MAE140 - Linear Circuits - Fall 2019
Midterm, November 19th

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
2. You have 70 minutes.
3. Write your answers on your "Blue Book".
4. Do not forget to write your name, student number, and instructor.
5. This exam has 3 questions, for a total of 23 points.

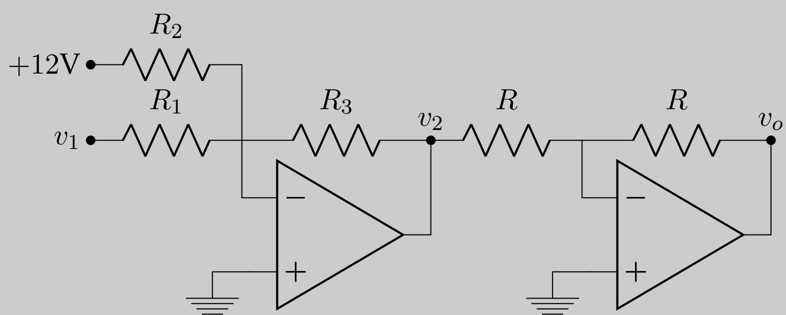
1. (6 points) **OpAmp Design**

   Assume that sources $+V_{cc} = 12V$ and $-V_{cc} = -12\text{V}$ are available and design an opAmp circuit that implements the following operation

$$v_o = 6\text{V} + 10\, v_1$$

without using any voltage or current source other than v_1 and the available voltages $+V_{cc}$ and $-V_{cc}$. You can use as many opAmps as you would like.

Solution: Perhaps the simplest possible solution is to use an inverting summer followed by an inverting opamp as in:



(+3 points)

In this circuit:

$$v_2 = -\frac{R_3}{R_2}12\text{V} - \frac{R_3}{R_1}v_1, \qquad v_o = -v_2 = \frac{R_3}{R_2}12 + \frac{R_3}{R_1}v_1 \qquad \text{(+2 points)}$$

so that any choice of resistors such that

$$\frac{R_3}{R_1} = 10, \qquad \frac{R_3}{R_2} = 1/2 \qquad \text{(+1 point)}$$

solve the question.

NOTE TO GRADERS: Any correct answer is valid. 3 points assigned to the correct circuit, 3 points to the correct design analysis and parameters.

2. (4 points) **Photodetector**

The circuit in Fig. 1 is used to produce a voltage in response to light. When the base of the phototransistor shown in the picture is exposed to light the transistor goes into saturation, that is it acts essentially as a short circuit. When the base is in the dark the transistor is cut, that is it is an open circuit.

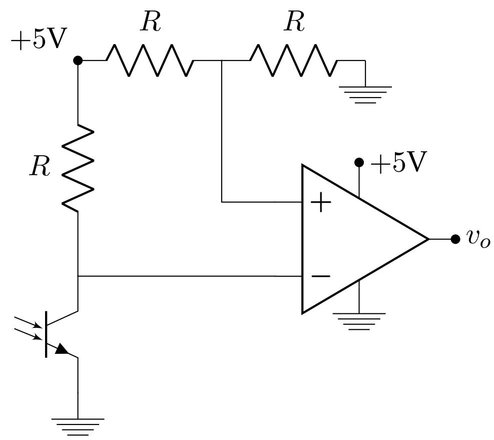


Figure 1: Phototransistor circuit

Explain how the output v_o responds in the presence of a light or dark environment.

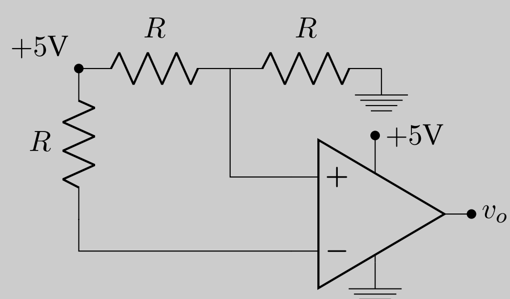
Solution:

The opAmp does not have a feedback resistor and therefore acts as a comparator:

$$v_o = \begin{cases} -V_{\text{cc}} = 0, & v_P - v_N < 0, \\ +V_{\text{cc}} = 5, & v_P - v_N > 0 \end{cases} \qquad \text{(+1 point)}$$

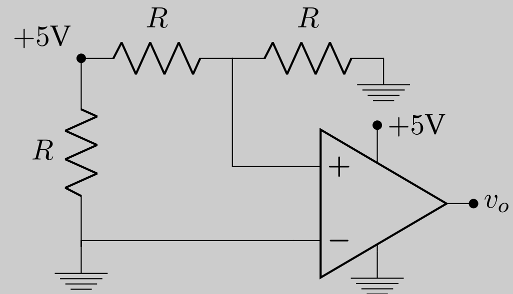
Note that $v_p = 2.5$V due to voltage division. (+1 point)

Each case is as follows. In the dark:

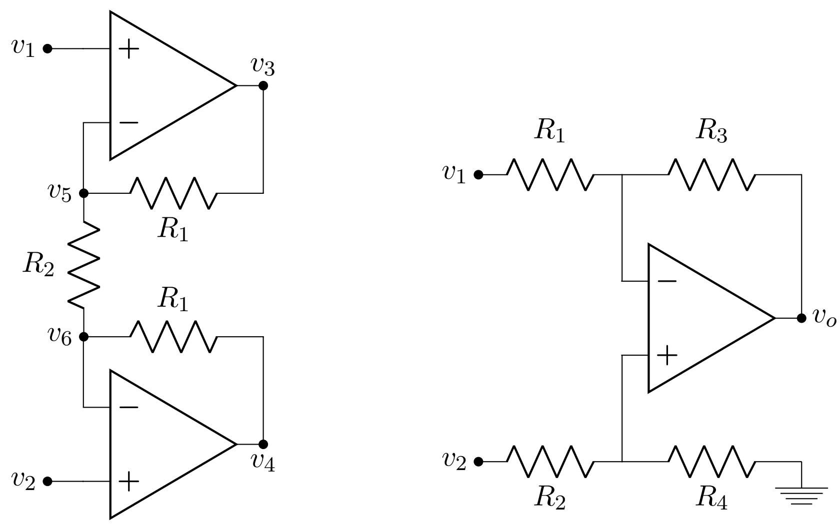


so that $v_N = 5$V, $v_P - v_N < 0$ and $v_o = 0$V. (+1 point)

In the light



so that $v_N = 0\text{V}$, $v_P - v_N > 0$ and $v_o = 5\text{V}$. (+1 point)



(a) Differential Stage

(b) Subtractor Stage

3. **Instrumentation Amplifier**.

   In the rest of the exam you will analyze a circuit called *instrumentation amplifier*, which is made of a combination of the circuits in Figs. 2(a) and 2(b).

   (a) (5 points) Formulate node voltage analysis equations relating the voltages v_1 through v_6 in the circuit in Fig. 2(a) and use those equations to show that

$$v_3 = \left(1 + \frac{R_1}{R_2}\right) v_1 - \frac{R_1}{R_2} v_2,$$
$$v_4 = -\frac{R_1}{R_2} v_1 + \left(1 + \frac{R_1}{R_2}\right) v_2.$$

Solution: We shall use method # 2 not to write KCL at nodes v_1, v_2, v_3 and v_4. (+1 point)

KCL at the remaining nodes can be obtained by inspection

$$\begin{bmatrix} -\frac{1}{R_1} & 0 & \frac{1}{R_1}+\frac{1}{R_2} & -\frac{1}{R_2} \\ 0 & -\frac{1}{R_1} & -\frac{1}{R_2} & \frac{1}{R_1}+\frac{1}{R_2} \end{bmatrix} \begin{pmatrix} v_3 \\ v_4 \\ v_5 \\ v_6 \end{pmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad \text{(+2 points)}$$

along with the opAmp relationships

$$v_5 = v_1, \qquad v_6 = v_2 \qquad \text{(+1 point)}$$

The expression in the problem statement can be obtained after multiplying by R_1 and solving for v_3 and v_4. (+1 point)

(b) (2 points) We have shown in class that

$$v_o = \frac{R_1 + R_3}{R_1}\frac{R_4}{R_2 + R_4}v_2 - \frac{R_3}{R_1}v_1$$

for the circuit in Fig. 2(b). Show that if the circuit in Fig. 2(b) is to have a gain of the form

$$v_o = K(v_2 - v_1), \quad K > 0,$$

then

$$K = \frac{R_3}{R_1} = \frac{R_4}{R_2}.$$

Solution: We need to have

$$\frac{R_1 + R_3}{R_1}\frac{R_4}{R_2 + R_4} = \frac{R_3}{R_1} = K.$$

That is

$$(1 + K)\frac{1}{1 + \frac{R_2}{R_4}} = K,$$

or

$$1 + \frac{R_2}{R_4} = \frac{1 + K}{K} = 1 + \frac{1}{K}$$

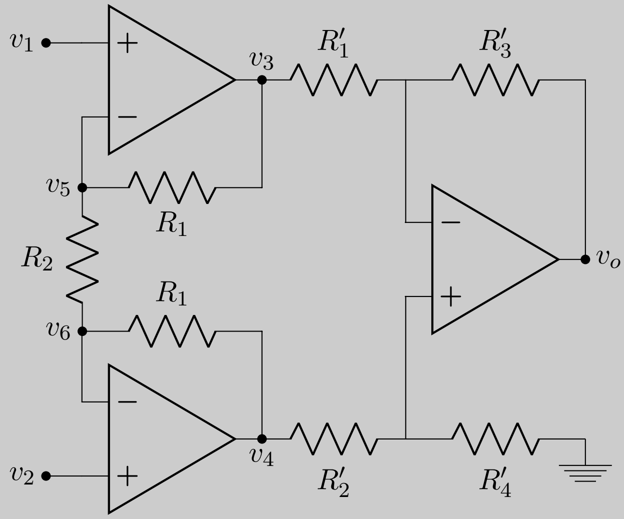
so that $K = \frac{R_4}{R_2}$. (+2 points)

(c) (4 points) Connect the outputs v_3 and v_4 from the circuit in Fig. 2(a) to the inputs v_1 and v_2 from the circuit in Fig. 2(b). The resulting circuit is an *instrumentation amplifier*. Show that in this new circuit

$$v_o = \left(1 + 2\frac{R_1}{R_2}\right)(v_2 - v_1).$$

Clearly explain your reasoning.

Solution:
The resulting circuit is the following:



(+1 point)

Because the nodes v_3 and v_4 are the output of opAmps we can simply use the formulas from parts (a) and (b). (+1 point)

Assuming that $K = R_3'/R_1' = R_4'/R_2'$ we obtain

$$\begin{aligned} v_o &= K(v_3 - v_4) \\ &= K\left(\left(1 + \frac{R_1}{R_2}\right)v_1 - \frac{R_1}{R_2}v_2 + \frac{R_1}{R_2}v_1 - \left(1 + \frac{R_1}{R_2}\right)v_2\right) \\ &= K\left(1 + 2\frac{R_1}{R_2}\right)(v_1 - v_2) \end{aligned}$$ (+1 point)

which reduces to the formula in the statement after a choice of $K = 1$, for example, $R_1' = R_2' = R_3' = R_4' = R$. (+1 point)

(d) (2 points) A student needs to measure a voltage out of a sensor that can be modeled as a voltage source in series with a 100Ω resistor. He intends to use either the circuit in Fig. 2(b) setup as in part (b) or the circuit you designed in part (c). Explain which circuit is better suited to this task?

Solution: Both circuits can have the same gain, however the circuit in part (c) is such that the input resistances are very large, that is the input current are very small. This will ensure that no voltage drop on the sensor resistor will affect the measurements, making the circuit from part (c) better suited for this task. (+2 points)