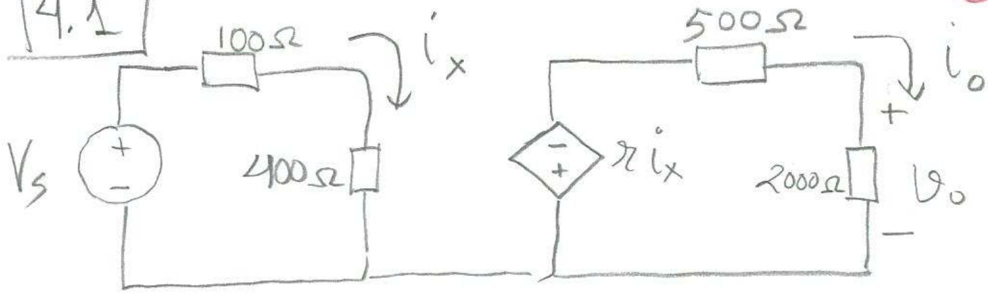
SOLUTIONS HW3

4.1



$r = 5K\Omega$

$\frac{v_o}{v_s}$? $\frac{i_o}{i_x}$?

$$i_x \overset{(1)}{=} \frac{V_s}{(0.1+0.4)K\Omega} \; ; \quad \boxed{v_o} = -\frac{2K\Omega}{0.5K\Omega+2K\Omega}\, r \cdot i_x =$$

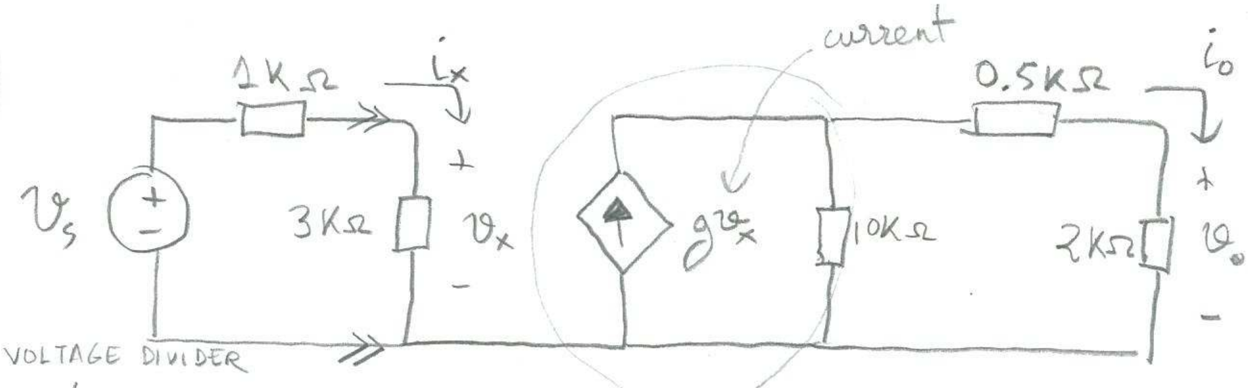
voltage divider

$$= -\frac{2\cancel{K\Omega}}{2.5\cancel{K\Omega}} \cdot 5\cancel{K\Omega} \cdot \frac{V_s}{0.5\cancel{K\Omega}} = \boxed{-8\,V_s}$$

The minus sign in the voltage divider comes from the polarity of the voltage dependent source

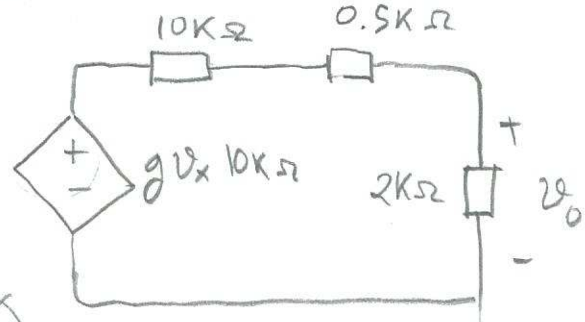
$$\boxed{i_o} = \frac{v_o}{2K\Omega} = \frac{-8}{2K\Omega} V_s \overset{(1)}{=} \frac{-8}{2\cancel{K\Omega}} \cdot 0.5\cancel{K\Omega}\, i_x = \boxed{-2\, i_x}$$

4.3



VOLTAGE DIVIDER

$$v_x \overset{\downarrow}{=} \frac{3}{1+3} v_s \underset{(2)}{=} 0.75 v_s$$



VOLTAGE DIVIDER

$$\boxed{v_o \overset{\downarrow}{=} \frac{2}{10+0.5+2} g v_x 10K\Omega}$$

cancel out

$$= \frac{2}{12.5} 2 \cdot \cancel{10^{-3} \frac{1}{\Omega}} 0.75 V_s \cdot 10 \cancel{K\Omega}$$

$$\overset{(1)}{=} \boxed{2.4 V_s}$$

(we use that $10^{-3} S = \frac{1}{10^3 \Omega} = \frac{1}{K\Omega}$)

$$\boxed{i_o} = \frac{v_o}{2K\Omega} \overset{(1)}{=} \frac{2.4 V_s}{2K\Omega} \overset{(2)}{=} \frac{2.4 \times \left(\frac{v_x}{0.75}\right)}{2K\Omega} = \frac{2.4}{2\cancel{K\Omega} \times 0.75} \cdot 3\cancel{K\Omega}\, i_x$$

$$i_x = \frac{v_x}{3K\Omega}$$

$$= \frac{2.4 \times 3}{2 \times 0.75} i_x = \boxed{4.8\, i_x}$$

$$\boxed{P_{\text{delivered by source}}} = v_s \cdot i_x = v_s \cdot \frac{i_o}{4.8} = v_s \cdot \frac{v_o / 2K\Omega}{4.8} \overset{(2)}{=} \frac{v_s\, 2.4\, v_s}{4.8 \times 2K\Omega} =$$

$v_s = 5V$

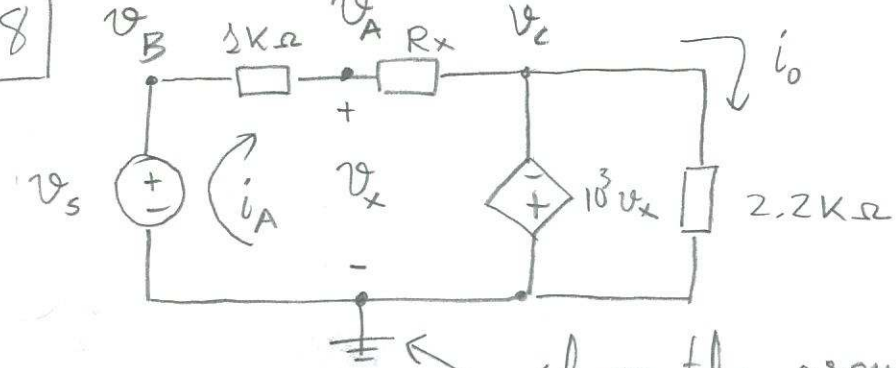
$$= \boxed{6.25 mV}$$

$$\boxed{P_{\text{consumed by } 2K\Omega \text{ load}}} = i_o v_o = \left(\frac{v_o}{2K\Omega}\right) \cdot v_o = \frac{v_o^2}{2K\Omega} \overset{(2)}{=} \frac{(2.4)^2 v_s^2}{2K\Omega}$$

$$= \boxed{72 mV}$$

$v_s = 5V$

4.8



choose the ground here because the $\boxed{v_B = v_s}$, $\boxed{v_c = -10^3 v_x}$ and $\boxed{v_x = v_A}$

Units $K\Omega$, mA, V ✓

The node equation at A is

$$\frac{v_A - v_B}{1} + \frac{v_A - v_c}{R_x} = 0 \iff \begin{bmatrix} \frac{1}{1K\Omega} + \frac{1}{R_x} & \frac{-1}{1K\Omega} & -\frac{1}{R_x} \end{bmatrix} \begin{bmatrix} v_A \\ v_B \\ v_c \end{bmatrix} = 0$$

and substituting the values in terms of v_s and v_x, we have

$$\left(\frac{1}{k\Omega} + \frac{1}{R_x}\right) v_x - \frac{v_s}{1K\Omega} - \frac{1}{R_x}\left(-10^3 v_x\right) = 0$$

$$v_x \left(\frac{1}{1K\Omega} + \frac{1}{R_x} + \frac{10^3}{R_x}\right) = \frac{v_s}{1K\Omega}$$

$$v_x \simeq \frac{v_s}{\frac{1001}{R_x} + \frac{1}{K\Omega}} = \frac{R_x v_s}{1001 + R_x}$$

$$\boxed{i_o = \frac{v_c - 0}{2.2} = \frac{-10^3 v_x}{2.2} = \frac{-10^3 R_x v_s}{2.2K\Omega\,(1.001 \times 10^3 + R_x)}}$$

$$\frac{i_o}{V_s} = -\frac{R_x}{2.2(1001 \times 10^6 + R_x)}$$

because I am using $K\Omega$ for R_x.

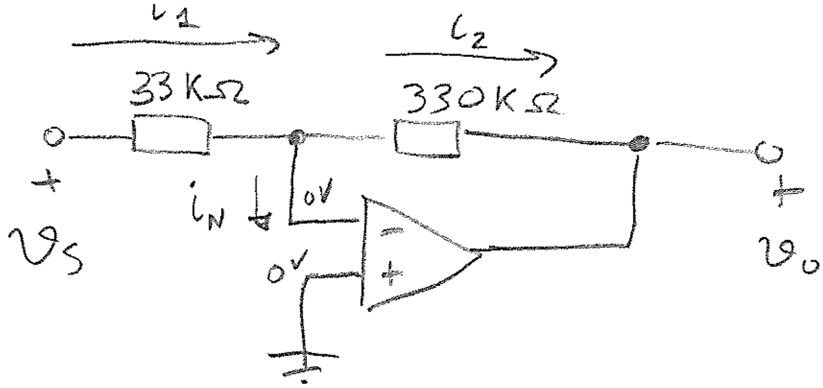
If we want the gain to be $-0.227 \frac{A}{V}$ $\left(-0.227 \frac{1}{\Omega}\right)$

we impose $\boxed{\frac{-10^3 R_x}{2.2 \times 10^3 + 2.2 R_x} = -0.227 \frac{1}{\Omega} = -227 \frac{1}{K\Omega}}$

to obtain $R_x\left(2.2(-227) + 10^3\right) = +2.2 \times 10^3 \times 227$; $R_x = \frac{499 \times 10^3}{500.6}$ K

$\boxed{R_x = 996\ K\Omega}$

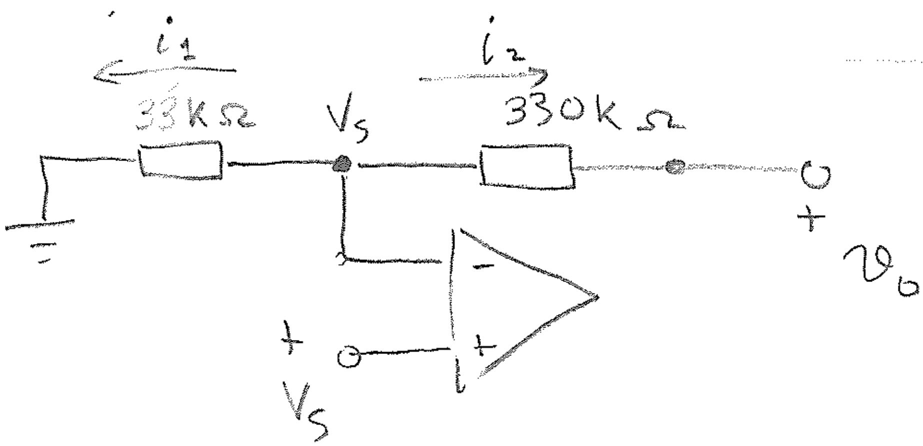
4.23



$$i_2 = \frac{v_o}{330K\Omega} \; ; \; i_1 = \frac{v_s}{33K\Omega}$$

Since $i_1 = i_2$ (because $i_N = 0$), we have

$$-\frac{v_o}{330\cancel{K\Omega}} = \frac{V_s}{33\cancel{K\Omega}} \; ; \; \boxed{v_o = -10V_s}$$



$$i_1 = \frac{V_s - 0}{33K\Omega} \; ; \; i_2 = \frac{V_s - v_o}{330K\Omega}$$

Since $i_1 = i_2$, we have

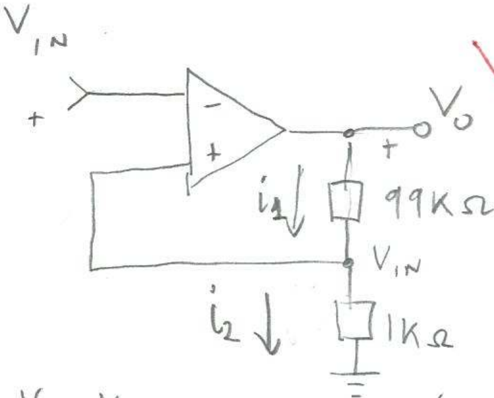
$$\frac{v_s}{33\cancel{K\Omega}} = \frac{v_o - v_s}{330\cancel{K\Omega}} \; ; \; \boxed{v_o = 330 v_s \left(\frac{1}{33} + \frac{1}{330}\right)}$$

$$= \cancel{330}\left(\frac{10+1}{\cancel{330}}\right) v_s$$

$$= \boxed{11 v_s}$$

4.25

a)

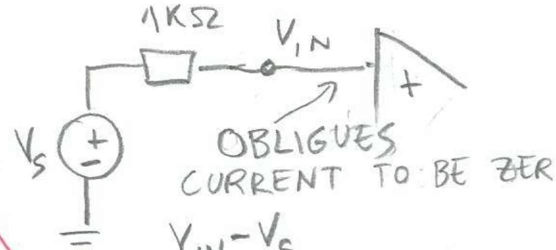


$i_1 = \dfrac{V_0 - V_{IN}}{99K\Omega}$; $i_2 = \dfrac{V_{IN} - 0}{1K\Omega}$;

Since $i_1 = i_2 \Rightarrow \dfrac{V_0 - V_{IN}}{99K\Omega} = \dfrac{V_{IN} - 0}{1K\Omega}$; $V_0 = 99V_{IN}\left(1 + \frac{1}{99}\right) = 99\,\frac{100}{99} \cdot V_{IN}$

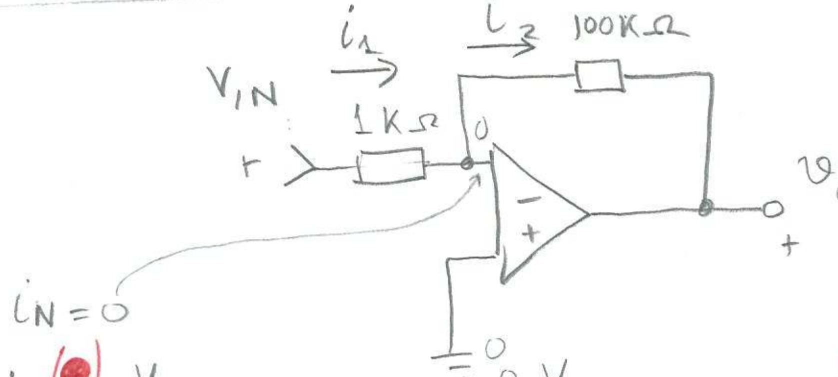
$\boxed{= 100 V_{IN}}$

(*) and with practical source



$\dfrac{V_{IN} - V_S}{1K\Omega} = 0$

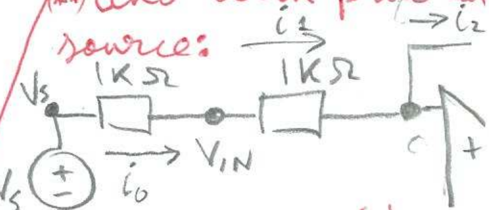
$\Rightarrow V_{IN} = V_S$



$i_1 = \dfrac{V_{IN}}{1K\Omega}$; $i_2 = \dfrac{0 - V_0}{100K\Omega}$;

Since $i_1 = i_2 \Rightarrow \boxed{V_0 = -100\ V_{IN}}$

(**) and with practical source:



$i_0 = \dfrac{V_S - V_{IN}}{1K\Omega} = i_1 = \dfrac{V_{IN} -}{1K\Omega}$

$\Rightarrow V_S = 2V_{IN} \neq V_{IN}$

TRUE: Both circuits have a gain of 100. and −100 respect

b) WITH PRACTICAL SOURCE

CIRCUIT 1 (*)

Same gain because $V_{IN} = V_S$.

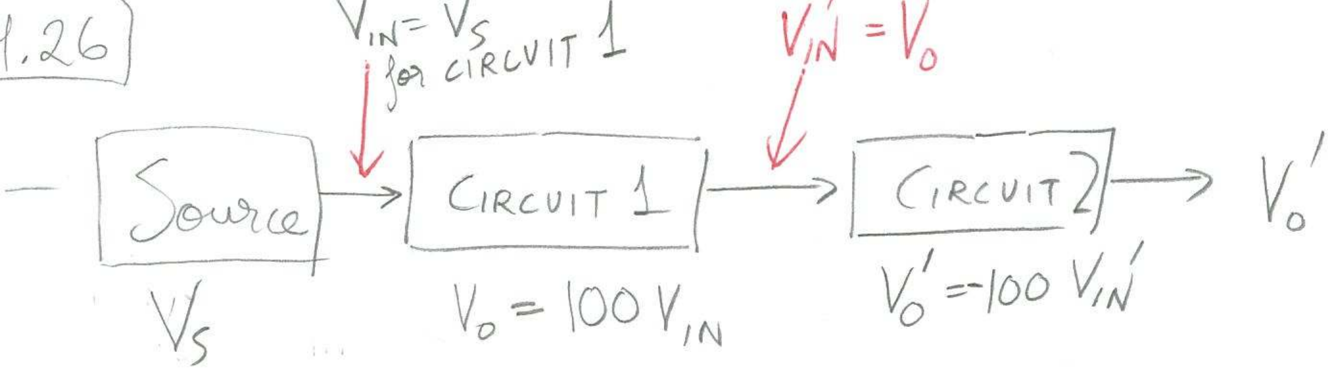
That is, $\boxed{V_0 = 100\,V_{IN} = 100\,V_S}$

CIRCUIT 2 (**)

Different gain because $V_S = 2\,V_{IN}$

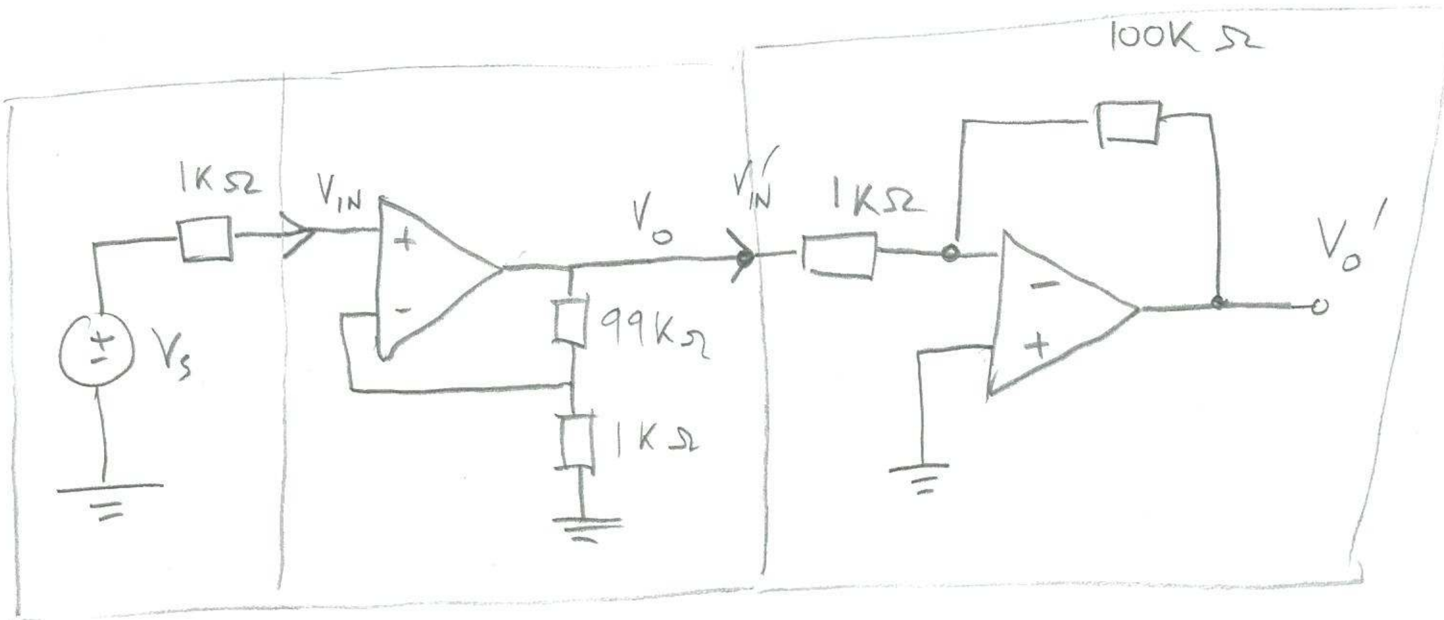
That is, $\boxed{V_0 = -100\,V_{IN} = -\frac{100}{2}\,V_S}$

4.26

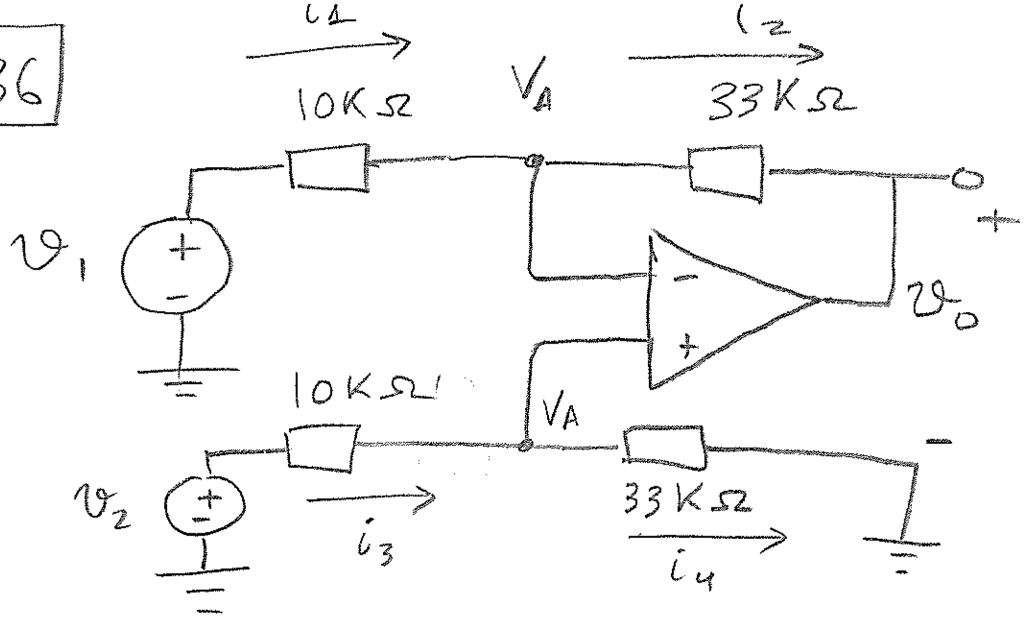


So $\boxed{V_o' = -100V_{IN}' = -100\,V_o = -100^2\,V_N}$

We need to connect circuit 1 first because its gain remains the same with respect to V_S.



4.36



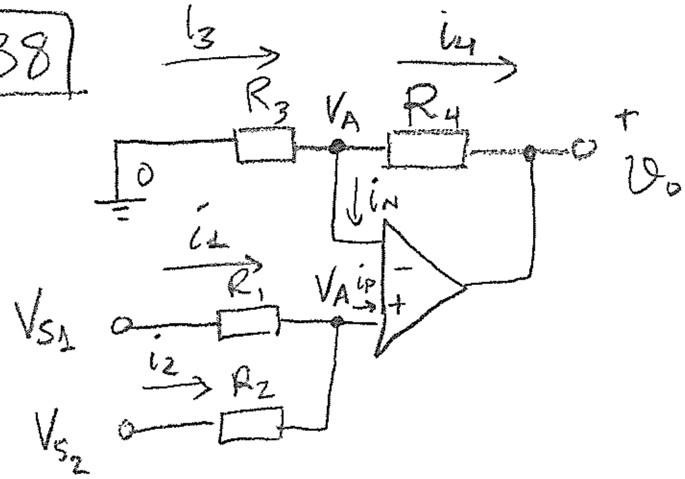
Since $i_1 = i_2 \Rightarrow \dfrac{V_1 - V_A}{10} = \dfrac{V_A - V_o}{33} \Rightarrow V_o = 33 * \left(\underbrace{V_A\left(\frac{1}{33} + \frac{1}{10}\right)} - \frac{V_1}{10} \right)$

since $i_3 = i_4 \Rightarrow \dfrac{V_2 - V_A}{10} = \dfrac{V_A - 0}{33} \Rightarrow \overbrace{V_A\left(\frac{1}{33} + \frac{1}{10}\right)} = \dfrac{V_2}{10}$

So we can solve for V_A and V_o but we just want to eliminate V_A and write V_o in terms of V_1 and V_2:

$$\boxed{V_o = 33\left(\frac{V_2}{10} - \frac{V_1}{10}\right) = -3.3\,V_1 + 3.3\,V_2}$$

4.38



$$i_N = 0 \Rightarrow i_3 = i_4 \Rightarrow \frac{0 - V_A}{R_3} = \frac{V_A - v_o}{R_4} \quad (1)$$

$$i_p = 0 \Rightarrow i_1 + i_2 = 0 \Rightarrow \frac{V_{S1} - V_A}{R_1} + \frac{V_{S2} - V_A}{R_2} = 0 \quad (2)$$

From (1) $V_A\left(\frac{1}{R_3} + \frac{1}{R_4}\right) = \frac{v_o}{R_4}$;

From (2) $\left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_A = \frac{V_{S1}}{R_1} + \frac{V_{S2}}{R_2}$;

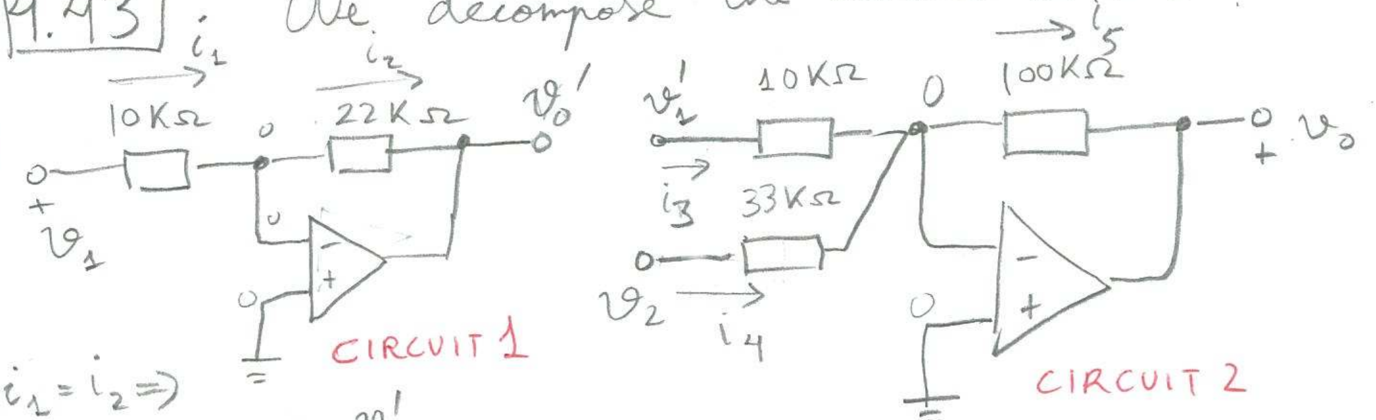
$$V_A = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \left(\frac{V_{S1}}{R_1} + \frac{V_{S2}}{R_2}\right) \quad (3)$$

Substituting (3) into (1), we get

$$\boxed{v_o = R_4 \frac{\frac{1}{R_3} + \frac{1}{R_4}}{\frac{1}{R_1} + \frac{1}{R_2}} \left(\frac{V_{S1}}{R_1} + \frac{V_{S2}}{R_2}\right)}$$

4.43 We decompose the circuit in two:

a)



$i_1 = i_2 \Rightarrow$

$$\frac{v_1 - 0}{10K\Omega} = \frac{0 - v_o'}{22K\Omega}$$

$$\Rightarrow \boxed{v_o' = -2.2\, v_1}$$

$i_3 + i_4 = i_5 \Rightarrow$

$$\frac{v_1' - 0}{10K\Omega} + \frac{v_2 - 0}{33K\Omega} = \frac{0 - v_o}{100K\Omega}$$

$$\Rightarrow \boxed{v_o = -100\left(\frac{v_1'}{10} + \frac{v_2}{33}\right)}$$

$$\boxed{= -10 v_1' - 3.03\, v_2}$$

Connecting the two (*) imposing

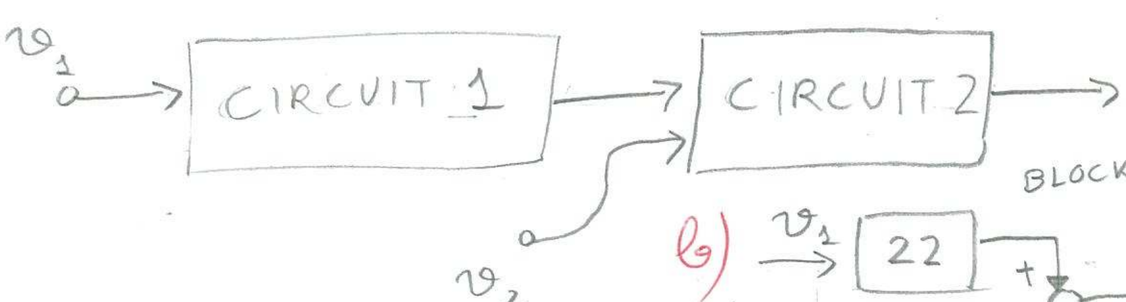
$$\boxed{v_1' = v_o'}$$

we get

$$\boxed{v_o = -10(-2.2\, v_1) - 3.03\, v_2}$$

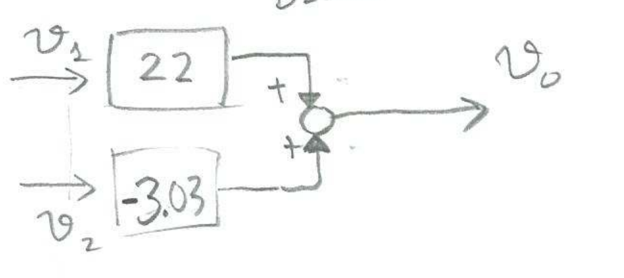
$$\boxed{= +22\, v_1 - 3.03\, v_2}$$

(*)

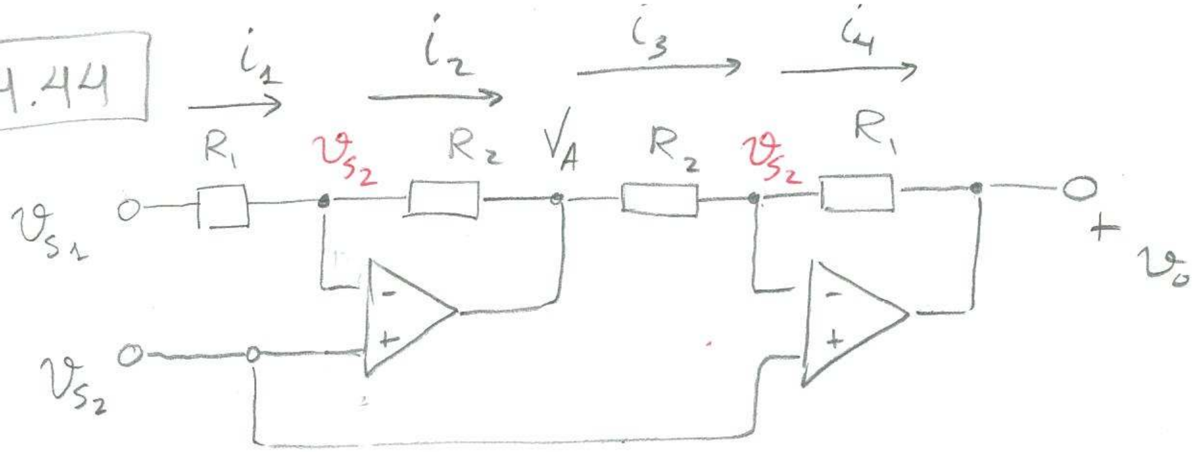


BLOCK DIAGRAM:

b)



4.44



$$i_1 = i_2 \Rightarrow \frac{v_{s1} - v_{s2}}{R_1} = \frac{v_{s2} - V_A}{R_2}$$

$$i_3 = i_4 \Rightarrow \frac{V_A - V_{s2}}{R_2} = \frac{V_{s2} - V_o}{R_1}$$

One quantity is equal to minus the other. Hence, it is easy to eliminate V_A

so we can equate

$$\frac{v_{s1} - v_{s2}}{\cancel{R_1}} = - \frac{v_{s2} - v_o}{\cancel{R_1}}$$

$$\Rightarrow \boxed{v_o = v_{s1}}$$