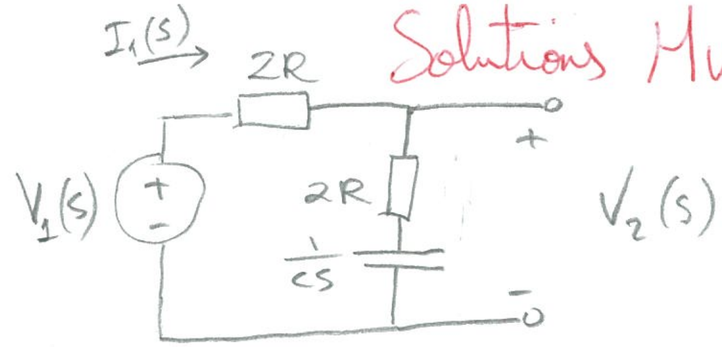
11.1

Solutions HW 8



Driving point impedance (impedance seen from the source)

$$Z(s) = \frac{V_1(s)}{I_1(s)} = 2R + 2R + \frac{1}{cs} = \frac{4Rcs+1}{cs}$$

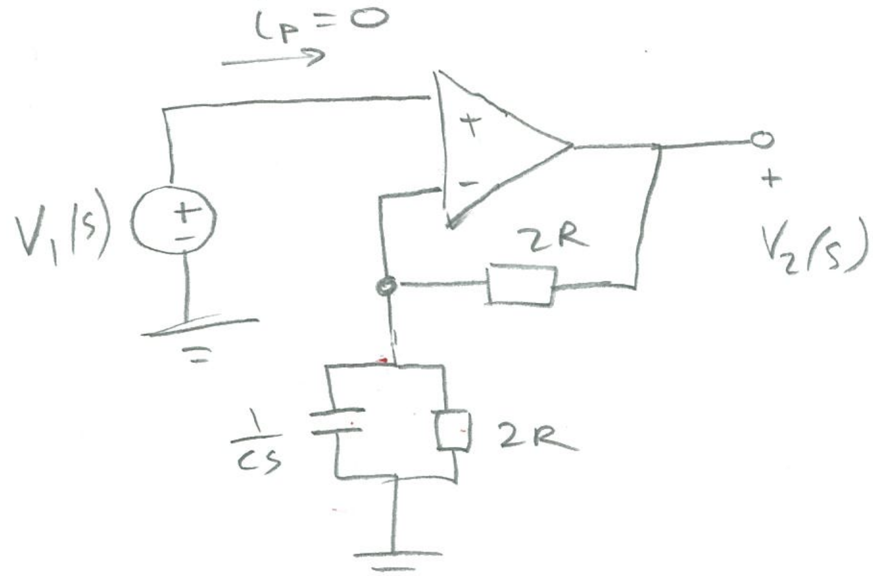
To compute $T_V(s) = \frac{V_2(s)}{V_1(s)}$ we do

$$V_2(s) = I_1 \cdot \left(2R + \frac{1}{cs}\right) = \frac{V_1(s)}{Z(s)}\, 2R + \frac{1}{cs}$$

$$= \frac{2R + \frac{1}{cs}}{Z(s)}\, V_1(s) = \frac{cs\left(2R + \frac{1}{cs}\right) V_1(s)}{4Rcs+1} = \boxed{\frac{2Rcs+1}{4Rcs+1}\, V_1}$$

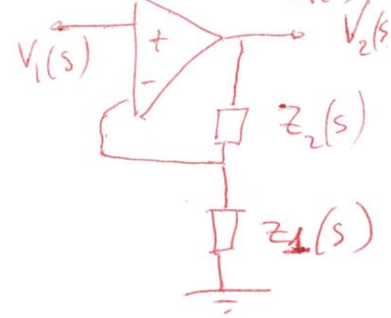
Alternatively, you can compute $\frac{V_2(s)}{V_1(s)}$ using voltage division.

11.7



Non inverting amplifier

$$\frac{V_2(s)}{V_1(s)} = \frac{Z_1(s)+Z_2(s)}{Z_1(s)}$$



Non inverting amplifier with $Z_2(s) = 2R$ and $Z_1(s) = \frac{1}{Cs + \frac{1}{2R}}$

(Mauricio's notes p.156)

$$T_V(s) = \frac{V_2(s)}{V_1(s)} = \frac{Z_1(s)+Z_2(s)}{Z_1(s)} = 1 + \frac{Z_2(s)}{Z_1(s)}$$

$$= 1 + \frac{2R}{\frac{1}{Cs+\frac{1}{2R}}} = 1 + \left(Cs + \frac{1}{2R}\right)2R$$

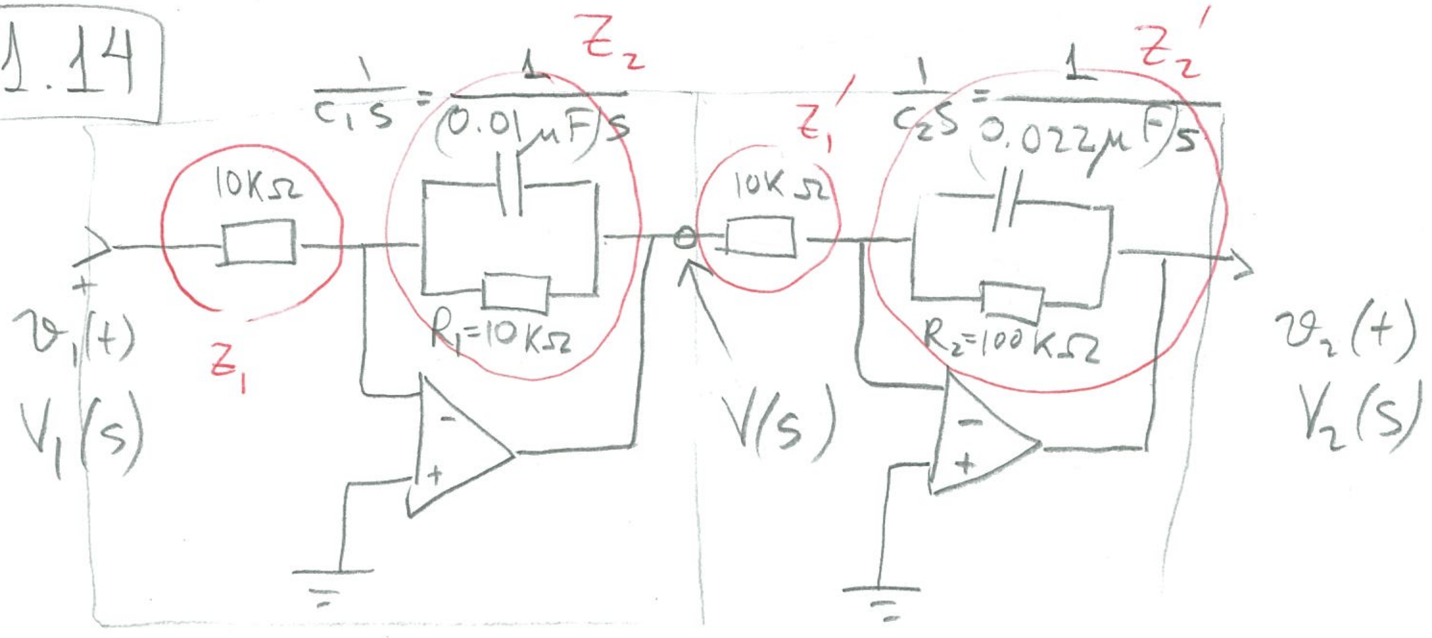
$$= 1 + 2RCs + 1 = 2 + 2RCs.$$

This is what we've been doing with resistive circuits and we called gain.

The driving point impedance (impedance seen from the source) is ∞ because $i_p = 0$, i.e.,

$$0 = I_p(s) = \frac{V_1(s)}{Z(s)} \Rightarrow Z(s) = \infty$$

11.14



Chain rule for two inverting amplifiers

$$\frac{V(s)}{V_1(s)} \overset{\text{Mauricio's notes p.155}}{=} -\frac{Z_2(s)}{Z_1(s)} \quad ; \quad \frac{V_2(s)}{V(s)} = \frac{-Z_2'(s)}{Z_1'(s)}$$

$$T_V = \frac{V(s)}{V_1(s)} \frac{V_2(s)}{V(s)} = \left(-\frac{Z_2(s)}{Z_1(s)}\right)\left(-\frac{Z_2'(s)}{Z_1'(s)}\right)$$

$$= \frac{\frac{1}{C_1 s + \frac{1}{R_1}}}{10} \; \frac{\frac{1}{C_2 s + \frac{1}{R_2}}}{10} =$$

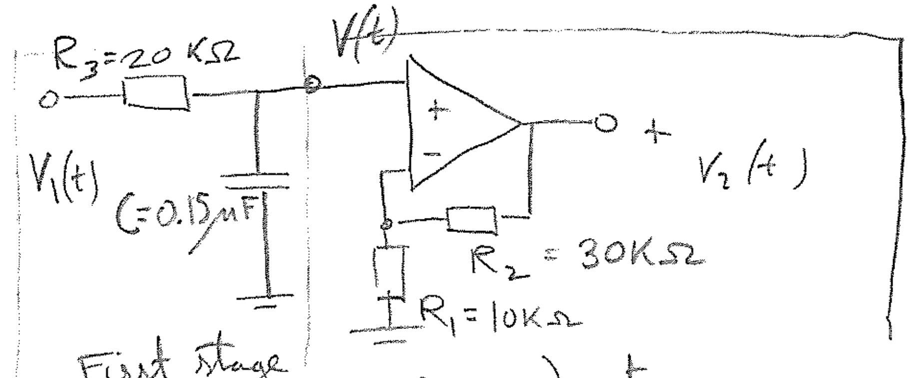
$\left(\text{Farad} = \frac{\text{sec}}{\text{Ohm}} ; \; \boxed{mF = \frac{\text{sec}}{K\Omega}} \quad C_1 = 10\,mF \quad C_2 = 22\,mF \quad R_1 = 10K\Omega \quad R_2 = 100K\Omega \right)$

$$= \frac{1}{100\left(C_1 s + \frac{1}{R_1}\right)\left(C_2 s + \frac{1}{R_2}\right)} = \boxed{\frac{1}{\frac{100}{C_1 C_2}\left(s + \frac{1}{R_1 C_1}\right)\left(s + \frac{1}{C_2 R_2}\right)}}$$

Poles: $-\frac{1}{R_1 C_1} = -\frac{1}{10 \cdot 10} = \boxed{-\frac{1}{100}\,\frac{1}{\text{sec}}}$; $-\frac{1}{C_2 R_2} = -\frac{1}{22 \cdot 100} = \boxed{-\frac{1}{2200}\,\frac{1}{\text{sec}}}$

zeros: two at ∞.

$\boxed{12\ 6}$



First stage
Low pass

Second stage
Inverting amplifier

$$\boxed{\frac{V(s)}{V_1(s)} = \frac{\omega_c}{\omega_c + s}} \quad \text{with } \omega_c = \frac{1}{CR_3}$$

$$\boxed{\frac{V_2(s)}{V(s)} = -\frac{R_2}{R_1}}$$

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{V(s)}{V_1(s)} \frac{V_2(s)}{V(s)} = \frac{\omega_c}{\omega_c + s}\left(-\frac{R_2}{R_1}\right)$$

- dc gain $= |T(0)| = \left| -\frac{R_2}{R_1} \frac{\omega_c}{\omega_c + 0} \right| = \boxed{\frac{R_2}{R_1}}$

$$|T(j\omega)| = \frac{\omega_c R_2}{R_1} \left| \frac{1}{\omega_c + j\omega} \right| = \frac{\omega_c R_2}{R_1} \frac{1}{\sqrt{\omega_c^2 + \omega^2}}$$

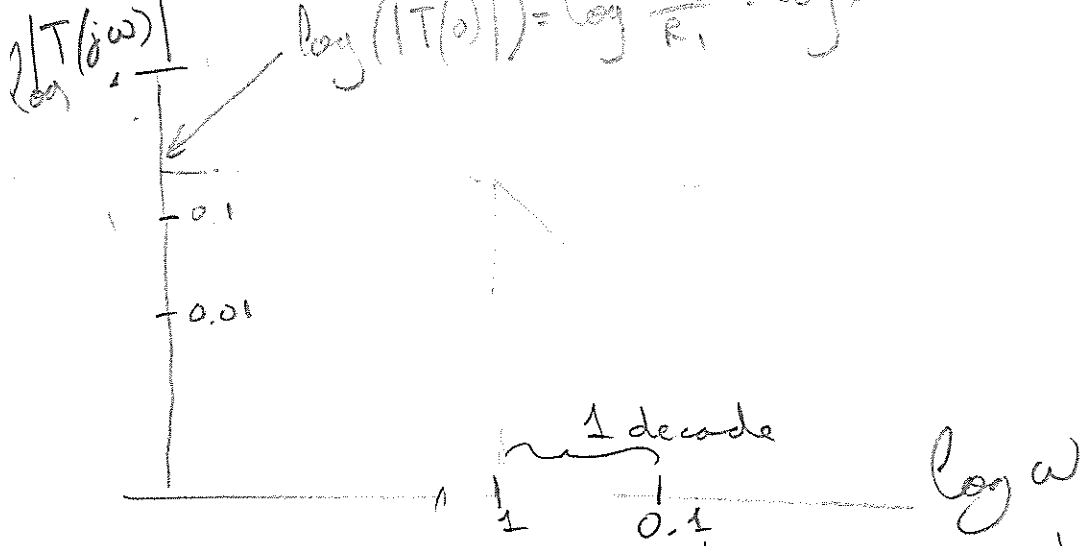
- infty frequency gain $= \lim_{\omega \to \infty} \frac{\omega_c R_2}{R_1} \frac{1}{\sqrt{\omega_c^2 + \omega^2}} = \boxed{0}$

- Cut off frequency $= \boxed{\omega_c}$ $= \frac{1}{CR_3} = \frac{1}{60.10^{-3} s}$ because this is the frequency such that $|T(j\omega_c)| = \frac{\sqrt{2}}{2} |T(0)|$

and ω_c is the cut off frequency of $\frac{\omega_c}{\omega_c + s}$ (note that $\left| \frac{\omega_c}{\omega_c + j\omega_c} \right| = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$).

- It is a low-pass filter

12.6 continued



$$\omega_c = \frac{1}{CR_3} = \frac{1}{0.15\mu F \cdot 20K\Omega} = \frac{1}{3\times10^{-6}\frac{s}{\Omega}\ 20\ 10^3\Omega} = \frac{1}{60.10^{-3}s}$$

For $\alpha = 0.1, 1, 10$

$$|T(j\alpha\omega_c)| = \frac{\omega_c R_2}{R_1}\frac{1}{\sqrt{\omega_c^2+\alpha\omega_c^2}} = \frac{\cancel{\omega_c} R_2}{R_1}\frac{1}{\cancel{\omega_c}\sqrt{1+\alpha}}$$

$$= \frac{R_2}{R_1}\frac{1}{\sqrt{1+\alpha}}$$ so we just need to evaluate

at $\alpha = 0.1, 1, 10$ to get $\frac{R_2}{R_1}\frac{1}{\sqrt{1.1}}, \frac{R_2}{R_1}\frac{1}{\sqrt{2}}, \frac{R_2}{R_1}\frac{1}{\sqrt{11}}$

e) Double the passband gain without changing the cutoff frequency.

We knew that $|T(0)| = \frac{R_2}{R_1}$

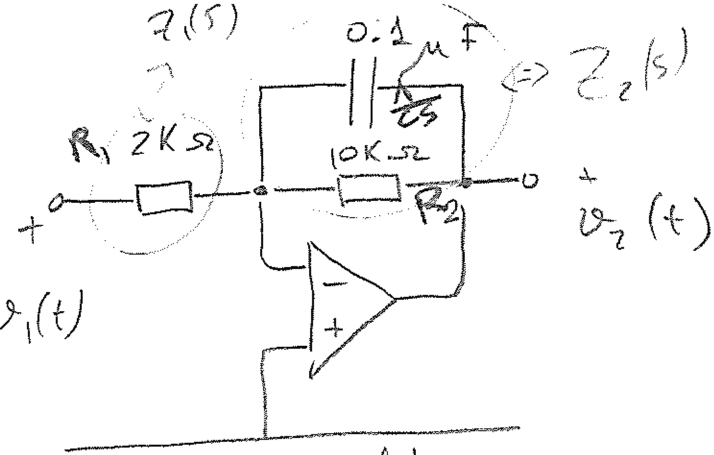
whereas the cutoff frequency is $\omega_c = \frac{1}{CR_3}$

Therefore, we just need to take $\boxed{\hat{R}_2 = 2R_2}$

f) Slope $\frac{R_2}{R_1}\cdot(-20\text{dB per decade})$, so

it is $-20\frac{R_2}{R_1}$ dB less $= -60$dB less.

12.7



Inverting amplifier

$$\frac{V_2(s)}{V_1(s)} = \frac{Z_2(s)}{Z_1(s)} = \frac{\frac{1}{Cs+\frac{1}{R_2}}}{R_1} = \frac{1}{CR_1 s + \frac{R_1}{R_2}} =$$

multiplying and dividing by R_2

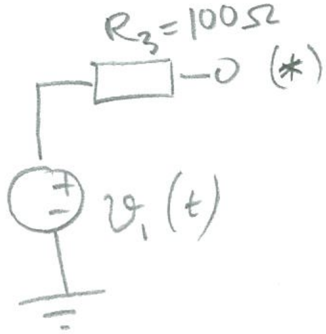
$$= \frac{1}{CR_1}\,\frac{1}{s+\frac{1}{CR_2}} = \frac{R_2}{R_2}\,\frac{1}{CR_1}\,\frac{1}{s+\frac{1}{CR_2}}$$

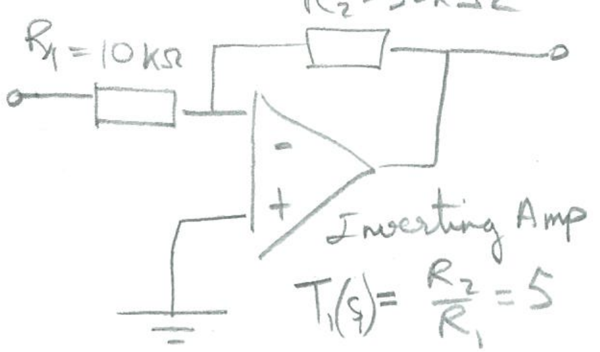
$$= \frac{R_2}{R_1}\,\frac{\frac{1}{CR_2}}{s+\frac{1}{CR_2}} = \frac{R_2}{R_1}\,\frac{\omega_c}{s+\omega_c} \quad \text{with } \omega_c = \frac{1}{CR_2}$$

This is the <u>same as in 12.6 except for the sign</u> and the specific numerical values.

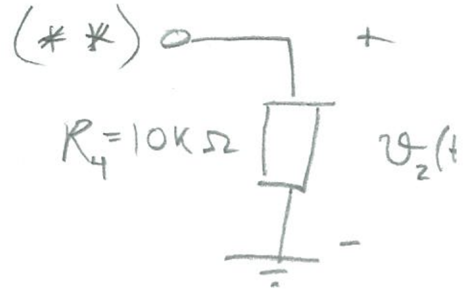
The sign <u>changes the phase</u> <u>but not $|T(j\omega)|$</u>

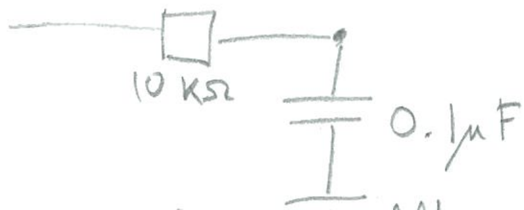
12.9





$T_1(s) = \frac{R_2}{R_1} = 5$



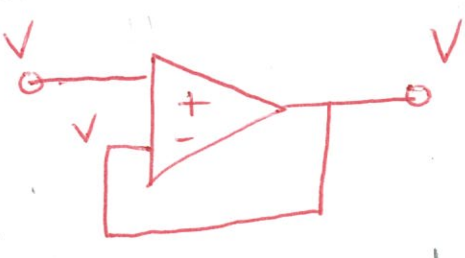


Low pass filter

$T_2(s)\,\frac{\omega_c}{s+\omega_c}$; $\omega_c = \frac{1}{RC} = \frac{1}{0.1\,10^{-6}\frac{s}{\Omega}\,10\,10^3\,\Omega} = 10^{-3}\,\frac{1}{s}$

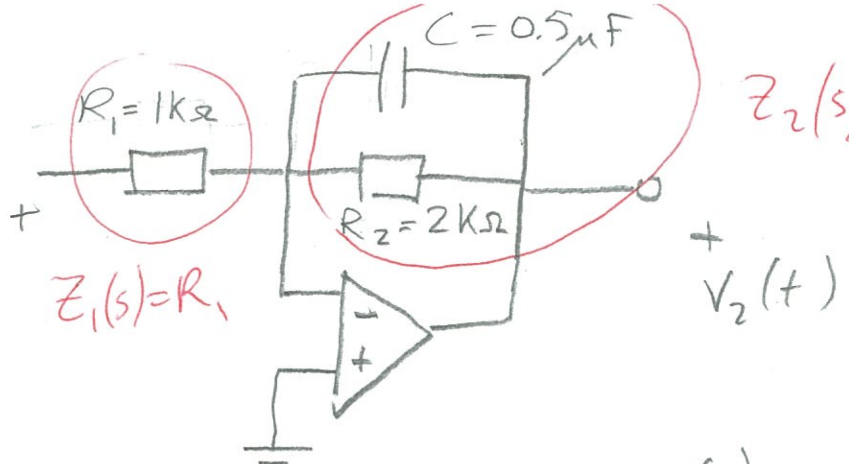
The problem is that R_4 is not ∞ (withdraws current) and R_3 is not 0 (part of the voltage $v_1(t)$ drops at R_3)

We saw in class the trick of using



in the interfaces (*) and (**).

12.35



$$Z_2(s) = \frac{1}{Cs + \frac{1}{R_2}}$$

Inverting amplifier : $T(s) = \dfrac{V_2(s)}{V_1(s)} = -\dfrac{Z_2(s)}{Z_1(s)}$

$$= -\frac{\frac{1}{Cs + \frac{1}{R_2}}}{R_1} = -\frac{1}{R_1 C s + \frac{R_1}{R_2}} = -\frac{1}{R_1 C}\,\frac{1}{s + \frac{1}{R_2 C}}$$

$$T(j\omega) = +\frac{1}{R_1 C}\,\frac{e^{i\pi}}{j\omega + \frac{1}{R_2 C}} = \frac{1}{R_1 C}\,\frac{e^{i\pi}}{\sqrt{\omega^2 + \frac{1}{R_2^2 C^2}}\; e^{i\tan^{-1}\left(\frac{\omega}{1/R_2 C}\right)}}$$

$$= \boxed{\frac{1}{R_1 C\sqrt{\omega^2 + \frac{1}{R_2^2 C^2}}}}\; e^{i\underbrace{\left(\pi - \tan^{-1}(\omega R_2 C)\right)}_{\measuredangle T(j\omega)}} \equiv |T(j\omega)|\, e^{i\measuredangle T(j\omega)}$$

$$|T(j\omega)| = \frac{1}{R_1 C\sqrt{\omega^2 + \frac{1}{(R_2 C)^2}}} = \overset{|T(j\omega)|}{\frac{1}{10^3 \times 5 \times 10^{-7}\frac{sec}{\Omega}\sqrt{\omega^2 + \left(\frac{1}{2\times 10^3_{\Omega} \times 5\times 10^{-7}\frac{sec}{\Omega}}\right)^2}}} =$$

FOR CONVENIENCE $\omega = \alpha 10^3$

$$= \boxed{\frac{1}{5\times 10^{-4}\frac{1}{sec}\sqrt{\omega^2 + 10^6\frac{1}{sec^2}}}} = \frac{1}{5\times 10^{-4}\sqrt{\alpha^2 10^6 + 10^6}} = \frac{2}{\sqrt{1+\alpha^2}}$$

$$\measuredangle T(j\omega) = \pi - \tan^{-1}(\omega R_2 C) = \boxed{\pi - \tan^{-1}(\omega 10^{-3})}$$

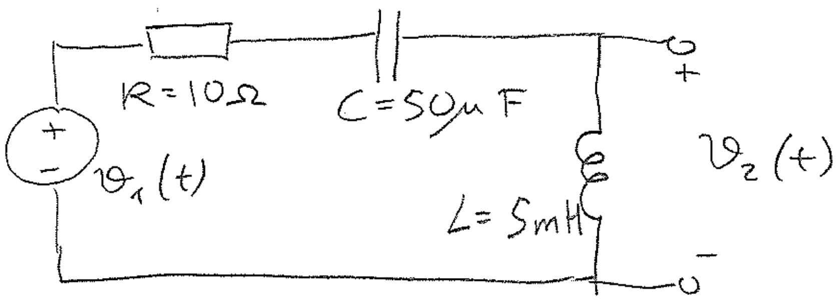
so $v_1(t) = A\cos\omega t \implies v_{2ss} = A\,|T(j\omega)|\cos(\omega t + \measuredangle T(j\omega))$

$10\cos 500t$ $\omega = 500 = \overset{\alpha}{0.5}\times 10^3$ $\implies$ $10\,\frac{2}{\sqrt{1+0.5^2}}\cos\left(\omega t + \pi - \tan^{-1}\left(\frac{1}{2}\right)\right)$

$10\cos 10^3 t$ $\omega = \overset{\alpha}{1}\times 10^3$ $\implies$ $10\,\frac{2}{\sqrt{2}}\cos\left(\omega t + \pi - \tan^{-1}(1)\right)$

$10\cos 10^4 t$ $\omega = \overset{\alpha}{10}\times 10^3$ $\implies$ $10\,\frac{2}{\sqrt{101}}\cos\left(\omega t + \pi - \tan^{-1}(10)\right)$

11.36



Voltage divider in s-domain

$$T(s) = \frac{V_2(s)}{V_1(s)} = \frac{Ls}{R + \frac{1}{Cs} + Ls} = \frac{LCs^2}{RCs + 1 + LCs^2} =$$

$$= \frac{s^2}{\frac{1}{LC} + \frac{R}{L}s + s^2} = \frac{s^2}{\frac{1}{5\times10^{-3}\times 50\times10^{-6}} + \frac{10}{5\times10^{-6}}s + s^2} = \frac{s^2}{\frac{10^8}{25} + \frac{10^7}{5}s + s^2}$$

$$= \frac{s^2}{4\times10^6 + 2\times10^6 s + s^2}$$ $\boxed{\text{with poles aprox. } -2\times10^6 \text{ and } 0}$

$$T(j\omega) = \frac{1}{10^6}\,\frac{-\omega^2}{4 + 2j\omega - \frac{\omega^2}{10^6}} = \frac{1}{10^6}\,\frac{-\omega^2}{4 - \frac{\omega^2}{10^6} + j2\omega}$$

$$= \frac{1}{10^6}\,\frac{\omega^2 e^{i\pi}}{\sqrt{\left(4 - \frac{\omega^2}{10^6}\right)^2 + 4\omega^2}\; e^{i\tan^{-1}\left(\frac{2\omega}{4 - \frac{\omega^2}{10^6}}\right)}}$$

so $$|T(j\omega)| = \frac{1}{10^6}\,\frac{\omega^2}{\sqrt{\left(4 - \frac{\omega^2}{10^6}\right)^2 + 4\omega^2}}$$

and $$\measuredangle T(j\omega) = \pi - \tan^{-1}\left(\frac{2\omega}{4 - \frac{\omega^2}{10^6}}\right)$$

$$v(t) = A\cos(\omega t) \Longrightarrow A\,|T(j\omega)|\cos(\omega t + \measuredangle T(j\omega))$$

$v_1(t) = 25\cos(2000t) \Longrightarrow$ see next page

$v_2(t) = 25\cos(10^4 t) \Longrightarrow$

We need to evaluate $|T(j\omega)|$ and $\angle T(j\omega)$ at $\omega = \alpha 10^3$ for $\alpha = 2, 10$.

$$T(j\alpha 10^3) = \frac{1}{\cancel{10^6}} \frac{\alpha^2 \cancel{10^6}}{\sqrt{(4-\alpha^2)^2 + 4\alpha^2 10^6}} \simeq \frac{\alpha^2}{2\alpha 10^3} = \frac{\alpha}{2000}$$

$$\angle T(j\alpha 10^3) = \pi - \tan^{-1}\left(\frac{2\alpha 10^3}{4-\alpha^2}\right)$$

$$= \begin{cases} \pi - \tan^{-1}(\infty) = \pi - \frac{\pi}{2} & \text{if } \alpha = 2 \\ \pi - \tan^{-1}\left(\frac{2\times 10 \times 10^3}{4-100}\right) \simeq \pi - \tan^{-1}\left(\frac{2\times 10^4}{-100}\right) \\ = \pi + \tan^{-1}(200) & \text{if } \alpha = 10 \end{cases}$$

$$v(t) = A \cos \omega t \implies v_{ss} = A|T(j\omega)| \cos(\omega t + \angle T(j\omega))$$

$$v_1(t) = 25 \cos 2000t \Rightarrow v_{2ss}(t) = \frac{25}{1000} \cos\left(2000t + \pi - \frac{\pi}{2}\right)$$

$$v_2(t) = 25 \cos 10^4 t \implies v_{2ss}(t) = \frac{25}{200} \cos\left(2000t + \pi + \tan^{-1} 200\right)$$

12.45 $T(s) = \frac{10(s+100)}{s+1}$ See next page for a shorter version.

The standard form for a bode plot is

$$T(j\omega) = \frac{10 \times 100\left(\frac{j\omega}{100}+1\right)}{j\omega+1} = 10^3 \frac{1+\frac{j\omega}{100}}{1+j\omega}$$

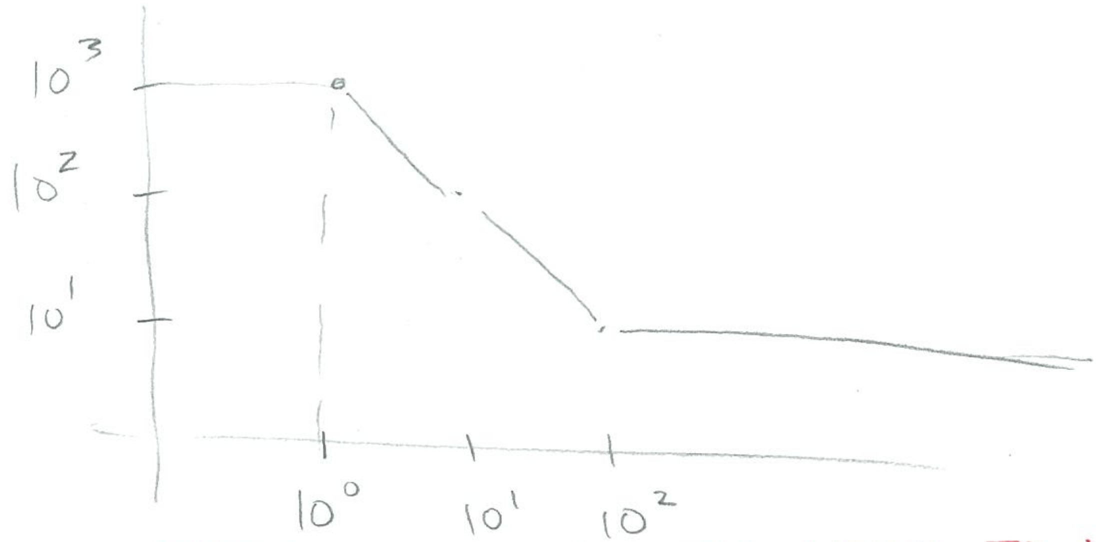
The scale factor is $K_0 = 10^3$ and the corner frequencies $\omega_c = 1$ (because there is a pole at -1) and $\omega_c = 100$ (because there's a zero at -100)

At low frequency (for $\omega < 1$ because that is the first corner) we have $|T(j\omega)| \approx 10^3 \frac{1}{1} = 10^3$ so the low frequency baseline is $|T(j\omega)|_{SL} = 10^3$ for $\omega \leq 1$.

At $\omega = 1$ rad/s, we encounter the pole -1, so the slope changes by -1 (or -20dB/decade). The straigh line gain becomes $|T(j\omega)|_{SL} = 10^3 \frac{1}{\omega}$ for $\omega \in [1, 100]$.

At $\omega = 100$ rad/s, we encounter the zero -100, so the slope changes by $+1$ (or 20dB/decade). I.e., the plot becomes horizontal. After this point the gain contribution due to the factor $1 + j\frac{\omega}{100}$ is represented by the high frequency asymptote $\frac{\omega}{100}$ and $|T(j\omega)|_{SL} = 10^3 \frac{1}{\omega} \cdot \frac{\omega}{100} = 10$. See next page

$$|T(j\omega)|_{SL} = \begin{cases} 10^3 & \text{if } 0 < \omega < 1 \\ \frac{10^3}{\omega} & \text{if } 1 < \omega < 100 \\ 10 & \text{if } 100 < \omega \end{cases}$$



THIS IS WHAT YOU NEED TO KNOW

Alternatively, we compute

$$|T(j\omega)| = 10 \frac{|j\omega+100|}{|j\omega+1|} = 10 \frac{\sqrt{\omega^2+100^2}}{\sqrt{\omega^2+1}}$$

and compute limits:

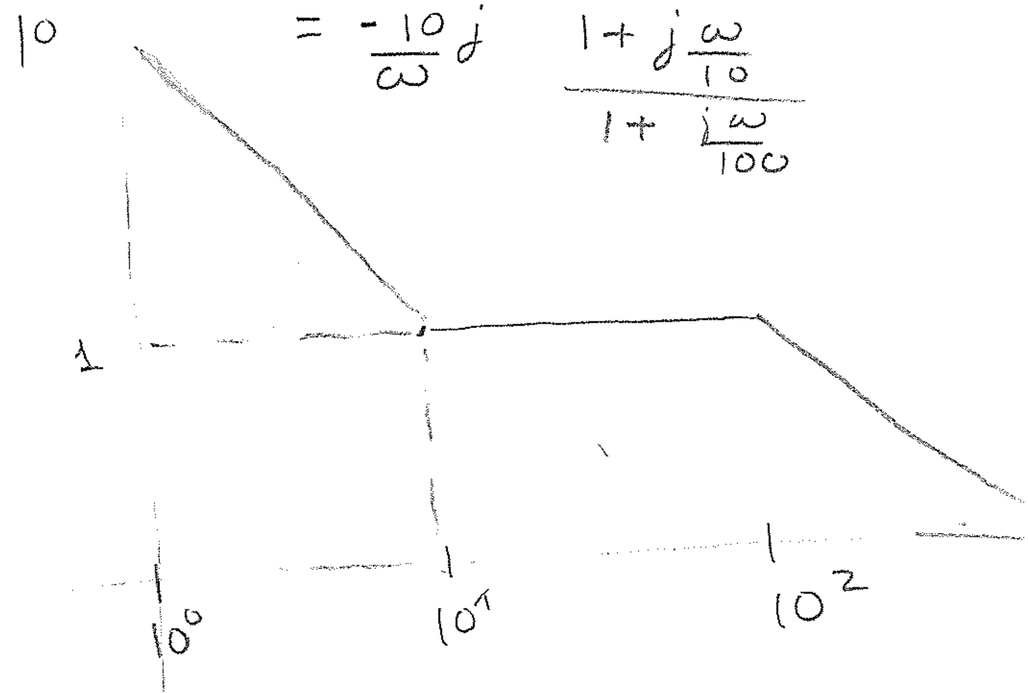
$$|T(0)| = 1000$$

$$\lim_{\omega\to\infty} |T(j\omega)| = 10$$

12.46 $T(s) = \dfrac{100(s+10)}{s(s+100)}$

$$T(j\omega) = \frac{\cancel{100}\times 10}{j\omega \times 10\cancel{0}} \cdot \frac{\left(1 + j\frac{\omega}{10}\right)}{1 + j\frac{\omega}{100}}$$

$$= \frac{-10}{\omega} j \quad \frac{1 + j\frac{\omega}{10}}{1 + j\frac{\omega}{100}}$$



$$|T(j\omega)| = \begin{cases} \dfrac{10}{\omega} & \text{if } 0 < \omega < 10 \\ \dfrac{10}{\omega} \cdot \dfrac{\omega}{10} = 1 & \text{if } 10 < \omega < 100 \\ 1 / \left(\dfrac{\omega}{100}\right) = \dfrac{100}{\omega} & \text{if } 100 < \omega \end{cases}$$

Or you can compute

$$|T(j\omega)| = 100 \frac{|j\omega + 10|}{|j\omega|\,|j\omega + 100|} = 100 \frac{\sqrt{\omega^2 + 10^2}}{\omega \sqrt{\omega^2 + 100^2}}$$

so $\lim_{\omega \to \infty} |T(j\omega)| = 0$ and $\lim_{\omega \to 0} |T(j\omega)| = \infty$