MAE143A - Signals and Systems - Fall 2019
Midterm 2, November 21st

Instructions

1. This exam is open book. You may use whatever written or printed materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
2. Please no visible or at reach cell phone, computer, tablet or electronic device. A visible device will get you dismissed from this exam.
3. You have 70 minutes.
4. Write your answers on your "Blue Book".
5. Do not forget to write your name, student number, and instructor.
6. This exam has 3 questions, for a total of 27 points and 2 bonus points.

HINT #1: Do not get stuck!
HINT #2: If a question gives you the answer needed to do another question use it!

1. **Fourier Transform**

   Consider a periodic signal $u(t)$ for which one period is defined as

   $$u(t) = |t|, \quad -1 \leq t \leq 1$$

   and answer the following questions:

   (a) (2 points) Sketch the signal $u(t)$.

   (b) (1 point (bonus)) Show that the coefficients of the exponential Fourier series of $u(t)$ are

   $$a_k = \begin{cases} \frac{1}{2}, & k = 0 \\ 0, & k \neq 0 \text{ and even} \\ \frac{-2}{\pi^2 k^2}, & k \text{ odd} \end{cases}$$

   (c) (2 points) Sketch the line spectrum $|a_k|$ of $u(t)$ as a function of k.

   (d) (2 points) Use part (b) to calculate the Fourier transform of the signal $u(t)$.

2. **Frequency-domain Filtering**

Consider a linear time-invariant filter defined by the frequency response function

$$G(j\omega) = \begin{cases} 1 - |\omega|/(2\pi) & |\omega| \leq 2\pi, \\ 0 & |\omega| \geq 2\pi, \end{cases}$$

and answer the following questions:

(a) (3 points) Sketch the magnitude $|G(j\omega)|$ and phase $\angle G(j\omega)$ of the frequency response.

(b) (1 point) What type of filter is G? E.g low-pass, high-pass, etc.

(c) (1 point (bonus)) Show that the impulse response of the above filter is given by

$$g(t) = \frac{\sin(\pi t)^2}{\pi^2 t^2}.$$

(d) (1 point) Is this filter causal? Explain.

(e) (2 points) Is this filter asymptotically stable? Explain.
Hint: $|g(t)| \leq 1/(\pi t)^2$, $|t| \geq 1/\pi$ *and* $|g(t)| \leq 1$, $|t| \leq 1/\pi$.

(f) (2 points) Calculate the steady-state response of the filter to the periodic input $u(t)$ from Problem 1.

3. **Time-domain Filtering**

Consider the linear time-invariant system defined by the ODE

$$\dot{y} = -y + u - \dot{u}$$

and answer the following questions:

(a) (2 points) Assume zero initial-conditions and use the Laplace or the Fourier transform to calculate the system's transfer-function $G(s)$ or $G(j\omega)$.

(b) (3 points) Sketch the magnitude $|G(j\omega)|$ and phase $\angle G(j\omega)$ of the frequency response.

(c) (1 point) The type of filter of the above G is known as *all-pass*. Can you explain why?

(d) (2 points) Use the inverse Laplace or Fourier transform to calculate the impulse response $g(t)$.

(e) (1 point) Is this system causal? Explain.

(f) (1 point) Is this system asymptotically stable? Explain.

(g) (2 points) Calculate the the steady-state response of the system to the periodic input $u(t) = \cos(t)$.