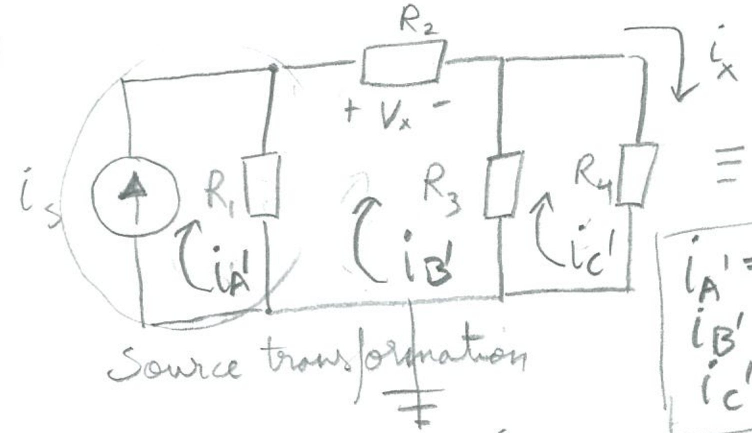
SOLUTIONS HW4

$\boxed{3.2}$



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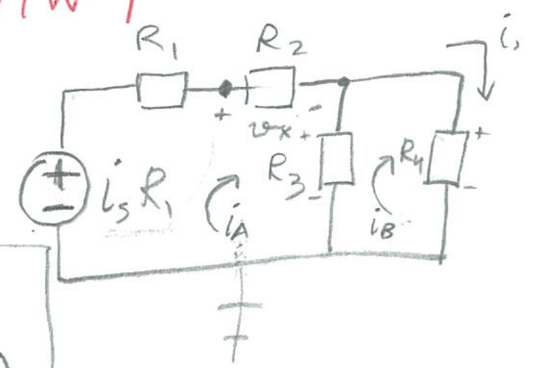


$$\boxed{\begin{aligned} i_A' &= i_s \\ i_B' &= i_A \\ i_C' &= i_B \end{aligned}}$$

$$\begin{cases} -i_s R_1 + i_A R_1 + i_A R_2 + (i_A - i_B) R_3 \\ -(i_A - i_B) R_3 + i_B R_4 = 0 \end{cases} \iff \underbrace{\begin{bmatrix} R_1+R_2+R_3 & -R_3 \\ -R_3 & R_3+R_4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_A \\ i_B \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} i_s R_1 \\ 0 \end{bmatrix}}_{b}$$

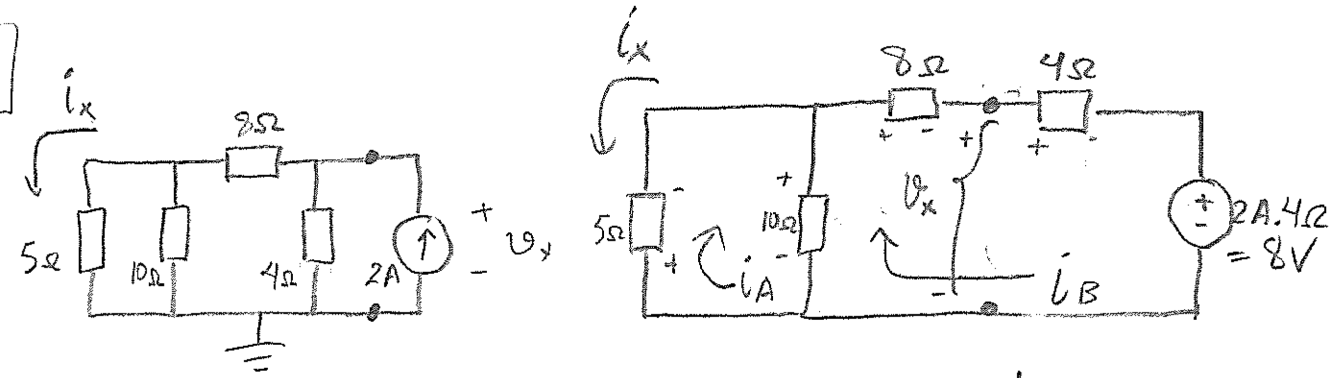
$$i_A = \frac{\begin{vmatrix} i_s R_1 & -R_3 \\ 0 & R_3+R_4 \end{vmatrix}}{|A|} = \boxed{\frac{i_s R_1 (R_3+R_4)}{|A|}}$$

$$i_B = \frac{\begin{vmatrix} R_1+R_2+R_3 & i_s R_1 \\ -R_3 & 0 \end{vmatrix}}{|A|} = \boxed{\frac{i_s R_1 R_3}{|A|}}$$

$$\underline{v_x} \overset{\text{(ohm's law)}}{=} i_A R_2 = \boxed{\frac{i_s R_1 R_2 (R_3+R_4)}{|A|}}$$

$$\underline{i_x} = i_B = \boxed{\frac{i_s R_1 R_3}{|A|}}$$

3.3



We use units Ω, A, V

Mesh (A): $5i_A + 10(i_A - i_B) = 0$

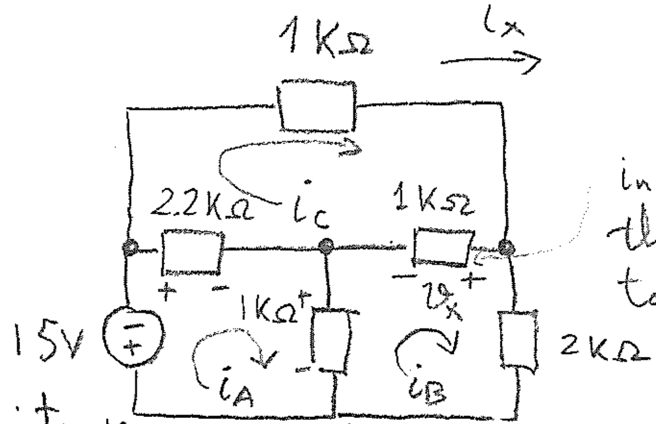
(B): $-10(i_A - i_B) + 8i_B + 4i_B + 8 = 0$

$$\Leftrightarrow \underbrace{\begin{bmatrix} +5+10 & -10 \\ -10 & 10+8+4 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_A \\ i_B \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 0 \\ -8 \end{bmatrix}}_{b} \Rightarrow \begin{array}{l} i_A = -0.347\text{ A} \\ i_B = -0.52\text{ A} \end{array}$$

$$i_x = -i_A = \boxed{0.347\text{ A}}$$

$$v_x = i_B \cdot 4\Omega + 8\text{V} = -0.52 \cdot 4 + 8 = \boxed{5.92\text{ V}}$$

3.14



in the mesh analysis the polarity "assigned" to the device is the opposite

Using units KΩ, mA, V

Mesh (A): $15 + 2.2(i_A - i_C) + 1(i_A - i_B) = 0$

" (B): $-1(i_A - i_B) + 1(i_B - i_C) + 2\,i_B = 0$

" (C): $-2.2(i_A - i_C) + 1\,i_C - 1(i_B - i_C) = 0$

$$\Longleftrightarrow \begin{bmatrix} 2.2+1 & -1 & -2.2 \\ -1 & 1+1+2 & -1 \\ -2.2 & -1 & 2.2+1+1 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix} = \begin{bmatrix} -15 \\ 0 \\ 0 \end{bmatrix}$$

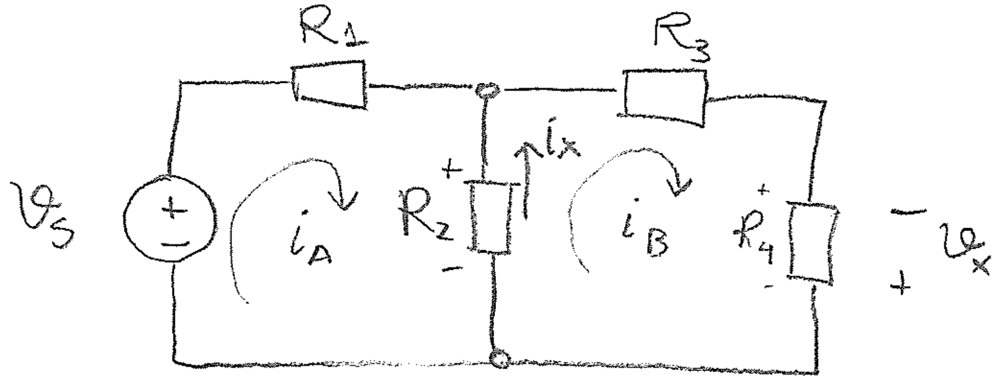
$\Rightarrow \quad i_A = -10.48\,mA \quad i_B = -4.25\,mA \quad i_C = -6.50\,mA$

$i_x = i_C = \boxed{-6.50\,mA}$

$v_x = 1\,K\Omega\,(i_C - i_B) = 1\,K\Omega\,(-6.5 - (-4.25))\,mA =$

$= \boxed{-2.25\,V}$

$\boxed{3.16}$



Mesh (A) $\quad -v_s + R_1 i_A + R_2(i_A - i_B) = 0$

" (B) $\quad -R_2(i_A - i_B) + R_3 i_B + R_4 i_B = 0$

$$\Longleftrightarrow \begin{bmatrix} R_1+R_2 & -R_2 \\ -R_2 & R_2+R_3+R_4 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \end{bmatrix} = \begin{bmatrix} v_s \\ 0 \end{bmatrix}$$

$$(A \quad x \quad = \quad b)$$

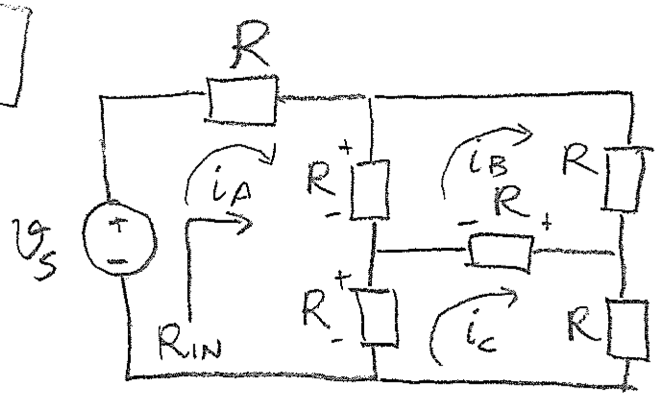
$$i_A = \frac{\begin{vmatrix} v_s & -R_2 \\ 0 & R_2+R_3+R_4 \end{vmatrix}}{|A|} = \frac{v_s(R_2+R_3+R_4)}{|A|}$$

$$i_B = \frac{\begin{vmatrix} R_1+R_2 & v_s \\ -R_2 & 0 \end{vmatrix}}{|A|} = \frac{v_s R_2}{|A|}$$

$$i_x = i_B - i_A = -\frac{v_s}{|A|}(R_3+R_4)$$

$$v_x = -R_4 i_B = \frac{v_s R_2 R_4}{|A|}$$

3.23



(A) $$-v_s + R i_A + R(i_A - i_B) + R(i_A - i_C) = 0$$

(B) $$-R(i_A - i_B) + R i_B + R(i_B - i_C) = 0$$

(C) $$-R(i_A - i_C) - R(i_B - i_C) + R i_C = 0$$

$$\Leftrightarrow \underbrace{\begin{bmatrix} 3R & -R & -R \\ -R & 3R & -R \\ -R & -R & 3R \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} i_A \\ i_B \\ i_C \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} v_s \\ 0 \\ 0 \end{bmatrix}}_{b}$$

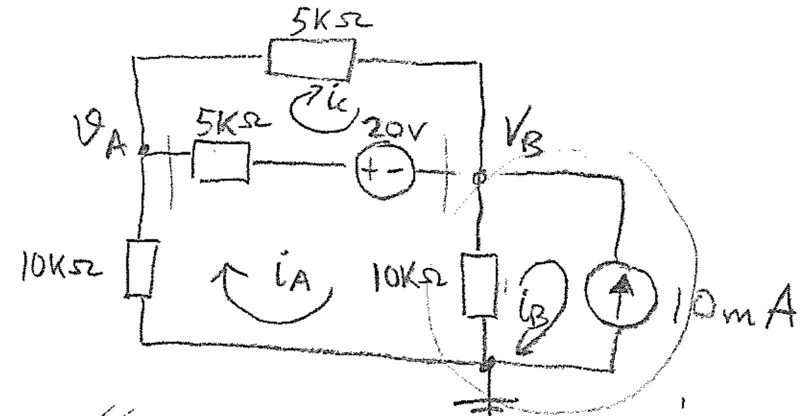
$$\Rightarrow \quad i_A = \frac{V_s}{2R} \qquad i_B = \frac{V_s}{4R} \qquad i_C = \frac{V_s}{4R}$$

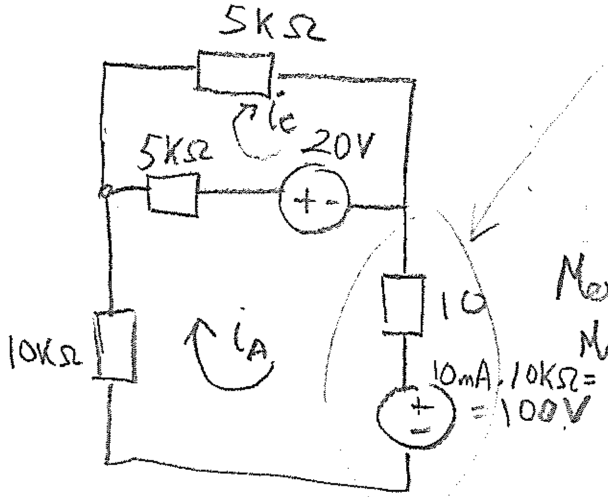
Ohm's law

$$R_{IN} = \frac{V_s}{i_A} = \frac{\cancel{V_s}}{\cancel{V_s}/2R} = \boxed{2R}$$

3.26

a)





Mesh-current equations

i_A remains the same and $\boxed{i_B = -10\,mA}$ *

Mesh (c) $5 i_c - 20 + 5(i_c - i_A) = 0$

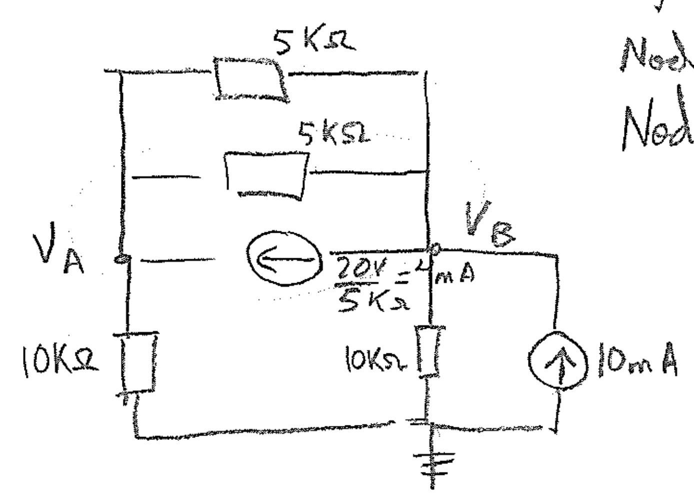
Mesh (A) $10 i_A + 5(i_c - i_A) + 20 + 10 i_A + 100 = 0$

Systems of same size

$$\Leftrightarrow \begin{bmatrix} 5+5 & -5 \\ -5 & 10+5+10 \end{bmatrix} \begin{bmatrix} i_c \\ i_A \end{bmatrix} = \begin{bmatrix} 20 \\ -20-100 \end{bmatrix}$$

$$\left(i_A = -4.89\,mA \qquad i_B = -10\,mA^{*} \qquad i_c = -0.44\,mA \right)$$

b) To formulate the node-voltage equations, we compute another source transformation

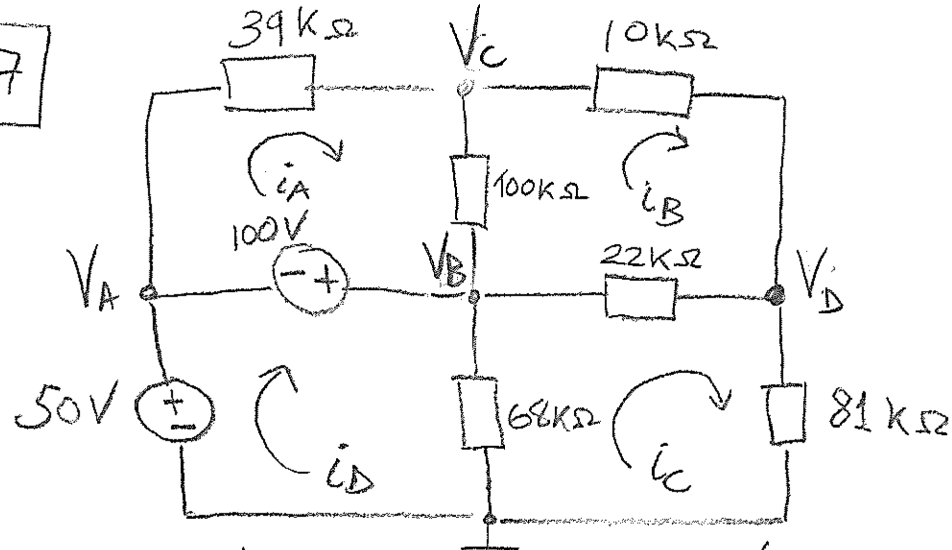


Node (A) $\frac{V_A}{10} + \frac{(V_A - V_B)}{5} + \frac{(V_A - V_B)}{5} = 4$

Node (B) $\frac{V_B}{10} + \frac{V_B - V_A}{5} + \frac{V_B - V_A}{5} = 10 - 4$

$$\Leftrightarrow \begin{bmatrix} \frac{1}{10} + \frac{2}{5} & -\frac{2}{5} \\ -\frac{2}{5} & \frac{1}{10} + \frac{2}{5} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

$\boxed{3.27}$



using units KΩ, mA, V ; $1\,mA = \frac{1V}{1K\Omega}$

Mesh (A): $39\,i_A + 100(i_A - i_B) + 100$

(B): $-100(i_A - i_B) + 10\,i_B + 22(i_B - i_c) = 0$

(C): $68(i_c - i_D) - 22(i_B - i_c) + 81\,i_c = 0$

(D): $-50 - 100 - 68(i_c - i_D) = 0$

$$\Leftrightarrow \begin{bmatrix} 39+100 & -100 & 0 & 0 \\ -100 & 100+10+22 & -22 & 0 \\ 0 & -22 & 68+22+81 & -68 \\ 0 & 0 & -68 & 68 \end{bmatrix} \begin{bmatrix} i_A \\ i_B \\ i_c \\ i_D \end{bmatrix} = \begin{bmatrix} -100 \\ 0 \\ 0 \\ 100+50 \end{bmatrix}$$

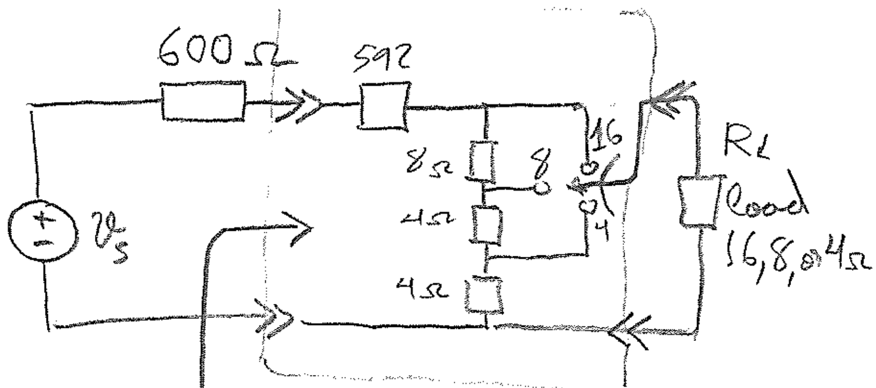
$$(A \quad x = b)$$

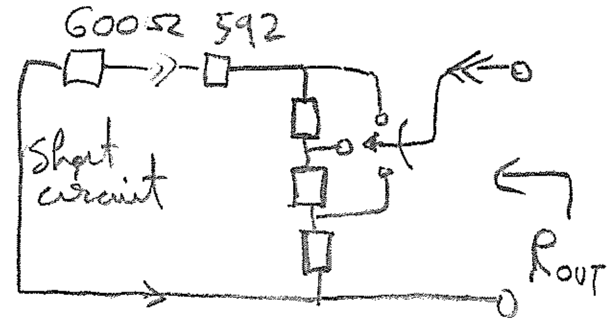
$\boxed{i_A = -1.23\,mA}$ $\boxed{i_B = -0.72\,mA}$ $\boxed{i_c = 1.30\,mA}$ $\boxed{i_D = 3.50\,mA}$

(In hw 3 we saw that we could determine right away the values of V_A and V_B, so we just needed to solve a system of 2 equations for V_c and V_D.)

3.91

NOTE FOR GRADER: be flexible; the statement was hard to interpret.





ROUT (from the perspective of the load with a short circuit where the source was)

$R_L = 16\Omega \Rightarrow R_{IN} = 592 + \left((8+4+4) \| R_L\right)$
(WITH SWITCH UP)

$= 592 + (16 \| 16) = 600 \in (600 - \underset{2\%(600)}{12}, 600+12)$

$R_{OUT} = 1192 \| (8+4+4) = \frac{1192 \cdot 16}{1208} = 15.78 \in (16 - \underbrace{0.32}_{2\%(16)}, 16+0.32)$

$R_L = 8\Omega \Rightarrow R_{IN} = 592 + 8 + \left((4+4) \| R_L\right)$
(WITH SWITCH IN THE MIDDLE)

$= 592 + 8 + 4 = 604 \in (600 - \underset{2\%600}{12}, 600+12)$

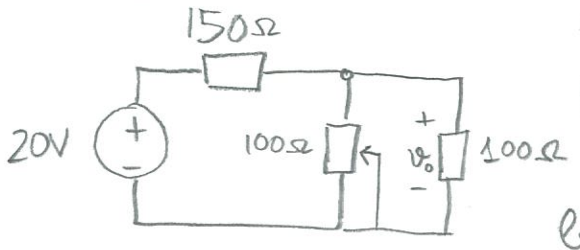
$R_{OUT} = (1192 + 8) \| (4+4) = \frac{1200 \times 8}{1208} = 7.94 \in (8 - 0.16, 8 + \underset{2\%(8)}{0.16})$

$R_L = 4\Omega \Rightarrow R_{IN} = 592 + 8 + 4 + (4 \| R_L) = 606 \in (600-12, 600+12)$
(WITH SWITCH DOWN)

$R_{OUT} = (1192 + 8 + 4) \| 4 = \frac{1204 \times 4}{1208} \simeq 3.98 \in (4 - \underbrace{0.08}_{2\%(4)}, 4+0.0$

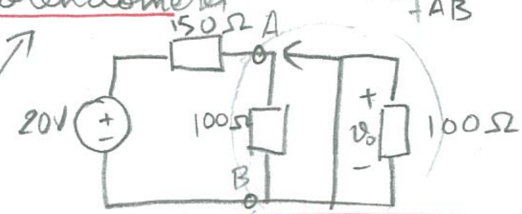
3.95

CIRCUIT 1



Position of the potenciometer

a)



$Req_{AB} = 0$

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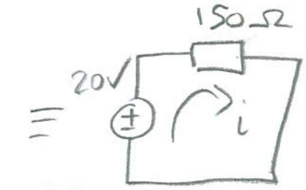
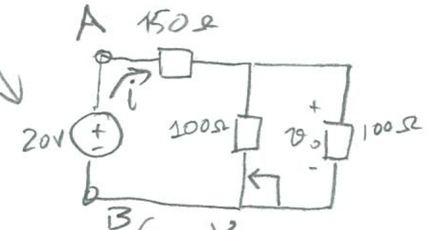


$\Rightarrow \boxed{v_o = 0\,V}$

$\boxed{P = iV = \frac{V^2}{R} = \frac{20^2}{150}} = \boxed{2.6\,W}$

b)



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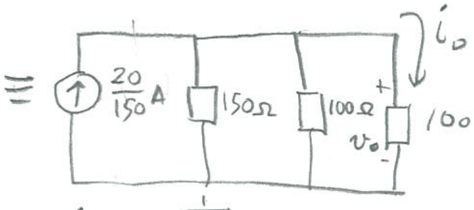


$\boxed{P = iV = \frac{V^2}{Req_{AB}}} = \frac{(20)^2}{(100 \| 100) + 150\,\Omega} = \frac{400}{200} = \boxed{2\,W}$

(power delivered by source)

$i_o = \frac{\frac{1}{100}}{\frac{1}{150} + \frac{1}{100} + \frac{1}{100}} \frac{20}{150}$

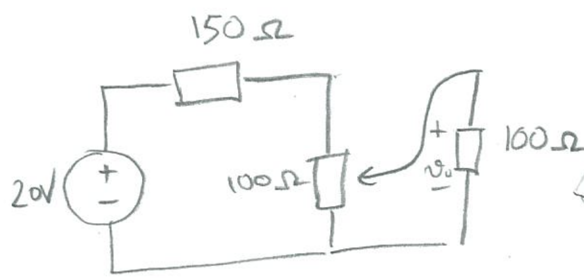
$= \frac{1}{\frac{10}{15} + 2} \frac{20}{15\cancel{0}} = \frac{2}{10+30}$

$= \frac{1}{20} = 0.05\,A$

$\boxed{v_o = i_o 100\,\Omega = 5\,V}$

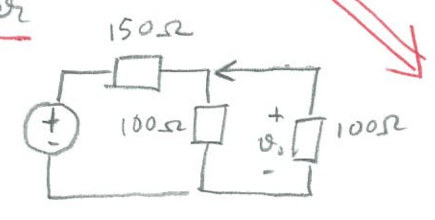
SAME TOPOLOGY IN THAT POSITION

CIRCUIT 2



Position of the potenciometer

a)

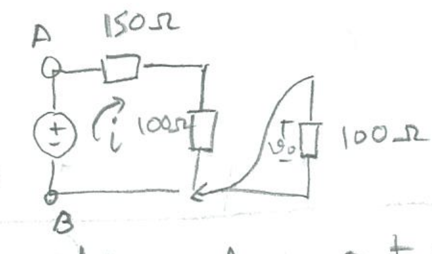


Same topology (as 1.b)

$\boxed{v_o = 5\,V}$

$\boxed{P = 2\,W}$

b)



$\boxed{v_o = 0\,V}$

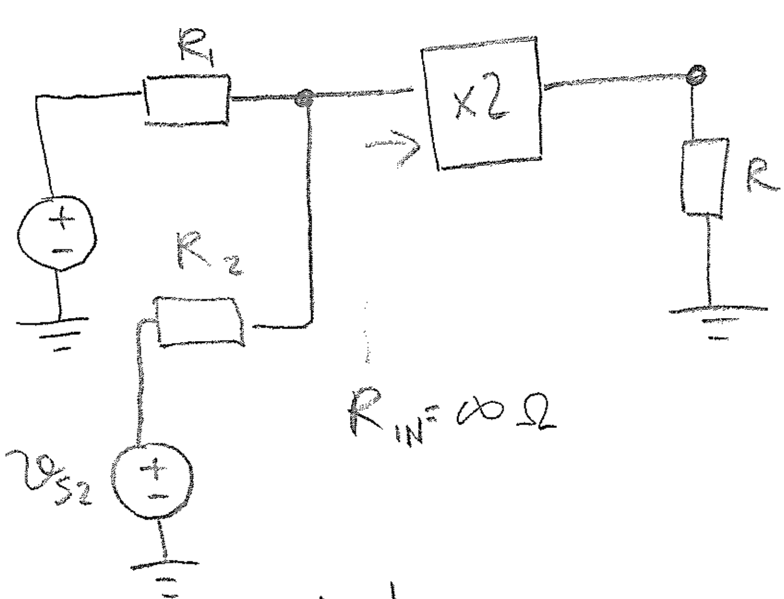
$\boxed{P = iV = \frac{V^2}{Req_{AB}}}$

$= \frac{20^2}{250} = \boxed{1.6\,W}$

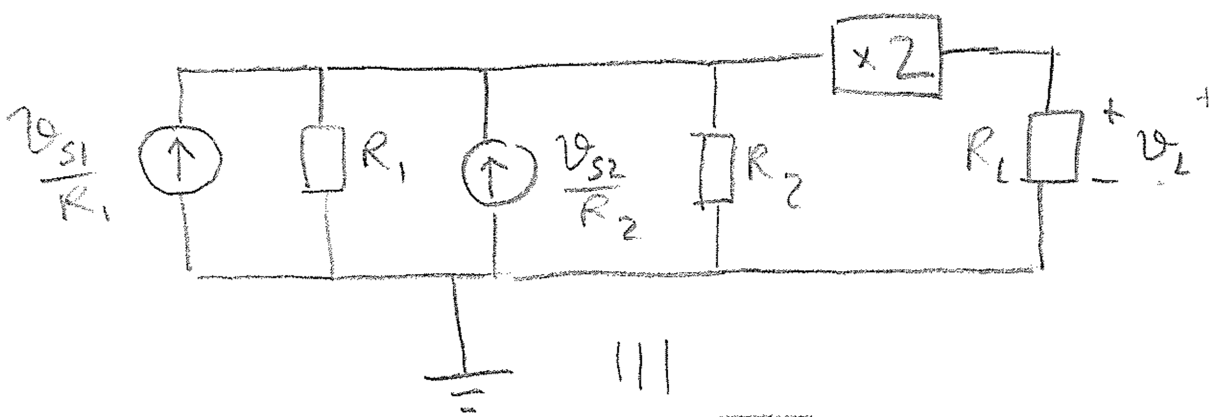
Both circuits drive the voltage from 0 to 5.
However, when $v_o = 0$ circuit 1 delivers 2.6 W
while circuit 2 delivers 1.6 W.

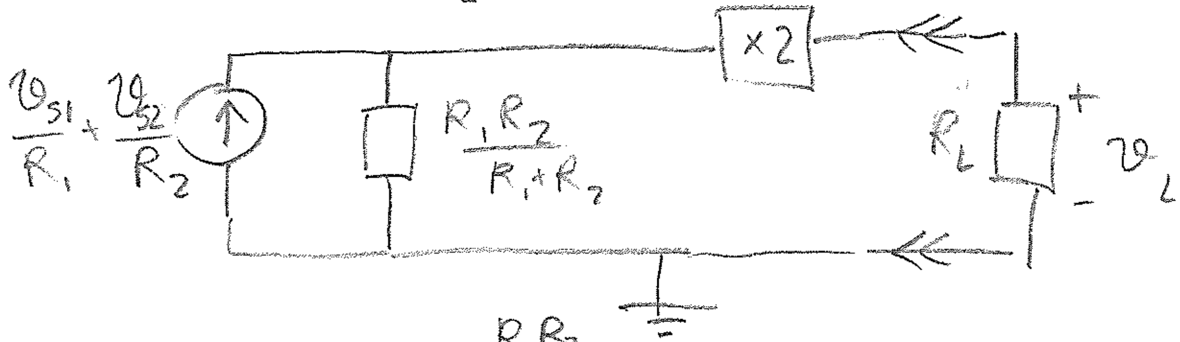
$\boxed{\text{CIRCUIT 2 consumes } \underline{\text{less}} \text{ to deliver the same voltage } v_o}$

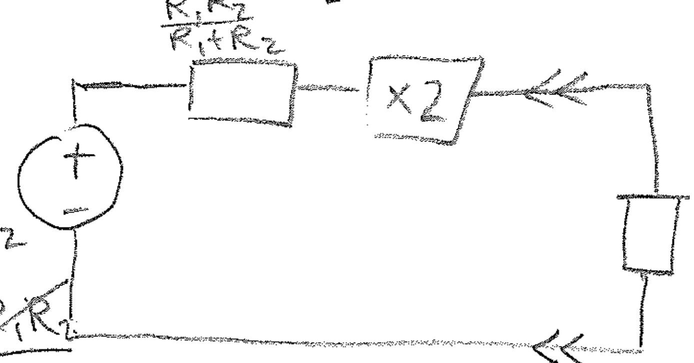
3.99



Computing equivalent sources







$$\left(\frac{v_{s1}}{R_1}+\frac{v_{s2}}{R_2}\right)\frac{R_1 R_2}{R_1+R_2}$$

$$= \frac{v_{s1}\cdot R_2 + v_{s2}\cdot R_1}{\cancel{R_1 R_2}}\cdot\frac{\cancel{R_1 R_2}}{R_1+R_2}$$

$$v_L = 2\left(\frac{v_{s1}R_2 + v_{s2}R_1}{R_1+R_2}\right)$$