MAE140 - Linear Circuits - Winter 2014
Midterm, February 4th

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities
2. You have 70 minutes
3. Write your answers on your “Blue Book”
4. Do not forget to write your name, student number, and instructor

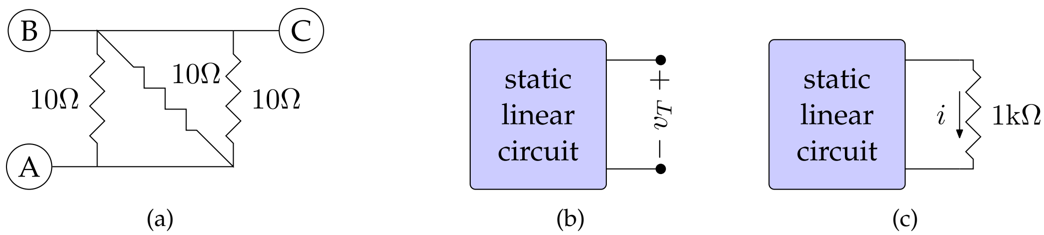


Figure 1: Circuits for Question 1

1. **Equivalent Circuits**

   (a) (5 points) For the circuit in Fig. 1(a), find the equivalent resistance as seen from terminals A and B, A and C, and B and C.

   Solution: Seen from A and B all three resistors are in parallel, therefore

   $$R_{AB} = \frac{1}{\frac{1}{10} + \frac{1}{10} + \frac{1}{10}} = \frac{10}{3}\Omega \qquad \text{(+2 points)}$$

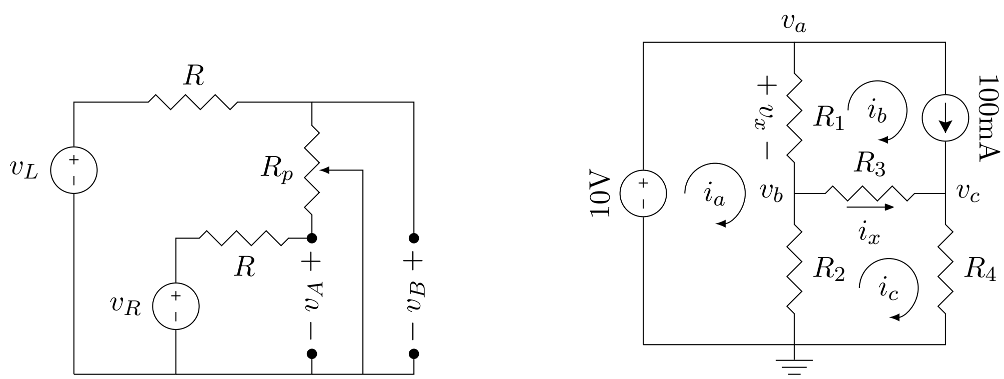
   Note that B and C are the same node, therefore (+1 point)

   $$R_{BC} = 0\Omega \qquad \text{(+1 point)}$$

   and

   $$R_{AC} = R_{AB} = \frac{10}{3}\Omega \qquad \text{(+1 point)}$$

   (b) (5 points) A student performed two experiments on a static linear circuit: first, the open-circuit voltage $v_T = 10$V was measured as indicated in Fig. 1(b); then a 1kΩ resistor was connected to the terminals of the circuit as shown in in Fig. 1(c) and a current of $i = 1$mA was measured. Use this information to construct the Thévenin and Norton equivalent circuits.



(a) (b)

Figure 2: Circuits for Questions 2 and 3

Solution: The Thévenin voltage is obtained directly from the first experiment:

$$v_T = 10\text{V} \qquad \text{(+1 point)}$$

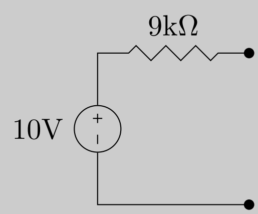
From the second experiment, we obtain

$$i = \frac{10}{R_T + 1\text{k}\Omega} = 1\text{mA} \qquad \text{(+1 point)}$$

and therefore

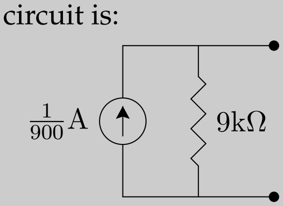
$$R_T = \frac{10}{0.001} - 1000 = 10000 - 1000 = 9000\Omega \qquad \text{(+1 point)}$$

The corresponding Thévenin equivalent is the circuit:



(+1 point)

and the Norton equivalent circuit is:



(+1 point)

2. **Voltage Division**

The circuit in Fig. 2(a) is used in (cheap) audio stereo amplifiers as a *balance control circuit*. It uses a single potentiometer to simultaneously control two audio channels: left, v_L, and right, v_R. Answer the following questions:

(a) (4 points) Use voltage division to show that

$$v_A = \frac{x}{R/R_p + x} v_R, \qquad v_B = \frac{1-x}{(R/R_p) + 1 - x} v_L$$

where R_p is the nominal resistance of the potentiometer, and $x \in [0,1]$ is used to represent the position of the potentiometer: when the potentiometer is at its center position $x = 1/2$; when $x = 0$ the potentiometer is at its lower end and when $x = 1$ it is on its upper end.

Solution:
The voltage v_A is the voltage across the "lower part" of the potentiometer that has resistance equal to xR_p. (+1 point)
Using voltage division:

$$v_A = \frac{xR_p}{R + xR_p} v_R = \frac{x}{(R/R_p) + x} v_R$$ (+1 point)

The voltage v_B is the voltage across the "upper part" of the potentiometer that has resistance equal to $(1-x)R_p$. (+1 point)
Using voltage division:

$$v_B = \frac{(1-x)R_p}{R + (1-x)R_p} v_L = \frac{1-x}{(R/R_p) + 1 - x} v_L$$ (+1 point)

(b) (4 points) Let $R = 510\Omega$ and $R_p = 500\Omega$ and evaluate v_A/v_R and v_B/v_L for $x = 0$, $x = 1/2$, and $x = 1$. Sketch v_A/v_R and v_B/v_L as a function of x.

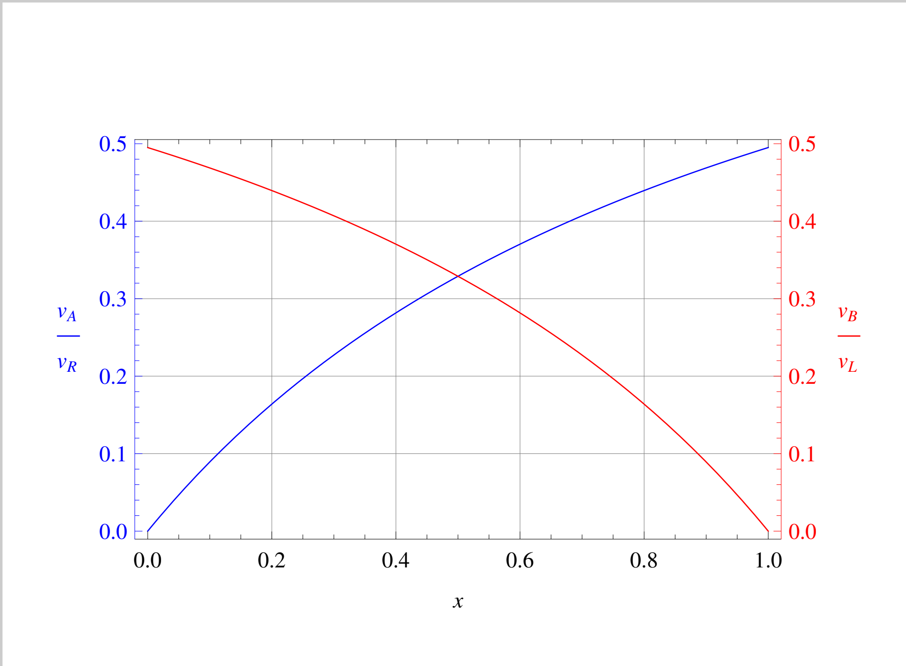
Solution: Substituting $R = 510\Omega$ and $R_p = 500\Omega$ we obtain and denoting $v_A(x)$ as a function of x we obtain

$$\frac{v_A(0)}{v_R} = \frac{0}{0 + 510/500} = 0,$$
$$\frac{v_A(1/2)}{v_R} = \frac{1/2}{1/2 + 510/500} \approx 0.33,$$
$$\frac{v_A(1)}{v_R} = \frac{1}{1 + 510/500} \approx 0.5.$$ (+1.5 points)

Repeating for v_B we obtain

$$\frac{v_B(0)}{v_L} = \frac{1}{1 + 510/500} \approx 0.5,$$
$$\frac{v_B(1/2)}{v_L} = \frac{1/2}{1/2 + 510/500} \approx 0.33,$$
$$\frac{v_B(1)}{v_L} = \frac{0}{0 + 510/500} = 0.$$ (+1.5 points)

A sketch of v_A/v_R and v_B/v_L should look like this:



(+1 point)

NOTE TO GRADERS: straight lines here are fine!

(c) (2 points) Explain how the circuit works.

Solution: When the potentiometer is at its "lowest" position $x = 0$ and $v_A/v_R = 0$ and $v_B/v_L = 0.5$ is at its maximum so you hear sound from the left channel v_L but no sound from the right channel v_R. (+2/3 point)
When the potentiometer is at its "highest" position $x = 1$ and $v_A/v_R = 0.5$ is at its maximum and $v_B/v_L = 0$ so you hear sound from the right channel v_R but no sound from the left channel v_L. (+2/3 point)
When the potentiometer is on an intermediate position you hear a combination of left and sound. In particular, when $x = 1/2$ and the potentiometer is at the middle you hear the two channels at equal levels of volume. (+2/3 point)

(d) (1 point (bonus)) Name one reason why R was chosen as 510Ω and not 500Ω in part (b)?

Solution: $500\Omega$is not a standard resistor value (+1 point)
P.S.: But is a commonly found potentiometer value!

3. **Node-Voltage and Mesh-Current Analysis**

(a) (4 points) Formulate node-voltage equations for the circuit in Fig. 2(b). Use the node-voltage and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations only involve node-voltages.

Solution: Using Method #2 we have that: (+1 point)

$$v_a = 10\text{V}. \tag{1}$$

Writing KCL at nodes b and c by inspection we obtain (+2 points)

$$\begin{bmatrix} -\frac{1}{R_1} & \frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3} & -\frac{1}{R_3} \\ 0 & -\frac{1}{R_3} & \frac{1}{R_3}+\frac{1}{R_4} \end{bmatrix} \begin{pmatrix} v_a \\ v_b \\ v_c \end{pmatrix} = \begin{pmatrix} 0 \\ 100\text{mA} \end{pmatrix} \tag{2}$$

In order to solve this circuit we need to solve the three equations in (1)-(2) on the node-voltage unknowns v_a, v_b, and v_c. (+1 point)

NOTE TO GRADERS: Students have to *clearly* indicate their node-voltage equations and the unkown voltages they need to solve for to get the final point. Any reasonable set of equations is fine.

(b) (1 point) Show how the current i_x and the voltage v_x are related to the node-voltages.

Solution: From the circuit:

$$v_x = v_a - v_b \qquad \text{(+1/2 point)}$$

Using the circuit and Ohm's law:

$$i_x = \frac{v_b - v_c}{R_3} \qquad \text{(+1/2 point)}$$

(c) (4 points) Formulate mesh-curent equations for the circuit in Fig. 2(b). Use the mesh currents and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations only involve mesh-currents.

Solution: Using Method #2 we have that: (+1 point)

$$i_b = 100\text{mA}. \tag{3}$$

Writing KVL at meshes a and c by inspection we obtain (+2 points)

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 \\ -R_2 & -R_3 & R_2+R_3+R_4 \end{bmatrix} \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} = \begin{pmatrix} 10\text{V} \\ 0 \end{pmatrix} \tag{4}$$

In order to solve this circuit we need to solve the three equations in (3)-(4) on the mesh-current unknowns i_a, i_b, and i_c. (+1 point)

NOTE TO GRADERS: Students have to *clearly* indicate their node-voltage equations and the unkown voltages they need to solve for to get the final point. Any reasonable set of equations is fine.

(d) (1 point) Show how the current i_x and the voltage v_x are related to the mesh-currents.

Solution: From the circuit:

$$i_x = i_c - i_b \qquad \text{(+1/2 point)}$$

Using the circuit and Ohm's law:

$$v_x = R_1(i_a - i_b) \qquad \text{(+1/2 point)}$$

(e) (2 points (bonus)) Suppose you compute the node-voltages and find out that $v_a > v_b > v_c > 0$. Which sources are delivering/absorbing power to/from the circuit? Clearly explain your reasoning to get the bonus points.

Solution:
Because $v_a > v_c$ we have that the power on the current source is

$$p_{i_s} = 100\text{mA} \times (v_a - v_c) > 0$$

so that the current source absorbs power from the circuit. (+1 point)
The power on the voltage source is

$$p_{v_s} = -10\text{V} \times i_a$$

so we need to find the sign of i_a. Since $v_a > v_b$ we have from the resistor R_1 that

$$i_a > i_b = 100\text{mA} > 0$$

therefore

$$p_{v_s} = -10\text{V} \times i_a < 0$$

so that the voltage source delivers power to the circuit. (+1 point)
An alternative simpler reasoning here could be that the voltage source must deliver power since we found out that the current source absorbs power.