MAE 143 A - Fall 2019

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Homework 4

1. Exercise 4.10 from Boulet FSS
2. Exercise 5.9 from Boulet FSS
3. A *Butterworth filter* of order N and *cutoff frequency* equal to one is a fitler in which the spectral density is given by

$$|G(j\omega)| = \frac{1}{\sqrt{1+\omega^{2N}}}.$$

Answer the following questions:

a) Show that $|G(j0)| = 1$ for all N.

b) Show that $|G(j1)| = 1/\sqrt{2}$ for all N.

c) Show that $\lim_{\omega\to\infty} |G(j\omega)| = 0$ for all N.

d) What type of filter is a Butterworth filter?

e) Use the fact that if $G(j\omega)$ is the transfer-function corresponding to a linear system with real coefficients then $G(j\omega)^* = G(-j\omega)$ and

$$|G(j\omega)|^2 = G(j\omega)^* G(j\omega) = G(-j\omega)G(j\omega)$$

to show that an *asymptotically stable* second-order Butterworth filter ($N = 2$) has a transfer-function of the form

$$G(j\omega) = \frac{1}{(s-s_1)(s-s_2)}$$

in which s_1 and s_2 are on the unit circle.

f) Explain how the roots s_1 and s_2 above can be chosen to have negative real part. Use the calculator of your choice to calculate the roots and plot the corresponding $|G(j\omega)|$ and $\angle G(j\omega)$.

g) Write the differential equation corresponding to the transfer-function of a second-order Butterworth filter and cutoff frequency one.

h) A *Butterworth filter* of order N and *cutoff frequency* equal to ω_c is a fitler in which the spectral density is given by

$$|G(j\omega)| = \frac{1}{\sqrt{1+(\omega/\omega_c)^{2N}}}$$

Explain how the different cutoff frequency affects the roots of the corresponding filter. How does it affect the other properties of the filter?