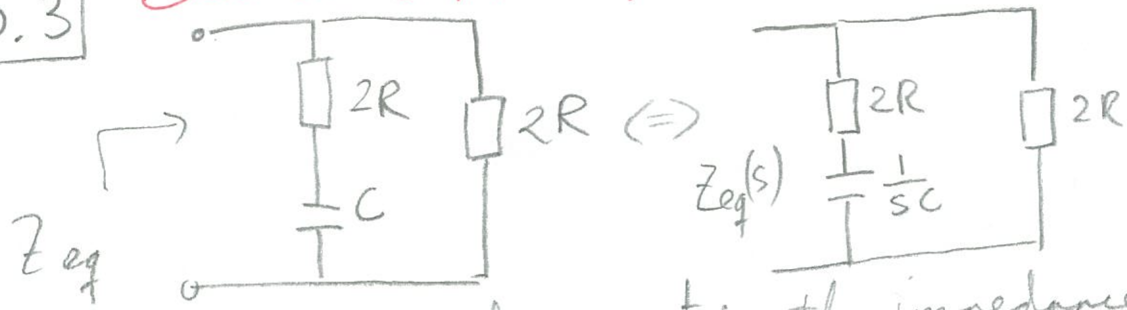
Solutions HW 7

$\boxed{10.3}$



For the purpose of computing the impedance, we set the initial condition of the capacitor to 0.

a) $Z_{eq}(s) = \left(2R + \frac{1}{sC}\right) \| 2R = \dfrac{\left(2R+\frac{1}{sC}\right)2R}{2R+\frac{1}{sC}+2R}$

$$= \frac{4R^2 s + \frac{2R}{C}}{4Rs + \frac{1}{C}} = \boxed{R\,\frac{\left(s+\frac{1}{2RC}\right)}{s+\frac{1}{4RC}}}$$

Zeros: $-\frac{1}{2RC}$; Poles: $-\frac{1}{4RC}$

Note:

$\boxed{\text{Henry} = \text{Ohm}\cdot\text{sec}}$ $(v = L i')$

$\boxed{\text{Farad} = \text{sec}/\text{Ohm}}$ $q = Cv \Updownarrow i = Cv'$

and $\frac{\text{rad}}{\text{sec}} = \frac{1}{\text{sec}}$

b) To have a pole at -640 rad/s we require

$-\frac{1}{4RC} = -640$ rad/s or $RC = \frac{1}{640}$ sec.

$\cancel{\text{Ohm}}\ \text{sec}/\text{Ohm}$

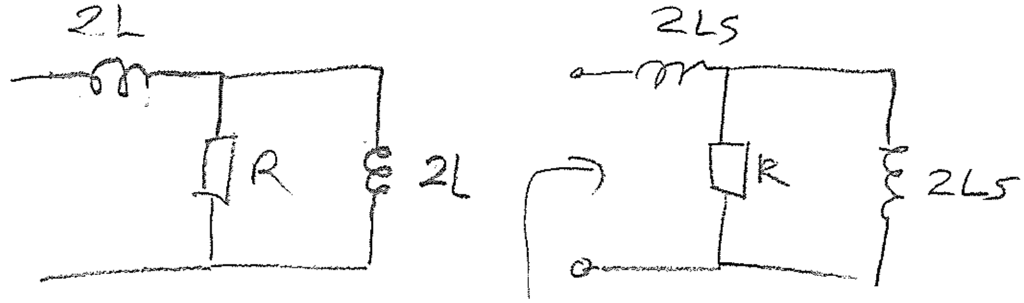
Selecting $R = 10K\Omega$ gives

$C = \frac{1}{6400}$ mF.

The resulting zero is then

$$s_0 = -\frac{1}{2RC} = -\frac{1}{4RC}\,2 = -640\cdot 2\,\frac{1}{s} = -1280 \text{ rad/s}$$

$\boxed{10.7}$



Zeq

$$Z_{eq}(s) = 2Ls + (R \,\|\, 2Ls)$$

$$= 2Ls + \frac{R2Ls}{R+2Ls} = \frac{2LRs + 4L^2s^2 + R2Ls}{R+2Ls}$$

$$= \frac{4LRs + 4L^2s^2}{R+2Ls} = \frac{s(2R + 2Ls)}{\frac{R}{2L} + s}$$

$$= 2L \frac{s\left(\frac{R}{L} + s\right)}{\frac{R}{2L} + s}$$

Zero: $-\frac{R}{L}$; poles $-\frac{R}{2L}$

So have a pole at -2 Krad/s, we impose

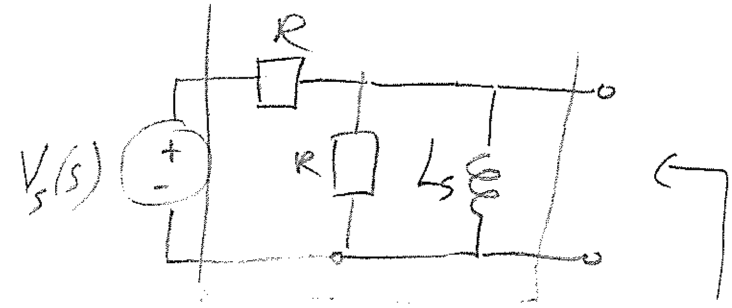
$-\frac{R}{2L} = -2$ Krad/s. Selecting $R = 2K\Omega$, we need $L = 0.5\,H$

and the resulting zero is $-\frac{R}{L} = -\frac{R}{2L}\cdot 2 =$

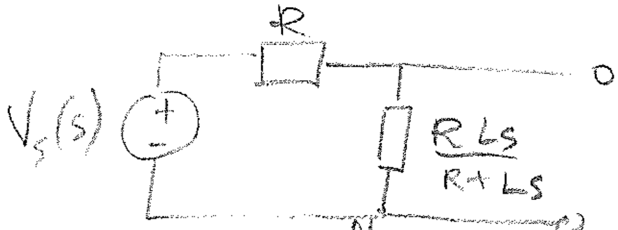
$= -2\text{Krad/s}\cdot 2$

$= -4$ Krad/s

10.17



$V_T(s)$ $Z_T(s)$



$$V_T(s) \overset{\text{voltage division}}{=} \frac{\frac{RLs}{R+Ls}}{R+\frac{RLs}{R+Ls}} \cdot V_s(s) = \frac{RLs}{(R+Ls)R+RLs}$$

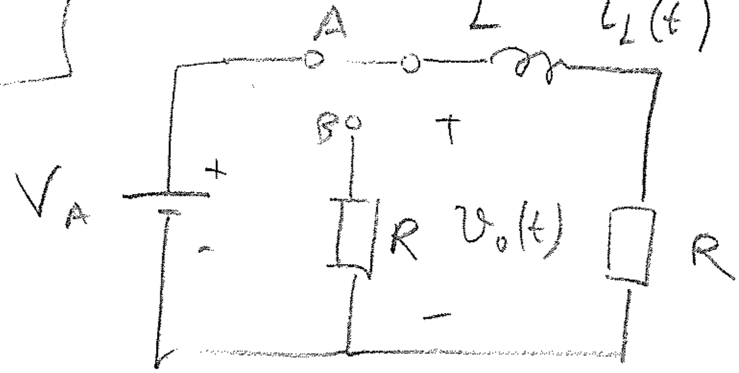
$$= \frac{RL\,s}{R^2+2RLs} = \boxed{\frac{1}{2}\,\frac{s}{\frac{R}{2L}+s}\,V_s(s)}$$

$$Z_T(s) = R \,\|\, R \,\|\, Ls = \frac{R}{2} \,\|\, Ls = \frac{\frac{R}{2}Ls}{\frac{R}{2}+Ls} =$$

$$= \boxed{\frac{\frac{R}{2}s}{\frac{R}{2L}+s}}$$

For $V_T(s)$ to have a pole at -12 Krad/s, we impose $-\frac{R}{2L} = -12$ Krad/s. Selecting $R = 12\,K\Omega$, we get $L = 0.5$ H

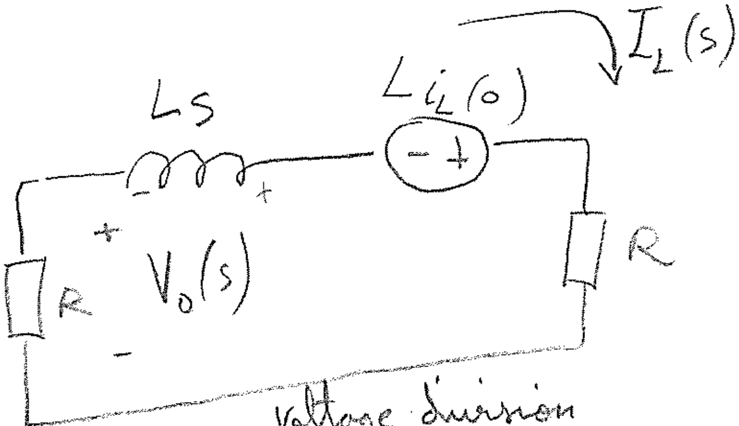
$\boxed{10.19}$



For a long time at A and then switch to B.

short circuit when switch is in position A

$$i_L(0) = \frac{V_A}{R}$$



voltage division

$$V_o(s) = -\frac{R}{R+Ls+R} \cdot L\, i_L(0) = -\frac{R}{2R+Ls} \cdot L \frac{V_A}{R}$$

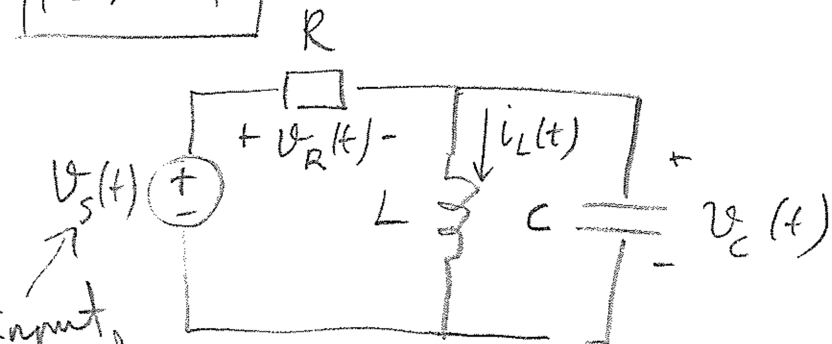
because of the polarity of source

$$\boxed{= -\frac{V_A}{\frac{2R}{L}+s}}$$

$$I_L(s) = \frac{L\, i_L(0)}{2R+Ls} = \frac{L \frac{V_A}{R}}{2R+Ls} = \frac{V_A/R}{\frac{2R}{L}+s}$$

So
$$V_o(t) = V_A\, e^{-\frac{2R}{L}t}$$
$$I_L(t) = \frac{V_A}{R}\, e^{-\frac{2R}{L}t}$$

$\boxed{10.29}$

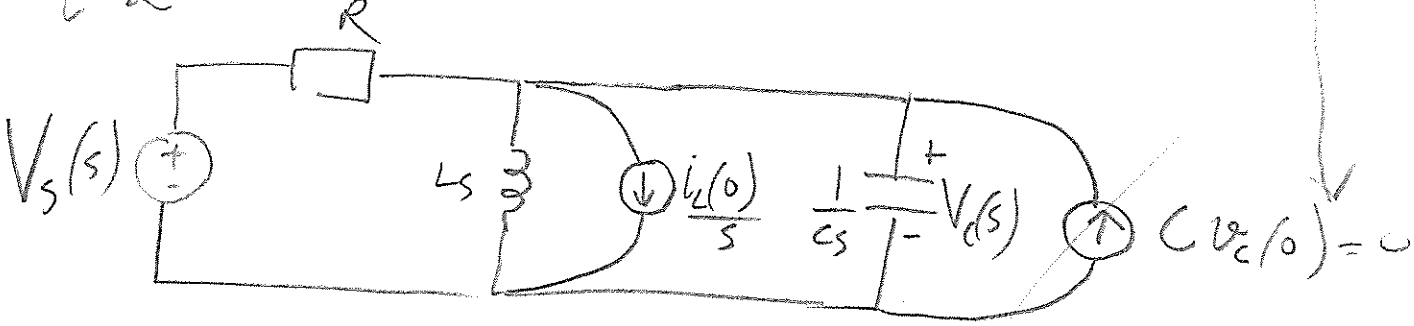


Input signal

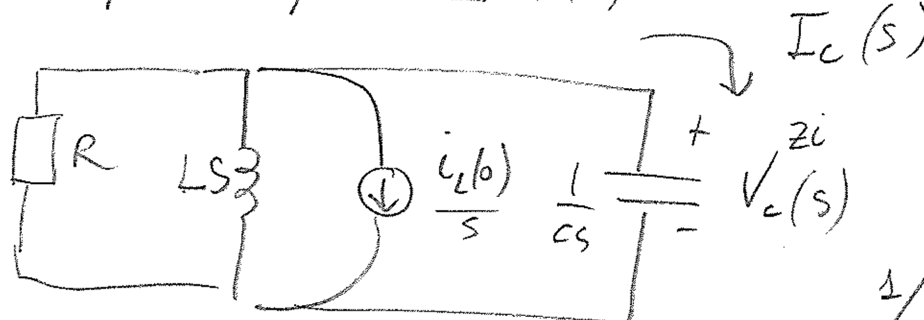
$i_L(0) = I_0$

$v_c(0) = 0$

$\Updownarrow \mathcal{L}$



Zero input response: $V_s(s) = 0$



Current division: $I_c(s) = \frac{1/\frac{1}{Cs}}{\frac{1}{R} + \frac{1}{Ls} + \frac{1}{1/Cs}}\left(-\frac{i_L(0)}{s}\right)$

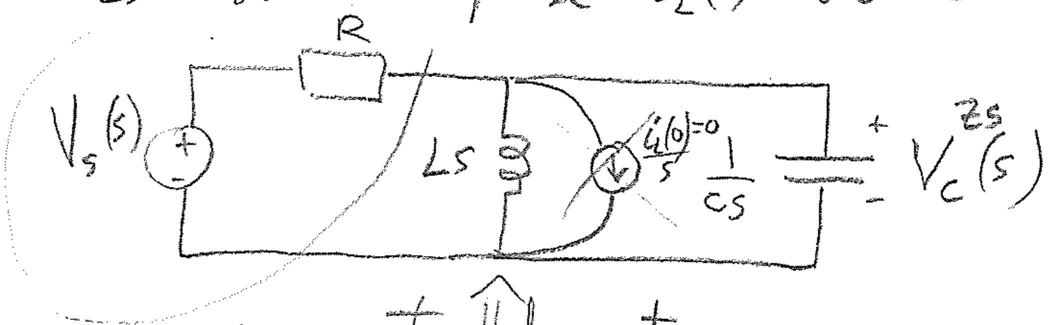
$$-\frac{Cs}{\frac{1}{R}+\frac{1}{Ls}+Cs}\,\frac{i_L(0)}{s} = -\frac{C\,i_L(0)}{\frac{Ls+R+RLCs^2}{RLs}} = -\frac{RLCs}{R+Ls+RLCs^2}\,I_0$$

$$V_c^{zi}(s) = I_c(s)\cdot\frac{1}{Cs} = -\frac{RL}{R+Ls+RLCs^2}\,I_0$$

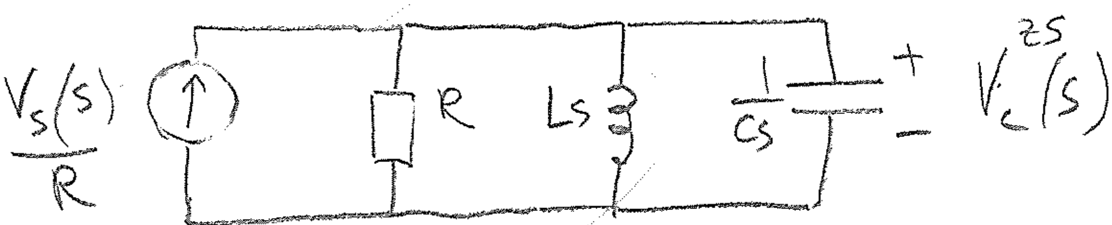
$$\boxed{= -\frac{\frac{1}{C}I_0}{\frac{1}{LC}+\frac{1}{RC}s+s^2}}$$

[10.29] continued

Zero state response $i_L(0) = I_0 = 0$



Source transformation



Source transformation

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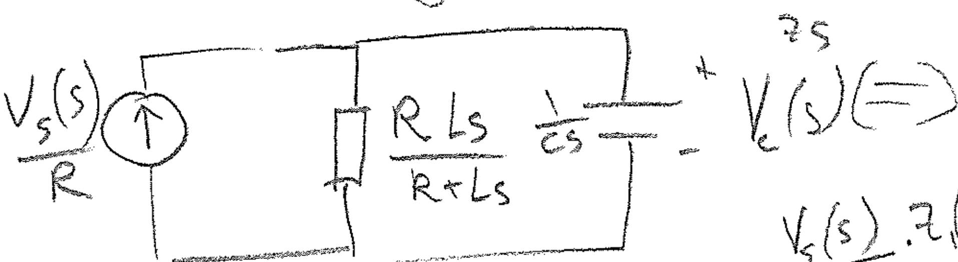
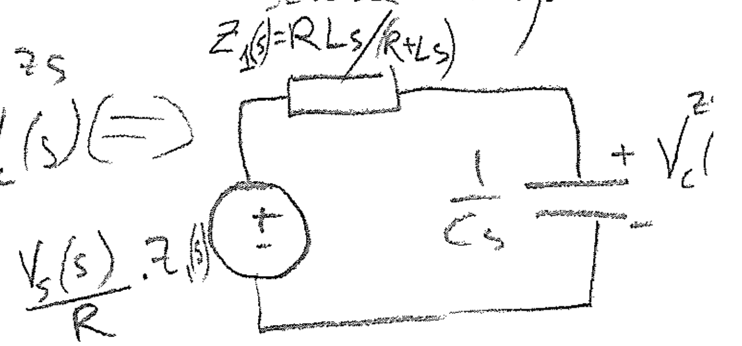




Now we use voltage division

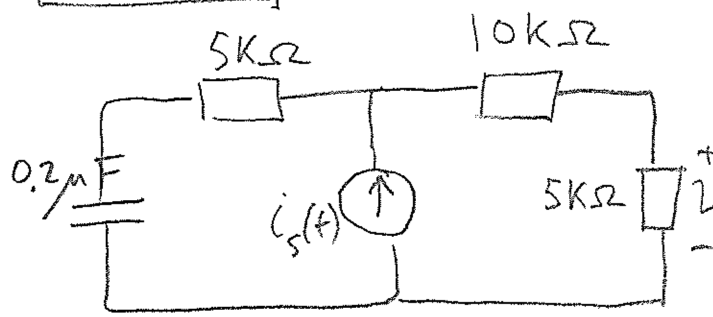
$$V_c^{zs}(s) = \frac{\frac{1}{Cs}}{\frac{1}{Cs} + Z_1(s)} \cdot \frac{V_s(s)}{R} \cdot Z_1(s) =$$

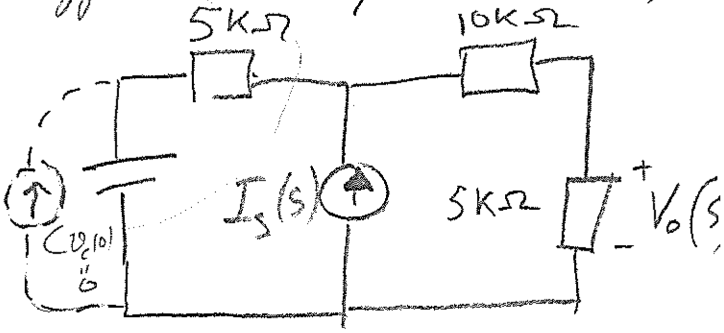
$$= \frac{V_s(s)\, Z_1(s)}{(1 + Z_1(s) Cs) R} = \frac{V_s(s)\, \cancel{R}Ls/\cancel{(R+Ls)}}{\left(1 + \frac{RLs}{(R+Ls)} Cs\right)\cancel{R}} =$$

$$= \frac{Ls}{(R+Ls) + RLCs^2} V_s(s) = \boxed{\frac{\frac{1}{RC} s}{\frac{1}{LC} + \frac{1}{RC} s + s^2} V_s(s)}$$

10.32

No energy stored in capacitor $\Leftrightarrow v_c(0)=0$



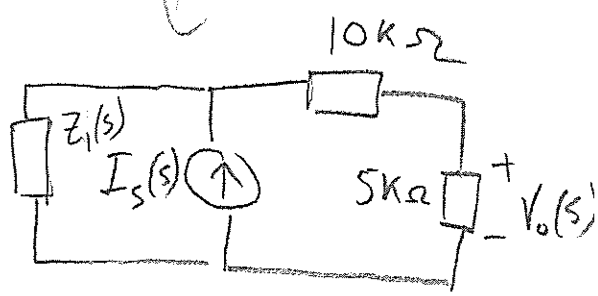


source transform.

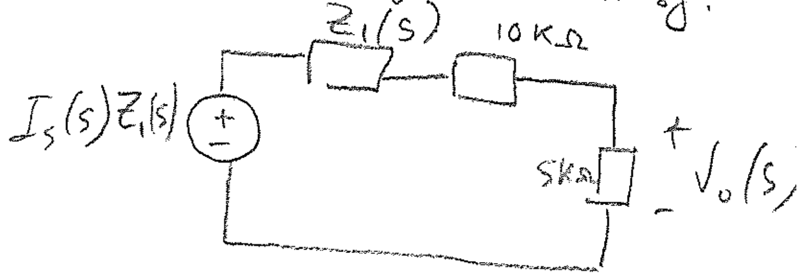
You can use current division there or voltage division later

$$Z_1(s) = 5K\Omega + \frac{1}{0.2\mu F\, s}$$

$$= 5K\Omega + \frac{5}{(10^{-3} mF)s} \quad \left(5 + \frac{5\times10^3}{s}\right) K\Omega$$



Source transf.



Voltage division :

$$V_o(s) = \frac{5K\Omega}{(5+10)K\Omega + Z_1(s)} \cdot Z_1(s) \cdot I_s(s)$$

$$\left(I_s(s) = \frac{5}{s+500}\, mA \cdot s\right) = \frac{5\cancel{K\Omega}}{\left(15 + 5 + \frac{5\times10^3}{s}\right)\cancel{K\Omega}} \left(5 + \frac{5\times10^3}{s}\right) K\Omega\ \boxed{I_s(s)}$$

$$= \frac{(25s + 25\times10^3)}{20s + 5\times10^3} \times K\Omega \times \frac{5}{s+500}\, mA.$$

$\boxed{10.32}$ continued.

volt. secons

$$V_o(s) = 25 \, \frac{s+10^3}{4s+10^3} \, \frac{1}{s+500} \; V.s$$

proper

$$V_o(s) = \frac{K_1}{s+250} + \frac{K_2}{s+500}$$

where $K_1 = \lim_{s \to -250} \frac{25}{4} \frac{(s+10^3)}{\cancel{(s+250)}(s+500)} \cdot \cancel{(s+250)}$

$$= \frac{25}{4} \frac{-250+10^3}{-250+500} = \frac{25}{4} \frac{750}{250}$$

$$= \frac{25 \times 3}{4} = 18.75$$

$$K_2 = \lim_{s \to -500} \frac{25}{4} \frac{s+10^3}{s+250} \frac{\cancel{(s+500)}}{\cancel{(s+500)}}$$

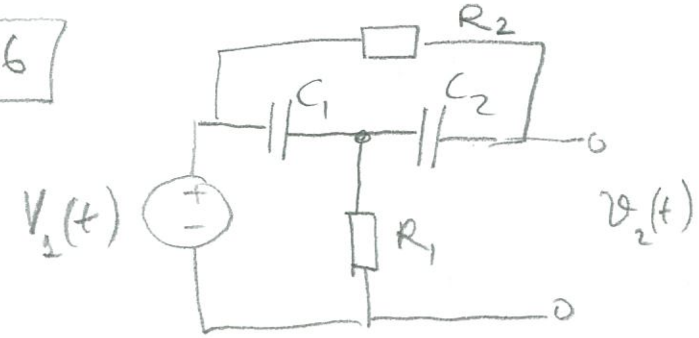
$$= \frac{25}{4} \frac{500}{-250} = -\frac{25}{2} = -12.5$$

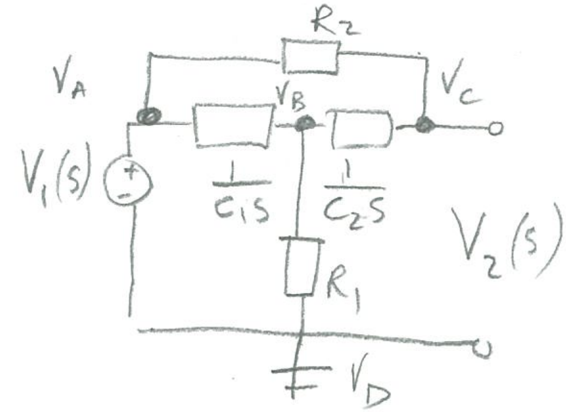
So $V_o(t) = 18.75 \times e^{-250t} - 12.5 e^{-500t}$ ✓

The forced pole comes from $i_s(t) = 5e^{-500t} \Leftrightarrow \frac{5}{s+50}$

I.e., the forced pole is -500.

10.46





$$V_D = 0\ ;\quad V_A = V_1(s)\ ;\quad V_2(s) = V_C$$

$$\begin{array}{cc} & \begin{array}{ccc} A & B & C \end{array} \\ \begin{array}{c} \text{Node } B \\ C \end{array} & \left[\begin{array}{ccc} -C_1 s & C_1 s + C_2 s + \frac{1}{R_1} & -C_2 s \\ -\frac{1}{R_2} & -C_2 s & C_2 s + \frac{1}{R_2} \end{array}\right] \end{array} \begin{bmatrix} \boxed{V_A} = V_1(s) \\ V_B \\ V_C = V_2(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Known V column

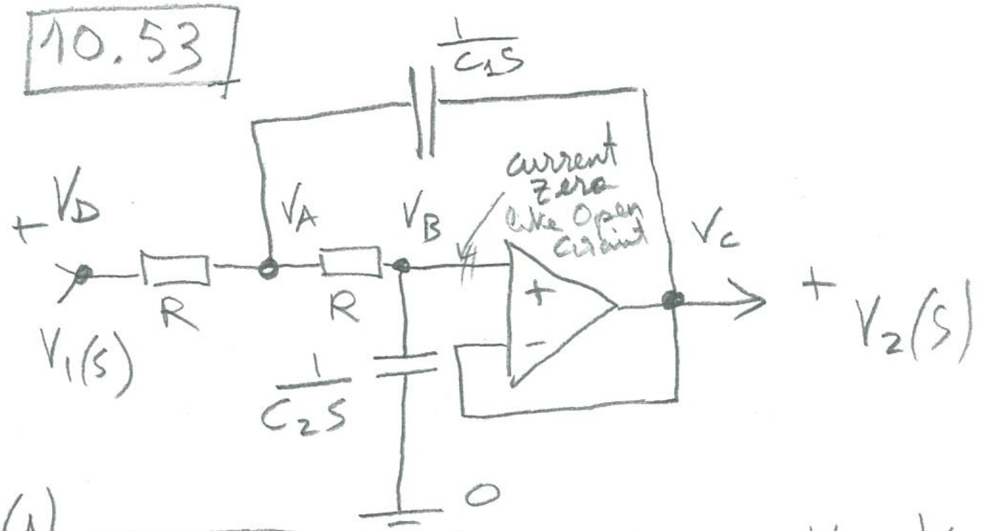
We need to solve for V_C and V_B in terms of V_A (although we just really need V_C)

$$\begin{array}{cc} & \begin{array}{cc} B & C \end{array} \\ \begin{array}{c} B \\ C \end{array} & \underbrace{\begin{bmatrix} C_1 s + C_2 s + \frac{1}{R_1} & -C_2 s \\ -C_2 s & C_2 s + \frac{1}{R_2} \end{bmatrix}}_{A} \end{array} \begin{bmatrix} V_B \\ V_2(s) \end{bmatrix} = \begin{bmatrix} C_1 s V_1(s) \\ \frac{1}{R_2} V_1(s) \end{bmatrix}$$

CRAMER:

$$V_2(s) = \frac{\begin{vmatrix} C_1 s + C_2 s + \frac{1}{R_1} & C_1 s V_1(s) \\ -C_2 s & \frac{1}{R_2} V_1(s) \end{vmatrix}}{|A|}$$

10.53



(1) $\boxed{V_B = V_C}$; $V_D = V_1(s)$; $V_C = V_2(s)$

$$
\begin{array}{r c}
 & \begin{array}{cccc} \quad A \quad & \quad B \quad & \quad C \quad & \quad D \quad \end{array} \\
\begin{array}{r} (2)\ \text{Node } A \\ (3)\ \text{Node } B \end{array} &
\begin{bmatrix} \frac{1}{R}+\frac{1}{R}+C_1 s & -\frac{1}{R} & -C_1 s & -\frac{1}{R} \\ -\frac{1}{R} & \frac{1}{R}+C_2 s & 0 & 0 \end{bmatrix}
\end{array}
\begin{bmatrix} V_A \ ? \\ V_B \ ? \\ V_C = V_2(s)\ ? \\ V_D = V_1(s) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}
$$

The unknowns are V_A, V_B, V_C.
(since we are given $V_D = V_1(s)$ as input)
We need three equations
(1), (2) and (3)

Using $V_B = V_C$ and moving V_D to the right hand side

Add columns B & C ← because $V_B = V_C$

$$
\begin{array}{cc} A & \text{Add columns } B \,\&\, C \end{array}
$$

$$
\begin{bmatrix} \frac{1}{R}+\frac{1}{R}+C_1 s & -\frac{1}{R}-C_1 s \\ -\frac{1}{R} & \frac{1}{R}+C_2 s \end{bmatrix}
\begin{bmatrix} V_A \\ V_2(s) \end{bmatrix} = \begin{bmatrix} \frac{1}{R} V_D = V_1(s) \\ 0 \end{bmatrix}
$$

with

$$
\Delta(s) = \left(\frac{2}{R}+C_1 s\right)\left(\frac{1}{R}+C_2 s\right) - \left(\frac{1}{R}+C_1 s\right)\frac{1}{R}
$$

$$
= \frac{2}{R^2} + \frac{2C_2 s}{R} + \cancel{\frac{C_1 s}{R}} + C_1 C_2 s^2 - \frac{1}{R^2} - \cancel{\frac{C_1 s}{R}} =
$$

$$
= \frac{1}{R^2} + \frac{2C_2}{R} s + C_1 C_2 s^2
$$

Cramer

$$
V_2(s) = \frac{\begin{vmatrix} \frac{2}{R}+C_1 s & \frac{1}{R} V_1(s) \\ -\frac{1}{R} & 0 \end{vmatrix}}{\Delta(s)} = \frac{V_1(s)}{R^2 \Delta(s)}
$$

$\boxed{10.53}$ continued.

We have seen that

$$\Delta(s) = \frac{1}{R^2} + \frac{2C_2}{R} s + C_1 C_2 s^2$$

The circuit determinant is then

$$\frac{\Delta(s)}{C_1 C_2} = s^2 + \frac{2}{C_1 R} s + \frac{1}{C_1 C_2 R^2}$$

$$= s^2 + 2\zeta\omega_0 s + \omega_0^2$$

so $\omega_0^2 = \dfrac{1}{C_1 C_2 R^2}$ / $\omega_0 = \dfrac{1}{\sqrt{C_1 C_2}\, R}$

$$\cancel{2}\zeta\omega_0 = \frac{\cancel{2}}{C_1 R} \; ; \; \zeta = \frac{1}{C_1 R \omega_0} = \frac{\sqrt{C_1 C_2}\,\cancel{R}}{C_1 \cancel{R}}$$

$$= \sqrt{\frac{C_2}{C_1}}$$

If we need $\omega_0 = 10 \text{K rad/s}$ and $\zeta = 1$

we impose $\dfrac{1}{\sqrt{C_1 C_2}\, R} = 10 \text{K rad/s}$ $\sqrt{\dfrac{C_2}{C_1}} = 1$

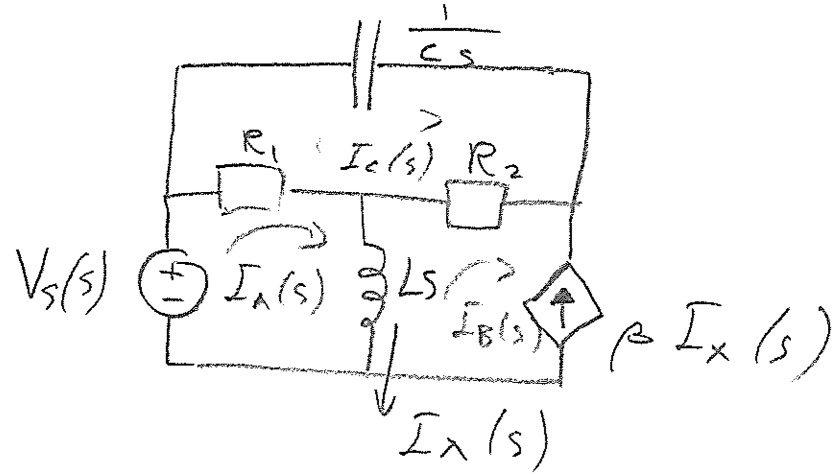
For instance, we can take $C_1 = C_2 = C$

which yields $\dfrac{1}{\sqrt{C_1 C_2}\, R} = \dfrac{1}{CR} = 10 \text{K rad/s}$.

If we select $C = 0.1\,\mu F$, we need $R = 1\,K\Omega$

because then $\dfrac{1}{CR} = \dfrac{1}{0.1\,\mu F \cdot 1 K\Omega} = \dfrac{1}{0.1 \times 10^{-6} F \cdot 10^3 \Omega} = 10^4 \text{K rad/s}$

$\boxed{10.56}$



Only $I_c(s)$ and $I_A(s)$ or $I_c(s)$ and $I_B(s)$ are independent because

$$I_B(s) = -\beta I_x(s) = -\beta\left(I_A(s) - I_B(s)\right).$$

I.e., $\boxed{(1-\beta)\, I_B(s) = -\beta I_A(s)}$

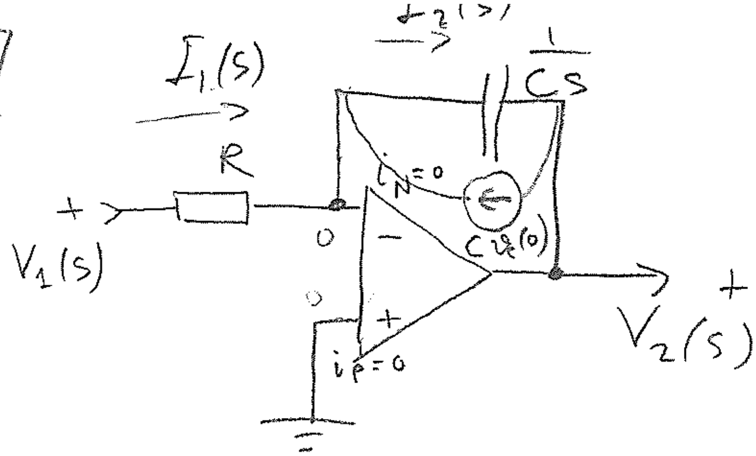
Mesh current equations

$$\begin{bmatrix} \frac{1}{Cs} + R_1 + R_2 & -R_1 & -R_2 \\ -R_1 & R_1 + Ls & -Ls \end{bmatrix} \begin{bmatrix} I_C(s) \\ I_A(s) \\ I_B(s) = -\frac{\beta}{1-\beta} I_A(s) \end{bmatrix} = \begin{bmatrix} 0 \\ V_s(s) \end{bmatrix}$$

In terms of just $I_c(s)$ and $I_A(s)$:

$$\begin{bmatrix} \frac{1}{Cs} + R_1 + R_2 & -R_1 - R_2\left(-\frac{\beta}{1-\beta}\right) \\ -R_1 & R_1 + Ls - Ls\left(-\frac{\beta}{1-\beta}\right) \end{bmatrix} \begin{bmatrix} I_c(s) \\ I_A(s) \end{bmatrix} = \begin{bmatrix} 0 \\ V_s(s) \end{bmatrix}$$

$\boxed{10.61}$



$v_1(t) = \cos 1000t$ $\Updownarrow \mathcal{L}$ We desire $v_2(t) = -\sin 1000t$

$$V_1(s) = \frac{s}{s^2+10^6} \qquad V_2(s) = -\frac{1000}{s^2+10^6}$$

Since $I_1(s) = I_2(s)$ (by KCL), we have

$$\frac{V_1(s)-0}{R} = \frac{0-V_2(s)}{\frac{1}{Cs}}. \quad \text{I.e.,} \quad \boxed{V_2(s) = -\frac{1}{RCs}V_1(s)}$$

Using the desired inputs and outputs, we obtain the constraint

$$-\frac{1000}{\cancel{s^2+10^6}} = -\frac{1}{RC\cancel{s}}\frac{\cancel{s}}{\cancel{s^2+10^6}} \iff \boxed{\frac{1}{RC} = 1000}$$

If $v_c(0) \neq 0$, we have instead

$$I_1(s) = I_2(s) - C v_c(0). \quad \text{I.e.,}$$

$$\frac{V_1(s)}{R} = \frac{-V_2(s)}{\frac{1}{Cs}} - C v_c(0), \text{ or } V_2(s) = \frac{1}{Cs}\Big(\frac{V_1(s)}{R} + C \underbrace{v_c(0)}_{\text{10V}}\Big)$$

$$= \boxed{\underbrace{\frac{1}{CRs}V_1(s)}_{\text{zero state}} + \underbrace{\frac{10}{s}}_{\text{zero input}}}$$