MAE140 - Linear Circuits - Winter 2015
Midterm, February 13th

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities
2. You have 50 minutes
3. Write your answers on your "Blue Book"
4. Do not forget to write your name, student number, and instructor

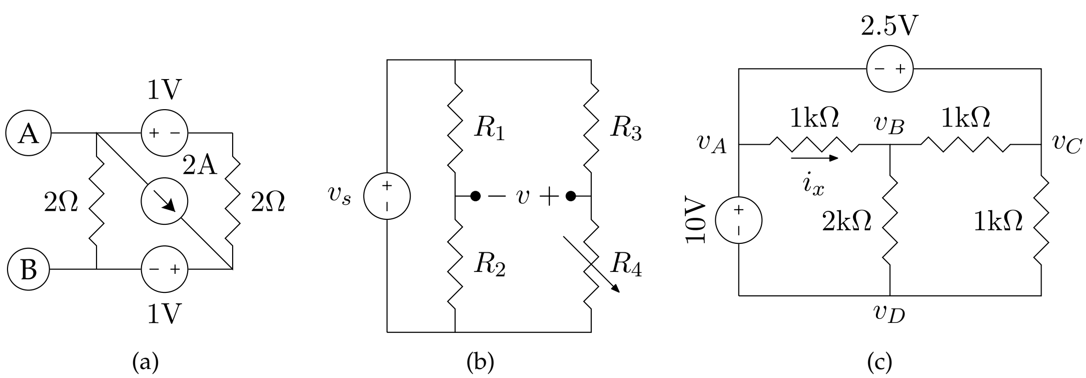
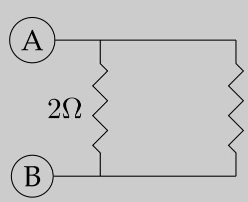


Figure 1: Circuits for all questions

1. **Equivalent Circuits**
   (a) (5 points) For the circuit in Fig. 1(a), find the Thevenin and Norton equivalent circuits as seen from terminals A and B.

Solution:

When the voltage sources are zero, i.e. replaced by a short circuit, and the current source is replaced by an open circuit the circuit becomes simply the parallel connection of two 2Ω resistors:

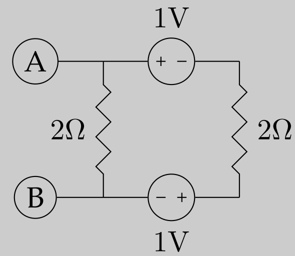


(+0.5 point)

The equivalent resistance is therefore

$$R_{EQ} = R_{AB} = \frac{1}{\frac{1}{2} + \frac{1}{2}} = 1\Omega \qquad \text{(+0.5 point)}$$

Now zero only the current source to obtain the circuit:

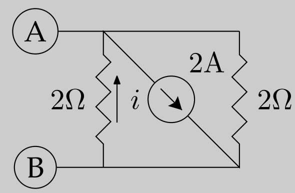


(+0.5 point)

from which

$$V_T^1 = V_{AB} = \frac{2}{2+2}2 = 1\text{V} \qquad \text{(+0.5 point)}$$

is obtained using voltage division.
Zeroing both voltage sources we obtain the circuit:



(+0.5 point)

in which the current

$$i = \frac{\frac{1}{2}}{\frac{1}{2}+\frac{1}{2}}2 = 1\text{A} \qquad \text{(+0.5 point)}$$

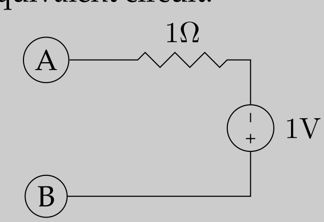
is obtained using current division and

$$V_T^2 = V_{AB} = -1\text{A} \times 2\Omega = -2\text{V} \qquad \text{(+0.5 point)}$$

Using superposition

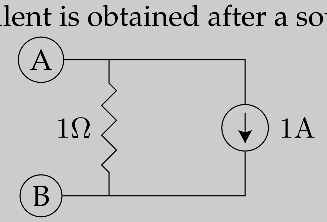
$$V_T = V_T^1 + V_T^2 = 1 + (-2) = -1\text{V} \qquad \text{(+0.5 point)}$$

This produces the Thevenin equivalent circuit:



(+0.5 point)

from which the Norton equivalent is obtained after a source transformation:



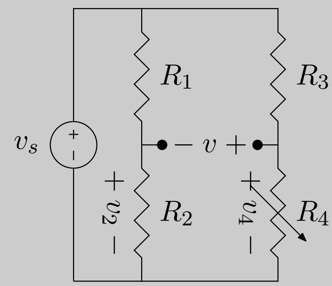
(+0.5 point)

NOTE TO GRADERS: Other solutions are possible. A popular one may be a solution by source transformations. Assign 1 point to each source transformation in this case.

2. **Voltage Division**

(a) (3 points) The circuit in Fig. 1(b) is known as a *bridge circuit*. Use voltage division to compute an expression for the voltage v.

Solution:
Refer to the following labels:



Using voltage division on the left branch we compute the voltage across the resistor R_2:

$$v_2 = \frac{R_2}{R_1 + R_2} v_s \qquad \text{(+1 point)}$$

Using voltage division on the right branch we compute the voltage across the resistor R_4:

$$v_4 = \frac{R_4}{R_3 + R_4} v_s \qquad \text{(+1 point)}$$

Using KVL we obtain that

$$v = v_4 - v_2 = \frac{R_4}{R_3 + R_4} v_s - \frac{R_2}{R_1 + R_2} v_s \qquad \text{(+1 point)}$$

(b) (2 points) When $v = 0$ we say that the bridge is *balanced*. Show that when $R_4 = R_3R_2/R_1$ the bridge is balanced.

Solution: Substitute $R_4 = R_3R_2/R_1$ into the expression from part (a):

$$v = \frac{R_3R_2/R_1}{R_3 + R_3R_2/R_1} v_s - \frac{R_2}{R_1 + R_2} v_s \qquad \text{(+1 point)}$$

and simplify to obtain:

$$v = \frac{R_2/R_1}{1 + R_2/R_1} v_s - \frac{R_2}{R_1 + R_2} v_s = \frac{R_2}{R_1 + R_2} v_s - \frac{R_2}{R_1 + R_2} v_s = 0 \qquad \text{(+1 point)}$$

NOTE TO GRADERS: one can also show that value of $v = 0$ or $v_4 = v_2$ with the provided expression for full credit.

(c) (1 point (bonus)) Explain why the answer to part (b) does not depend on v_s.

Solution: For balancing we need that each voltage divider divides v_s by the exact same ratio. If the ratio is kept the same but v_s is made different v will still be zero hence the answer must not depend on v_s! (+1 point)

(d) (1 point (bonus)) Let $R_1 = R_2 = R_3 = R$ and $R_4 = R(1+x)$. Show that $v \approx (v_s/4)x$ when x is small.

Solution:
When $R_1 = R_2 = R_3 = R$ and $R_4 = R(1+x)$

$$\begin{aligned} v &= \frac{R(1+x)}{R+R(1+x)}v_s - \frac{R}{R+R}v_s = \frac{1+x}{2+x}v_s - \frac{1}{2}v_s \\ &= \frac{2(1+x)-(2+x)}{2(2+x)}v_s = \frac{x}{4+2x}v_s \end{aligned} \qquad \text{(+0.5 point)}$$

We know that $v = 0$ when $x = 0$ and that

$$v = \frac{x}{4+2x}v_s \approx \frac{x}{4}v_s \qquad \text{(+0.5 point)}$$

when x is small. Formally $v_s/4$ is the derivative of v evaluated at $x = 0$.

3. **Node Voltage Analysis**

(a) (4 points) Choose the ground wisely and formulate node-voltage equations for the circuit in Fig. 1(c). Use the node-voltage and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations only involve node-voltages. You do not need to solve any equations.

Solution: The most convenient choice of ground is

$$v_A = 0 \qquad \text{(+1 point)}$$

Using Method #2 we have that:

$$v_C = 2.5\text{V}, \qquad v_D = -10\text{V}. \qquad \text{(+1 point)}$$

Writing KCL at node B by inspection we obtain

$$10^{-3}\left[\tfrac{1}{1}+\tfrac{1}{1}+\tfrac{1}{2} \quad -\tfrac{1}{1} \quad -\tfrac{1}{2}\right]\begin{pmatrix} v_B \\ v_C \\ v_D \end{pmatrix} = 0 \qquad \text{(+1 point)}$$

In order to solve this circuit we need to solve the above three equations on the node-voltage unknowns v_B, v_C, and v_D with the choice of ground $v_A = 0$. (+1 point)
NOTE TO GRADERS: Students have to *clearly* indicate their node-voltage equations and the unkown voltages they need to solve for to get the final point. Any reasonable set of equations is fine.
NOTE TO GRADERS: Other solutions are possible with a different choice of ground.

(b) (1 point) Show how the current i_x is related to the node-voltages.

Solution: From the circuit:

$$i_x = \frac{v_A - v_B}{1k} \qquad \text{(+1 point)}$$

With the choice of ground $v_A = 0$ then $i_x = -v_B/1k$.