rightarrow \boxed{A_3 = \frac{12.5}{26}(3-2j)}$ (iv)

Plugging (iv), (iii), (ii) back into (i), we get:

$$V_o(s) = \frac{-15}{104}\left(\frac{1}{s+250}\right) + \frac{12.5}{26}\left(\frac{3+2j}{s+2000j} + \frac{3-2j}{s-2000j}\right)$$

$$= \frac{-15}{104}\left(\frac{1}{s+250}\right) + \frac{12.5}{26}\left(\frac{6s+8000}{s^2+(2000)^2}\right) = \frac{-15}{104}\left(\frac{1}{s+250}\right) + \frac{12.5}{26}\left(\frac{6s}{s^2+(2000)^2}\right) + \frac{12.5\times 4}{26}\left(\frac{2000}{s^2+(2000)^2}\right)$$

$$\Rightarrow \boxed{V_o(s) = \underbrace{\frac{-15}{104}\,\frac{1}{s+250}} + \underbrace{\frac{15}{13}\left(\frac{2000}{s^2+(2000)^2}\right)} + \underbrace{\frac{85}{26}\,\frac{s}{s^2+(2000)^2}}}$$

$$\Downarrow$$

$$v_o(t) = \frac{-15}{104}e^{-250t} + \frac{15}{13}\cos(2000t) + \frac{85}{26}\sin(2000t)$$