MAE 140 – Solutions of homework #1

Maurício de Oliveira and David Mateos-Núñez

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The problems are taken from the book "The Analysis and Design of Linear Circuits", 7th Edition, R. E. Thomas, A. J. Rosa and G. J. Toussaint, Wiley 2012.

1.13 An ideal voltage source of 12V with an output capacity of 100Ah is connected to a lamp that absorbs 200W. In this case,

(a) the current supplied by the battery is

$$i = \frac{p}{v} = \frac{200W}{12V} = \frac{50}{3}A \approx 16.6A\,,$$

and the time that the battery can power the headlight is

$$t = \frac{100Ah}{i} = \frac{100Ah}{50/3A} = 6h.$$

(b) When a second device that absorbs a power of $p' = 100$W is connected to the utility port in **parallel**, then the current i' through the parallel wire is

$$i' = \frac{p'}{v} = \frac{100W}{12V} = \frac{25}{3} \approx 8.3A\,,$$

and the time that the battery can power both is

$$t = \frac{100Ah}{i + i'} = \frac{100Ah}{(50/3 + 25/3)A} = 4h\,.$$

On the other hand, if the second device is connected in **series** (and assuming that the power absorbed by the device is independent of the voltage), then the current through both devices is

$$i = \frac{p + p'}{v} = \frac{200W + 100W}{12V} = 25A\,,$$

so the time that the battery can power both in this case is

$$t = \frac{100Ah}{25A} = 4h\,,$$

which interestingly is the **same time**.

1.14 Two different lamps produce the same amount of light when connected to an ideal voltage source of 12V, but one absorbs 100W and the other 16W.

They are operated over 1000 hours and the electricity costs 7.8 cents/kWh.

Therefore, the cost of operating each one is, respectively,

$$\begin{aligned} c_1 &= 10^2W \cdot 10^3h \cdot \frac{7.8 \text{ cents}}{kWh} = 10^2kWh \cdot \frac{7.8 \text{ cents}}{kWh} = 780 \text{ cents} = \$7.8 \\ c_2 &= 16W \cdot 10^3h \cdot \frac{7.8 \text{ cents}}{kWh} = 16kWh \cdot \frac{7.8 \text{ cents}}{kWh} = 124.8 \text{ cents} \approx \$1.25 \end{aligned}$$

That is,

$$\frac{c_1}{c_2} = \frac{780 \text{ cents}}{124.8 \text{ cents}} = 6.25\,.$$

In the light of the above formulas, we deduce that this index could have been computed with the quotient of the powers consumed by each device:

$$\frac{p_1}{p_2} = \frac{100W}{16W} = 6.25\,.$$

1.21 We observe that the **passive sign convention** is used with respect to the device B, so the power is transferred **from A to B** when the result is **positive**, and **from B to A** when the result is **negative**.

In each case, the power absorbed by the device B is

(a) $p_1 = 11V \cdot (-1.1)A = -12.1W$ (power is transferred to device A)

(b) $p_2 = 80V \cdot 0.02A = 1.6W$

(c) $p_3 = -120V \cdot -0.012A = 1.44W$

(d) $p4 = -1.5V \cdot (-600 \cdot 10^{-6})A = 9 \cdot 10^{-4}W = 0.9mW$

2.3 From Ohm's law, $i = \frac{v}{R}$, and the formula for the power, $p = vi$, we derive the formula $p = vi = i^2R$. Therefore,

$$i = \sqrt{\frac{p}{R}} = \sqrt{\frac{0.1W}{10^5\Omega}} = \sqrt{10^{-6}}A = 10^{-3}A\,.$$

2.6 By Ohm's law, we have

$$R_x = \frac{v}{i} = \frac{100V}{0.01A} = 10^4\Omega\,.$$

Since the **passive sign convention** has been used for the resistor, the power that the resistor absorbs is

$$p = vi = 100V \cdot 0.01A = 1W\,.$$

2.9 From Ohm's law, $i = \frac{v}{R}$, and the formula for the power, $p = vi$, we derive

$$p = vi = v\frac{v}{R} = \frac{v^2}{R}\,,$$

so the maximum voltage is

$$v_{\max} = \sqrt{p_{\max}R} = \sqrt{0.125W \cdot 10^5\Omega} \approx 111.8V$$

2.14 The purpose of this problem (and the next three) is to identify which set of equations **suffices** to compute the variables that are requested. In general, to obtain the current (through) and voltage (across) any device, one needs to **define** polarities and reference current directions (one or the other, because they are related through the **passive sign convention**, which says that the reference direction points towards the positive terminal), and then write

- the **KCL connection equations** in a set of N-1 **nodes** (where a node is a juncture of two or more devices);
- the **KVL connection equations** in a set of independent **loops** (whose direction you define as you wish and has nothing to do with the current reference directions). If E is the **number of two-terminal devices** and N is the number of **nodes**, there are $E-N+1$ **independent KVL equations** (one for each **loop**, which is a closed path formed by tracing through an ordered sequence of nodes without passing through any node more than once).
- the **element constraints**, one for each device, which for the time being obey **Ohm's law**.

In this problem, it **suffices with the KVL along three loops**. The only loop that is not obvious is the one that goes from one **ground to the other**, because the grounds are effectively the **same node**.

Once we assign the **direction** of each loop, the voltage is written with a **plus** if the loop is **traversed entering the positive mark** of the polarities already defined, and it is written with minus if the opposite is true.

The equations corresponding to the KVL along the three loops are then

$$\begin{aligned} -v_1 + v_2 + v_3 &= 0 \\ -v_3 + v_4 + v_5 &= 0 \\ \text{(between grounds)} \quad -v_3 + v_4 &= 0, \end{aligned}$$

and using the available values $v_1 = 3$ and $v_3 = 5$, we obtain

$$v_2 = -2\,, \qquad v_4 = 5\,, \qquad v_5 = 0\,.$$

(Note that we could have known right away that v_5=0 by the KVL around the loop that goes through this device and connects the two grounds.)

2.16 The KCL connection equations at each **node** give

$$\begin{aligned} \textbf{At node (A):} &\quad -i_1 - i_2 - i_3 = 0 \\ \textbf{At node (B):} &\quad i_3 - i_4 = 0 \\ \textbf{At node (C):} &\quad i_1 + i_2 + i_4 = 0, \end{aligned}$$

but we observe that the third equation **results from** adding the previous two with negative sign, so **one of the equations is redundant**. Any set of two is **independent** so you can get rid of any of the three (say the third).

Using the available information that $i_2 = -20mA$ and $i_4 = 10mA$, we get

$$i_1 = -10mA, \quad i_3 = 10mA.$$

2.21 The number of **independent KVL connection equations** is $E - N + 1 = 6 - 4 + 1 = 3$. The equations are

$$\begin{aligned} -v_2 + v_6 - v_4 &= 0 \\ -v_1 + v_2 + v_3 &= 0 \\ -v_3 + v_4 + v_5 &= 0\,. \end{aligned}$$

Using that $v_2 = 10V$, $v_4 = 5V$, and $v_5 = 15V$, we get

$$v_1 = 30\,, \qquad v_3 = 20\,, \qquad v_6 = 15\,.$$

(You can compute them by inspection if you substitute them in the right order.)

2.23

- As you know, under the **passive sign convention**, the reference direction of the current through a device points towards the positive terminal. Now, the number of **independent KVL connection equations** is $E - N + 1 = 5 - 3 + 1 = 3$, and one of the many possible choices gives

$$\begin{aligned} -v_3 - v_4 &= 0 \\ -v_5 - v_2 + v_4 &= 0 \\ v_2 + v_1 &= 0\,. \end{aligned}$$

- If $v_4 = 0$, we can deduce that

$$v_3 = 0, \quad v_2 = -v_5 = -v_1\,.$$