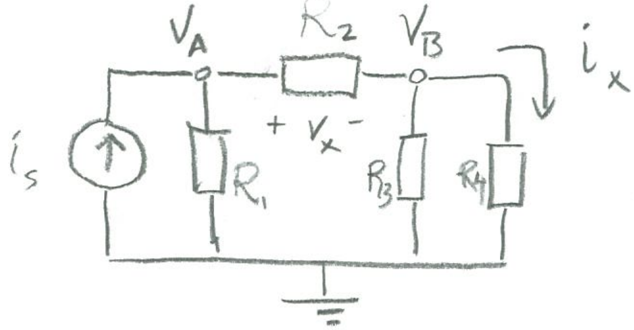
SOLUTIONS HW 3

$\boxed{3.2}$



A) $$\underbrace{(V_A - 0)\frac{1}{R_1} + (V_A - V_B)\frac{1}{R_2}}_{\text{leaving currents}} = \underbrace{i_s}_{\text{entering currents}}$$

B) $$(V_B - 0)\frac{1}{R_3} + (V_B - 0)\frac{1}{R_4} + (V_B - V_A)\frac{1}{R_2} = 0$$

System in form $\boxed{Ax = b}$ with A) and B)

$$\begin{bmatrix} \frac{1}{R_1} + \frac{1}{R_2} & -\frac{1}{R_2} \\ -\frac{1}{R_2} & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \end{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} = \begin{bmatrix} i_s \\ 0 \end{bmatrix}$$

Using Cramer, $$V_A = \frac{\begin{vmatrix} i_s & -\frac{1}{R_2} \\ 0 & \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \end{vmatrix}}{|A|} = \frac{\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) i_s}{|A|}$$

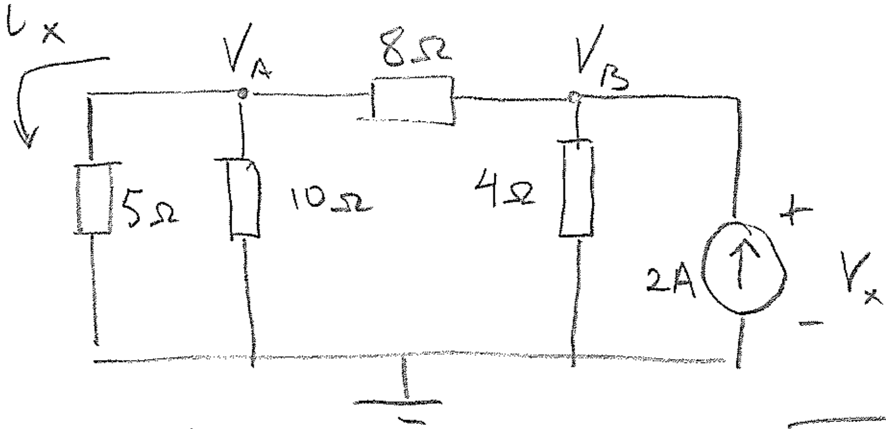
where $$|A| = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) - \frac{1}{R_2^2}$$

and $$V_B = \frac{\begin{vmatrix} \frac{1}{R_1} + \frac{1}{R_2} & i_s \\ -\frac{1}{R_2} & 0 \end{vmatrix}}{|A|} = \boxed{\frac{i_s}{|A| R_2}}$$

Then,

$$i_x = \frac{V_B}{R_4} = \frac{i_s}{|A| R_2 R_4}, \quad V_x = V_A - V_B = \frac{i_s}{|A|}\left(\frac{1}{R_3} + \frac{1}{R_4}\right)$$

3.3



This circuit is the same as in 3.2 (except the definition of V_x) with the (nonunique) $V_B = V_A^{\boxed{3.2}}$, $V_A = V_B^{\boxed{3.2}}$, $i_s = 2A$, $R_1 = 4\Omega$, $R_2 = 8\Omega$, $R_3 = 10\Omega$, $R_4 = 5\Omega$. Therefore, using the previous expression for $Ax = b$, we have

$$|A| = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) - \frac{1}{R_2^2} \cong 0.144$$

$$V_A = V_B^{\boxed{3.2}} = \frac{i_s}{|A| R_2} \simeq \frac{2A}{0.144 \cdot 8\Omega} \simeq \boxed{1.73\,V}$$

$$V_B = V_A^{\boxed{3.2}} = \frac{\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right) i_s}{|A|} \simeq \frac{0.425 \cdot 2}{0.144} = \simeq \boxed{5.9\,V}$$

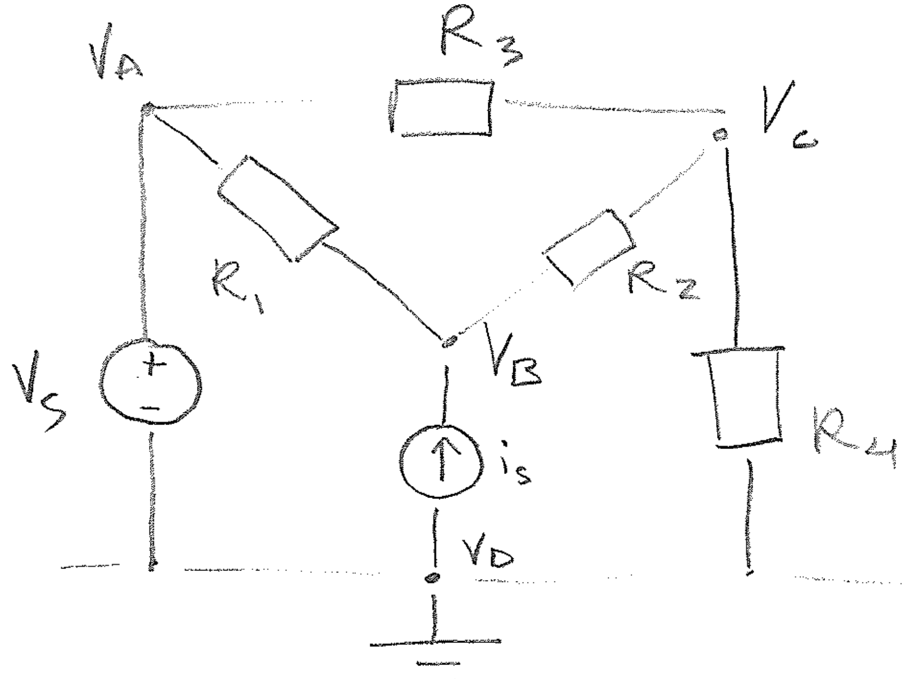
$$i_x = \frac{V_A}{R_4} = \frac{1.73\,V}{5\,\Omega} = \boxed{0.346\,A}$$

$$V_x = V_B = \boxed{5.9\,V}$$

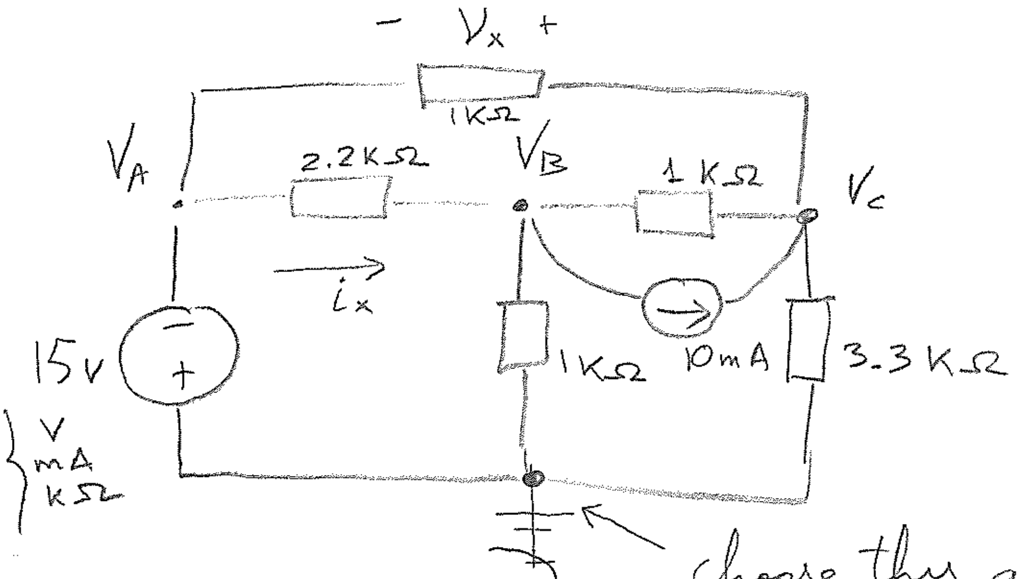
$\boxed{3.7}$

$$
\begin{cases}
V_A = V_S \\
\dfrac{V_B - V_A}{R_1} + \dfrac{V_B - V_C}{R_2} - I_S = 0 \\
\dfrac{V_C - V_A}{R_3} + \dfrac{V_C - V_B}{R_2} + \dfrac{V_C}{R_4} = 0 \\
V_D = 0
\end{cases}
$$

The circuit might be



$\boxed{3.8}$



choose this ground, for instance

We use units $\begin{cases} V \\ mA \\ K\Omega \end{cases}$

$$V_A = -15$$

$$\left.\begin{aligned} &\frac{V_B - V_A}{2.2} + \frac{V_B - V_C}{1} + \frac{V_B - 0}{1} = -10 \\ &\frac{V_C - V_B}{1} + \frac{V_C - 0}{3.3} + \frac{V_C - V_A}{1} = 10 \end{aligned}\right\} \Leftrightarrow \begin{bmatrix} \frac{1}{2.2}+1+1 & -1 \\ -1 & 1+\frac{1}{3.3}+1 \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -10-\frac{15}{2.2} \\ 10-15 \end{bmatrix}$$

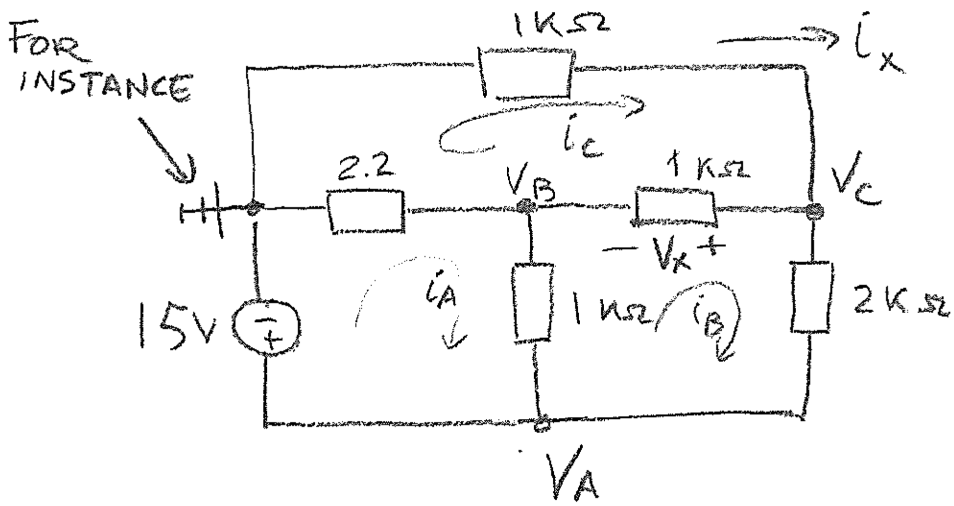
$$\Leftrightarrow \begin{bmatrix} 2.45 & -1 \\ -1 & 2.30 \end{bmatrix} \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -3.18 \\ 25 \end{bmatrix} \Rightarrow \begin{bmatrix} V_B \\ V_C \end{bmatrix} = \begin{bmatrix} -9.40\ V \\ -6.25\ V \end{bmatrix}$$

$$V_x = V_C - V_A = -6.25\ V - (-15V) = \boxed{8.75\ V}$$

$$i_x = \frac{V_A - V_B}{2.2} = \frac{-15-(-9.40)}{2.2} = \boxed{-2.55\ mA}$$

3-14

Units used V, KΩ, mA



$V_A = 15V$

(B) $\frac{V_B - 0}{2.2} + \frac{V_B - V_c}{1} + \frac{V_B - V_A}{1} = 0$

(C) $\frac{V_c - V_B}{1} + \frac{V_c - 0}{1} + \frac{V_c - V_A}{2} = 0$

So we get the linear system

$$\begin{bmatrix} \frac{1}{2.2} + 1 + 1 & -1 \\ -1 & 1+1+\frac{1}{2} \end{bmatrix} \begin{bmatrix} V_B \\ V_c \end{bmatrix} = \begin{bmatrix} 15 \\ \frac{15}{2} \end{bmatrix} \Rightarrow \begin{bmatrix} V_B \\ V_c \end{bmatrix} = \begin{bmatrix} 8.76 \text{ V} \\ 6.50 \text{ V} \end{bmatrix}$$

$i_c = \frac{0 - V_c}{1} = \boxed{-6.50 mA}$

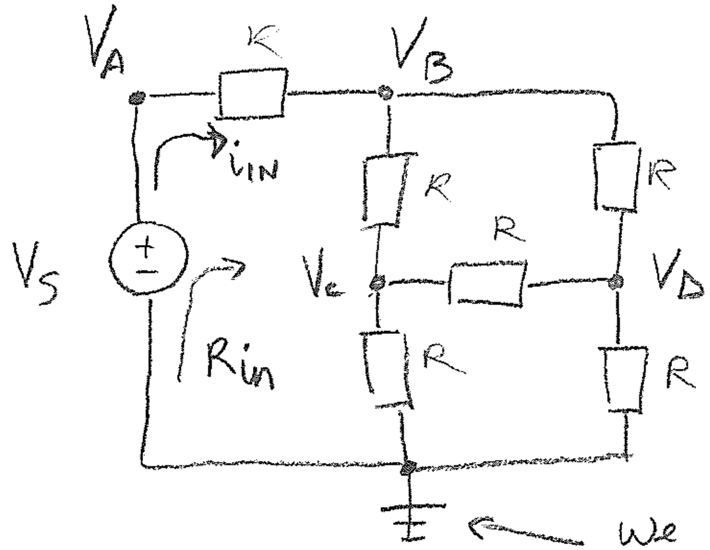
$i_B = \frac{V_c - V_A}{2} = \frac{6.50 - 15}{2} = \boxed{-4.25 mA}$

$i_A = \frac{V_B - V_A}{1} + i_B = \frac{8.76 - 15}{1} + -4.25 = -10.49 m$

$i_x = i_c = -6.50 mA$

$V_x = V_c - V_B = 6.50 - 8.76 = -2.26 \text{ V}$

$\boxed{3.23}$



← we can choose the ground here

$V_A = V_S$

$$\begin{bmatrix} \frac{3}{R} & -\frac{1}{R} & -\frac{1}{R} \\ -\frac{1}{R} & \frac{3}{R} & -\frac{1}{R} \\ -\frac{1}{R} & -\frac{1}{R} & \frac{3}{R} \end{bmatrix} \begin{bmatrix} V_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} \frac{V_S}{R} \\ 0 \\ 0 \end{bmatrix}$$

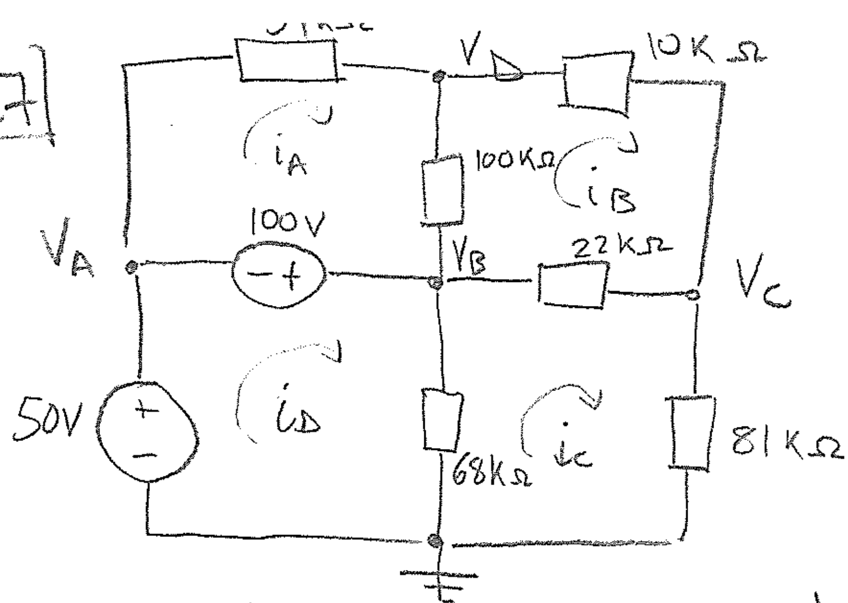
$$\Longrightarrow \begin{bmatrix} V_B \\ V_C \\ V_D \end{bmatrix} = \begin{bmatrix} V_S/2 \\ V_S/4 \\ V_S/4 \end{bmatrix}$$

To compute the equivalent resistance, we can compute the current through V_S and then use Ohm's law:

$$i_N = \frac{V_A - V_B}{R} = \frac{V_S - \frac{V_S}{2}}{R} = \frac{V_S}{2R}$$

$$R_{IN} = \frac{V_S}{i_N} = \frac{\cancel{V_S}}{\frac{\cancel{V_S}}{2R}} = \boxed{2R}$$

3.27



Node analysis (using the units V, KΩ, mA)

(A) $\boxed{V_A = 50\ V}$

(B) $V_B - V_A = 100\ V \Rightarrow \boxed{V_B = 100 + 50 = 150\ V}$

(C) $$\frac{V_C - V_B}{22} + \frac{V_C - 0}{81} + \frac{V_C - V_D}{10} = 0$$

(D) $$\frac{V_D - V_A}{39} + \frac{V_D - V_B}{100} + \frac{V_D - V_C}{10} = 0$$

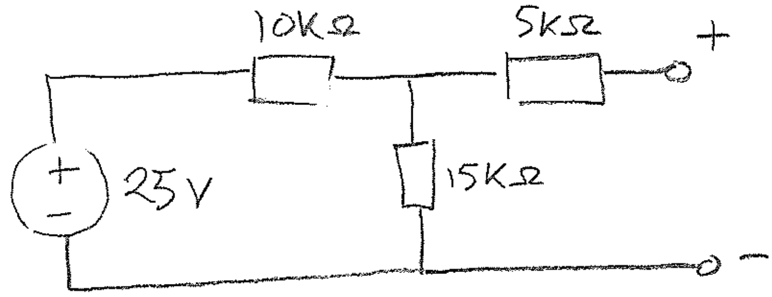
$$\begin{bmatrix} \frac{1}{22} + \frac{1}{81} + \frac{1}{10} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{1}{39} + \frac{1}{100} + \frac{1}{10} \end{bmatrix} \begin{bmatrix} V_C \\ V_D \end{bmatrix} = \begin{bmatrix} \frac{150}{22} \\ \frac{50}{39} + \frac{150}{100} \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} V_C \\ V_D \end{bmatrix} \simeq \begin{bmatrix} 105.49V \\ 98.28V \end{bmatrix}$$

$\boxed{i_A = \frac{V_A - V_D}{39\,K\Omega} \simeq -1.238mA}$ $\boxed{i_B = \frac{V_D - V_C}{10K\Omega} \simeq -0.721\,mA}$

$i_C = \frac{V_C}{81\,K\Omega} \simeq 1.30\ mA$ $\frac{V_B}{68} = i_D - i_C;\ i_D = i_C + \frac{V_B}{68} \simeq 3.51m$

$\boxed{3.47}$



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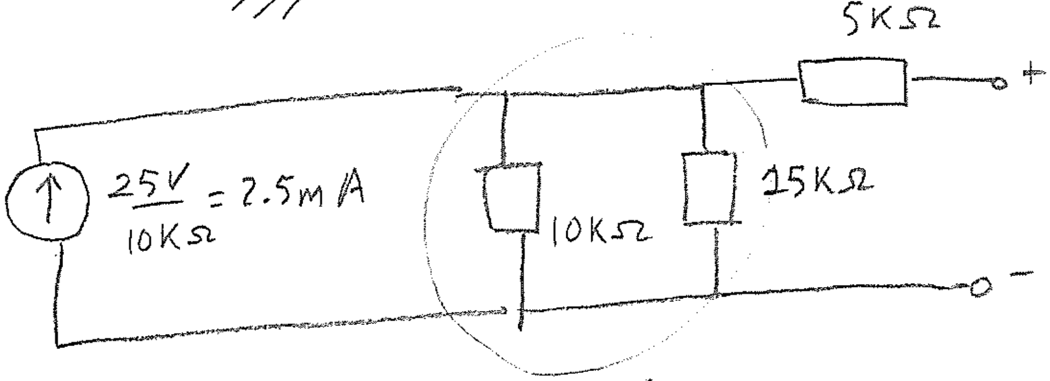

$$10 \| 15 = \frac{10 \cdot 15}{10+15} = 6 K\Omega$$

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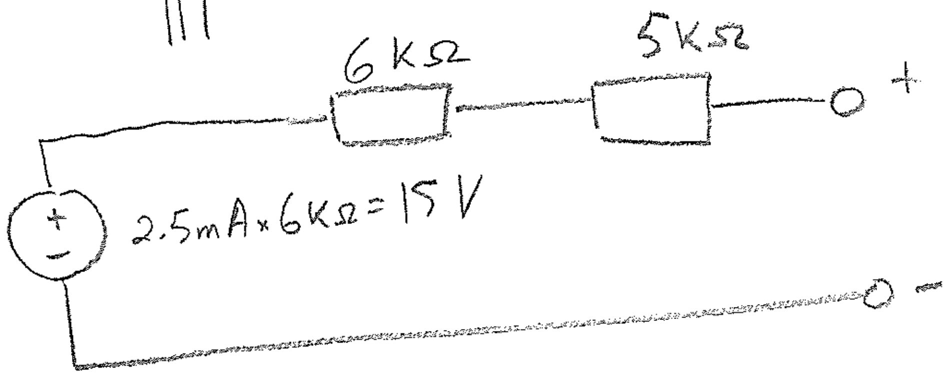
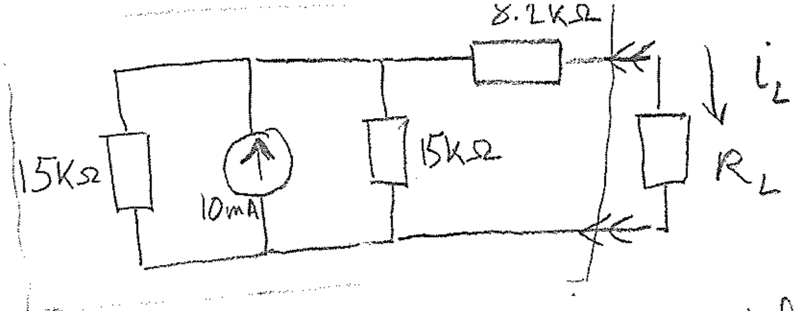
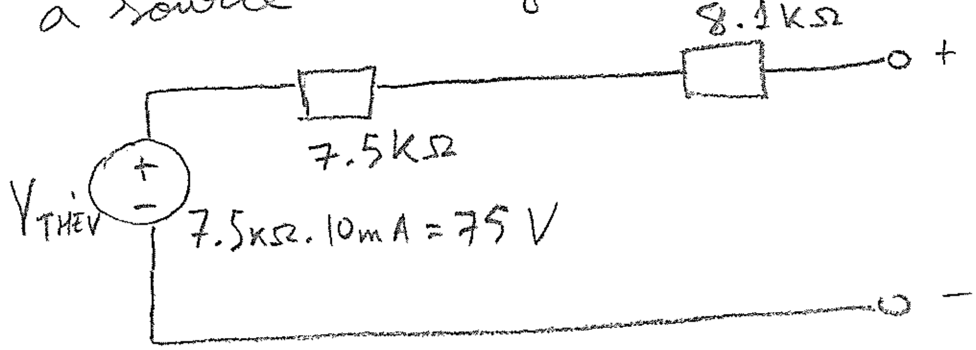


$$V_T = 15V \; ; \quad R_T = 11 K\Omega \; ; \quad i_N = \frac{V_T}{R_T} = \frac{15V}{11K\Omega} = 1.36m$$

3.52

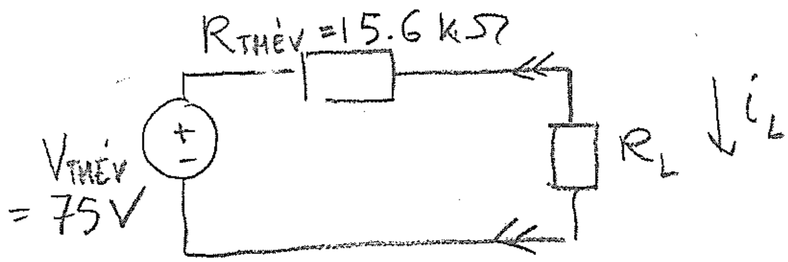


We compute the Thévenin equivalent circuit by combining the parallel resistances an computing a source transformation



$\boxed{V_{THEV} = 75\,V}$

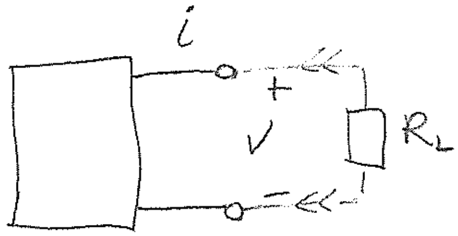
$R_{THEV} = 7.5 + 8.1$
$= 15.6\,k\Omega$



$$i_L = \frac{V_{THEV}}{R_{THEV} + R_L} = \frac{75V}{15.6k\Omega + R_L}$$

R_L	i_L
$4.7k\Omega$	$\frac{75}{15.6+4.7} \simeq 3.70\,mA$
$15\,k\Omega$	$\frac{75}{15.6+15} \simeq 2.45\,mA$
$68\,k\Omega$	$\frac{75}{15.6+68} \simeq 0.89\,mA$

$\boxed{3.59}$



$$5v + 500\,i = 100$$

$$\left\{ \begin{array}{l} \boxed{v = \frac{100}{5} - \frac{500}{5} i = 20 - 100\,i} \\ \& \\ \boxed{i = \frac{v}{R_L}} \quad \text{(Ohm's law for the load)} \end{array} \right.$$

$$\Downarrow$$

$$v = 20 - 100\left(\frac{v}{R_L}\right);$$

$$v = \frac{20}{1 + \frac{100}{R_L}} = \frac{20}{1 + \frac{100}{500}} = \frac{20}{1 + \frac{1}{5}} = \frac{20.5}{6}$$

$$\doteq \boxed{16.67\text{ V}}$$