

MAE143A - Signals and Systems - Fall 2021
Midterm #1, October 21st

Instructions

1. This exam is open book. You may use whatever written or printed materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities.
2. You have 70 minutes.
3. Write your answers on your “Blue Book”.
4. Do not forget to write your name, student number, and instructor.
5. This exam has 5 questions, for a total of 45 points.

1. Signals

Consider the signal

$$x(t) = \cos(2\pi t) (1(t) - 1(t - 1)),$$

and answer the following questions:

- (a) (3 points) Sketch the signal $x(t)$;
- (b) (3 points) Sketch the signal $x(1 - t)$;
- (c) (3 points) Sketch the signal $x[k] = x(kT)$, $T = 1/8$;

NOTE: Make sure your sketches contain all information needed to clearly identify the signals.

2. Continuous-Time Impulse Response

Consider a continuous-time LTI system with impulse response

$$g(t) = e^{-t} 1(t),$$

and answer the following questions:

- (a) (2 points) Is this system causal? Explain.
- (b) (4 points) Use convolution to calculate the response to a unit step, $u(t) = 1(t)$. Sketch the response.
- (c) (3 points) Is the system BIBO stable? Show your work.

3. ODE

Consider a causal LTI system modeled by the ODE

$$2\dot{y}(t) + y(t) = u(t)$$

and answer the following questions:

- (a) (3 points) Calculate the system's transfer-function and its region of convergence.
- (b) (2 points) Is this system asymptotically stable? Explain.
- (c) (4 points) Use the transfer-function from part (a) to calculate the response of the system to the input $u(t) = \cos(t)$. Explain the assumptions needed in your approach.

4. Difference Equation

Consider a causal discrete-time LTI system described by the difference equation

$$y[k+1] - y[k] = u[k+1] + u[k]$$

and answer the following questions:

- (a) (3 points) Calculate the system's transfer-function and its region of convergence.
- (b) (2 points) Is this system asymptotically stable? Explain.
- (c) (4 points) Calculate the first 5 terms of the system's impulse response.

5. Rational Transfer-Function

Consider the following transfer-function

$$G(s) = \frac{s}{s^2 + 3s + 2}$$

which is associated with a causal LTI continuous-time system and answer the following questions:

- (a) (3 points) What is the transfer-function's region of convergence?
- (b) (2 points) Is the system asymptotically stable?
- (c) (4 points) Expand the function in partial fractions.