MAE143A – Signals and Systems – Fall – 2019
Final, December 13th

Instructions

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a handheld calculator with no communication capabilities.
2. You have 170 minutes.
3. Do not forget to write your name and student number.
4. This exam has 7 questions, for a total of 70 points and 1 bonus points.

Have a great Winter break!

$$\int te^{-at}dt = -\frac{(1+at)}{a^2}e^{-at}, \qquad \mathcal{L}\{t\,e^{-at}1(t)\} = \frac{1}{(s+a)^2}, \qquad \mathcal{Z}\{a^n 1[n]\} = \frac{1}{1-az^{-1}}$$

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1. **Signal**

   Let

$$g(t) = \begin{cases} 0, & t < -1 \\ t+1, & -1 \le t < 0 \\ 1, & 0 \le t < 1 \\ -t+2, & 1 \le t < 2 \\ 0, & t > 2 \end{cases}$$

   Sketch the signals:
   (a) (2 points) $g(t)$
   (b) (2 points) $g(1-t)$
   (c) (2 points) $-g(2t)$
   (d) (2 points) $g(0.5n)$, $n \in \mathbb{Z}$
   (e) (2 points) $g(2n)$, $n \in \mathbb{Z}$

2. **Fourier Series**

   The signal

$$g_p(t) = \sum_{k=-\infty}^{\infty} g(t - 4k + 1/2)$$

   is periodic.
   (a) (2 points) Sketch $g_p(t)$.
   (b) (1 point) Calculate its period.
   (c) (5 points) Show that

$$a_k = \begin{cases} \frac{1}{2}, & k = 0 \\ \frac{8}{k^2\pi^2}\cos(\frac{k\pi}{4})\sin(\frac{k\pi}{4})^2, & k \neq 0 \end{cases}$$

   are the coefficients of the corresponding exponential Fourier Series.
   (d) (2 points) Sketch $|a_k|$.

3. **Convolution**

   Let $g(t)$ from Question 1 be the impulse response of a time-invariant continuous-time system and answer the following questions. Show your reasoning.
   (a) (2 points) Is the system causal?
   (b) (2 points) Is the system memoryless?
   (c) (2 points) Is the system asymptotically stable?
   (d) (4 points) Use the convolution formula

$$y(t) = \int_{-\infty}^{\infty} g(t-\tau)\,u(\tau)\,d\tau$$

   to calculate and plot the system's step response.

4. **Continuous-Time System**

   Consider a continuous-time system described by the differential equation

   $$\ddot{y} + 2\dot{y} + y = u$$

   and answer the following questions. Show your reasoning.

   (a) (1 point) Is the system time-invariant?

   (b) (2 points) Use the Laplace transform to calculate the system's transfer-function.

   (c) (2 points) Is the system asymptotically stable?

   (d) (3 points) Calculate and sketch the system's impulse response.

   (e) (2 points) Calculate and sketch the system's response to a zero input and initial conditions $\dot{y}(0) = 1$, $y(0) = 0$.

   Hint: This is simpler than you think!

5. **Frequency Response**

   Consider a continuous-time system described by the differential equation

   $$\ddot{y} + 2\dot{y} + y = u$$

   and answer the following questions.

   (a) (1 point) Calculate the system's frequency response function $G(j\omega)$.

   (b) (3 points) Evaluate $|G(j\omega)|$ and $\angle G(j\omega)$ at $\omega = \{0, 1, \infty\}$.

   (c) (3 points) Sketch $|G(j\omega)|$ and $\angle G(j\omega)$.

   (d) (1 point) What type of filter is this system?

   (e) (2 points) Calculate the steady-state response of the system to the input

   $$u(t) = \cos(t)1(t).$$

6. **Discretized System**

   Consider a continuous-time system described by the differential equation

   $$\ddot{y} + 2\dot{y} + y = u$$

   and answer the following questions. Show your reasoning.

   (a) (3 points) Apply the Tustin transformation

   $$s = \frac{2}{T_s}\frac{z-1}{z+1}, \quad z = \frac{1 + sT_s/2}{1 - sT_s/2}, \quad T_s > 0$$

   to calculate the discretized transfer-function $G[z]$.

   (b) (3 points) Calculate the difference equation associated with $G[z]$.

   (c) (2 points) Calculate the poles of $G[z]$.

   (d) (2 points) Is the discretized system asymptotically stable?

   (e) (1 point (bonus)) Assuming that the system will be subject to low-frequency inputs, what would you believe to be a suitable value for T_s?

7. **Discrete-Time System**

   Consider a discrete-time system described by the difference equation

   $$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = -u[n] - 2u[n-1] - u[n-2]$$

   and answer the following questions. Show your reasoning.

   (a) (1 point) Is the system causal?

   (b) (3 points) Use the Z-transform to calculate the system's transfer-function.

   (c) (2 points) Calculate the transfer-function poles and zeros.

   (d) (1 point) Is the system asymptotically stable?

   (e) (3 points) Calculate and sketch the system's impulse response.