MAE143A - Signals and Systems - Fall 2020
Final, December 15th

Instructions

1. You have 2 hours to finish this exam.
2. You have until 10:10AM to submit your answers on gradescope.
3. The exam is to be done individually.
4. All answers should be manuscript.
5. No typed answers of any form will be accepted.
6. All graphs and diagrams have to be sketched by hand. Include enough detail.
7. If you use a tablet to do the work make sure it is handwritten.
8. This exam has 3 questions, for a total of 30 points.

1. **Signals and Transforms**

   Consider the continuous-time signal

   $$x(t) = 2^{-t}\, 1(t)$$

   and answer the following questions.

   (a) (3 points) Sketch the signals $x(t)$, $x(t-1)$, $x(-t)$.

   (b) (1 point) What is the Laplace transform of $x(t)$ and its region of convergence?

   Hint #1: calculate a such that $2^{-t} = e^{-at}$.

   Hint #2: answer from a table is fine!

   (c) (1 point) Can you calculate the Fourier transform of $x(t)$ from its Laplace transform? Explain.

   (d) (1 point) Calculate the generalized derivative $\dot{x}(t)$.

   Hint: calculate a such that $2^{-t} = e^{-at}$.

   (e) (1 point) Calculate $\dot{x}(t)$ using the Laplace transform.

   Hint: calculate $\dot{x}(t) = \mathcal{L}^{-1}\{?\}$

   (f) (1 point) Sketch the signal $x[k] = x(k)$.

   (g) (1 point) What is the z-transform of $x[k] = x(k)$?

   Hint: answer from a table is fine!

2. **Continuous-time System**

The velocity of a vehicle traveling on a straight line, $y(t)$, is given by the following ODE:

$$m\,\dot{y}(t) + b\,y(t) = u(t)$$

in which $m > 0$ is the vehicle's mass, $b > 0$ is a coefficient of viscous friction, and $u(t)$ is an external input force.

(a) (2 points) Calculate the system's transfer-function $G(s)$ and its region of convergence.

(b) (1 point) Calculate the transfer-function's poles and zeros.

(c) (1 point) Calculate the system's impulse response.

(d) (1 point) Is the system asymptotically stable? Explain.

(e) (3 points) Calculate the system's response to a unit step force input. Do not assume zero initial conditions.

(f) (3 points) Let $m = b = 1$, calculate $|G(j\omega)|$ and $\angle G(j\omega)$. Evaluate $|G(j\omega)|$ for $\omega = \{0, 1, \infty\}$, and sketch the magnitude of the frequency response function $G(j\omega)$.

(g) (2 points) Let $m = b = 1$, and use the frequency response to calculate the steady-state component of the response to an input of the form

$$u(t) = \cos(t)$$

Explain why is this possible.

Hint: use answer from previous question.

3. **Discrete-time System**

A discrete-time system is modeled by the difference equation

$$y[k+1] = u[k+1] - u[k]$$

(a) (2 points) Calculate the system's transfer-function $G(z)$ and its region of convergence.

(b) (1 point) Calculate the transfer-function's poles and zeros.

(c) (1 point) Is the system causal? Explain.

(d) (1 point) Is the system asymptotically stable? Explain.

(e) (3 points) Assume zero-initial state and calculate the response of the system to a unit step input.