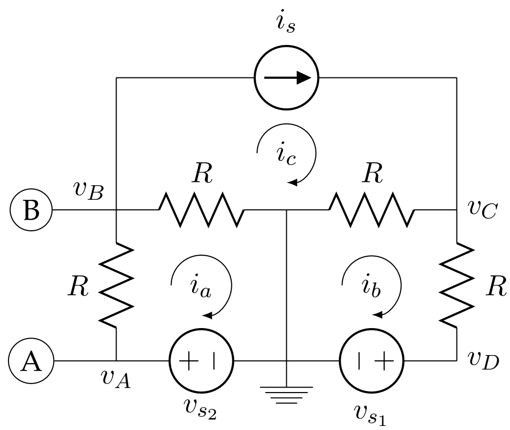
MAE140 - Linear Circuits - Fall 2019
Midterm, October 24th

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
2. You have 70 minutes.
3. Write your answers on your "Blue Book".
4. Do not forget to write your name, student number, and instructor.
5. This exam has 2 questions, for a total of 22 points.



1. **Systematic Circuit Analysis**

   (a) (6 points) Formulate either node-voltage or mesh-current equations for the circuit above. Use the node-voltages, mesh-currents and labels provided in the figure and clearly indicate the final equations and circuit variable unknowns. Make sure your final equations involve only node-voltages or mesh-currents. You do not need to solve any equations.

Solution:
One solution is via nodal analysis. One can use method # 2 to write KCL at nodes B and C:

$$\begin{bmatrix} -\frac{1}{R} & \frac{2}{R} & 0 & 0 \\ 0 & 0 & \frac{2}{R} & -\frac{1}{R} \end{bmatrix} \begin{pmatrix} v_A \\ v_B \\ v_C \\ v_D \end{pmatrix} = \begin{pmatrix} -i_s \\ i_s \end{pmatrix} \qquad \text{(+3 points)}$$

along with the two equations

$$v_A = v_{s_2}, \qquad v_D = v_{s_1}. \qquad \text{(+2 points)}$$

Those are all the equations that need to be solved in terms of the node voltages v_A, v_B, v_C, and v_D. (+1 points)

Alternatively, via mesh analysis. One can use method # 2 to write KVL at meshes a and b:

$$\begin{bmatrix} 2R & 0 & -R \\ 0 & 2R & -R \end{bmatrix} \begin{pmatrix} i_a \\ i_b \\ i_c \end{pmatrix} = \begin{pmatrix} +v_{s_2} \\ -v_{s_1} \end{pmatrix} \qquad \text{(+4 points)}$$

along with the one equation

$$i_c = i_s \qquad \text{(+1 point)}$$

Those are all the equations that need to be solved in terms of the mesh currents i_a, i_b, and i_c. (+1 point)

(b) (2 points) Explain how one could solve for the voltage across the current source i_s in terms of the indicated mesh-currents.

Solution:
The voltage across the current source i_s is the voltage $v_B - v_C$. From the circuit diagram:

$$v_B = R(i_a - i_c), \qquad v_C = -R(i_b - i_c) \qquad \text{(+1 point)}$$

hence

$$v_B - v_C = Ri_a + Ri_b - 2Ri_c \qquad \text{(+1 point)}$$

NOTE TO GRADERS: Multiple solutions are possible! Calculating $v_C - v_B$ is fine as well.
Here is another possible solution:

$$v_B = v_{s_2} - Ri_a, \qquad v_C = v_{s_1} + Ri_b \qquad \text{(+1 point)}$$

hence

$$v_B - v_C = v_{s_2} - v_{s_1} - Ri_a - Ri_b \qquad \text{(+1 point)}$$

(c) (2 points) Explain how one could solve for the current through the voltage source v_{s_1} in terms of the indicated node-voltages.

Solution:
The current through the voltage source v_{s_1} is the current through the resistor connected to noder C and D. From the circuit diagram:

$$i_{s_1} = \frac{v_C - v_D}{R} \qquad \text{(+2 points)}$$

NOTE TO GRADERS: Multiple solutions are possible! Calculating $v_D - v_C$ is fine as well.

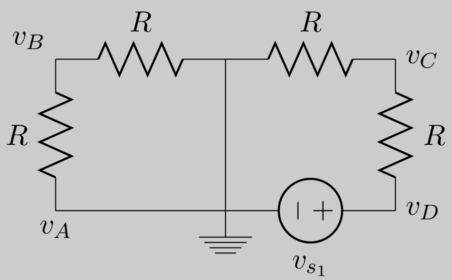
2. **Linearity and Equivalent Circuits**

(a) (6 points) Draw diagrams for the three circuits resulting from setting all sources in the above circuit to zero then turning each one of them on, one at a time.

Solution:

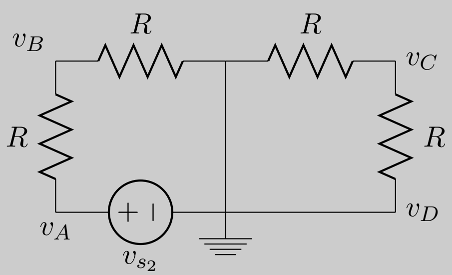
The three circuits are:

1) $i_s = 0$, $v_{s_2} = 0$:



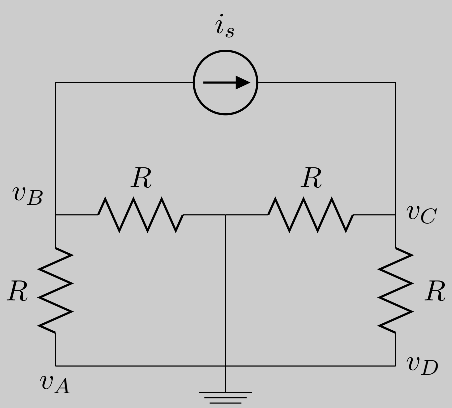
(+2 points)

2) $i_s = 0$, $v_{s_1} = 0$:



(+2 points)

3) $v_{s_1} = 0$, $v_{s_2} = 0$:



(+2 points)

(b) (4 points) Use linearity and superposition to calculate the voltage difference $v_B - v_A$ using the circuits you obtained in part (a). Clearly show how you solve each circuit and the final expression for $v_B - v_A$.

Solution:

On circuit 1), $v_A^1 = v_B^1 = 0$, since their loop does not have any sources. (+1 point)

On circuit 2), $v_B^2 - v_A^2 = -\frac{R}{2R} v_{s_2}$, using voltage division. (+1 point)

On circuit 3), $v_B^3 - v_A^3 = -\frac{R}{2} i_s$, after associating the resistors in parallel. (+1 point)

Using superposition:

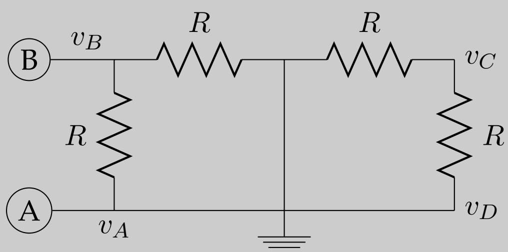
$$\begin{aligned} v_B - v_A &= v_B^1 - v_A^1 + v_B^2 - v_A^2 + v_B^3 - v_A^3 \\ &= 0 - \frac{v_{s_2}}{2} - \frac{Ri_s}{2} \end{aligned} \qquad \text{(+1 point)}$$

(c) (2 points) Calculate the Thevenin equivalent of the above circuit as seen from the terminals A – B. *Hint: Use the solution from part (b).*

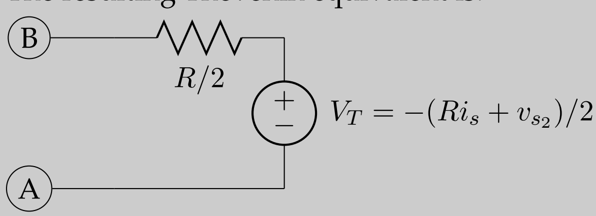
Solution:
Part (b) gives us the Thevenin voltage

$$V_T = v_B - v_A = -\frac{Ri_s}{2} - \frac{v_{s_2}}{2} \qquad \text{(+1/2 point)}$$

The resistance is obtained by zeroing all the sources as in:



which is simply R in parallel with R, or $R_{EQ} = R/2$ (+1 point)
The resulting Thevenin equivalent is:



(+1/2 point)