This note will help you fill up the gaps in FPE-N p. 5.35.

1 Models

With the 1:1 gear ratio, the mechanical model of the DC-motor is, from Newton's law,

$$(J_m + J_L)\,\ddot{\theta}_o = \tau. \tag{1}$$

The relationship between the torque τ and the motor current i_a is given by the *torque constant* K_t:

$$\tau = K_t i_a. \tag{2}$$

This is a positioning system and hence the velocities of the motor are small and the model for the DC-motor is given by Ohm's Law:

$$v_a = R_a i_a. \tag{3}$$

The transfer-function for the feedback filter can be approximated by a voltage-divider in the s-domain (use your MAE 140 skills). Let $R = 1$ K Ω. First associate the capacitor and resistor:

$$Z(s) = \frac{1}{sC + \frac{1}{R}} = R\frac{\frac{1}{RC}}{s + \frac{1}{RC}} = R\frac{\omega_c}{s + \omega_c}, \qquad \omega_c = \frac{1}{RC},$$

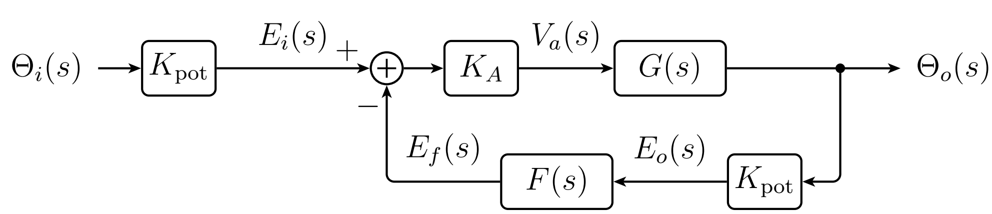
then use the voltage-divider

$$\frac{E_f(s)}{E_o(s)} = \frac{Z(s)}{R + Z(s)} = \frac{R\frac{\omega_c}{s+\omega_c}}{R + R\frac{\omega_c}{s+\omega_c}} = \frac{\omega_c}{s + 2\,\omega_c}. \tag{4}$$

2 Questions

Answers the following questions, instead of (a) and (b) given in the book!

(a) Use the equations provided in p. 5.35 and (1)–(4) to find $G(s)$ and $F(s)$ so that the servomechanism can be represented by the block-diagram:



Use this block-diagram to answer the next questions.
Hint: Use matlab's rlocus and rlocfind

(b) What is the range of the amplifier gain K_A for which the closed-loop system is stable?

(c) Replace K_A with a PD-type controller $K(s) = K_A(s+1)$. What is the range of the amplifier gain K_A for which the closed-loop system is stable?

(d) Select the smallest value of K_A with which a PD-type controller $K(s) = K_A(s+1)$ places two complex-conjugate roots with a corresponding $\zeta = 0.7$? Locate all three roots for this value of K_A.

(e) (Bonus question) Using the value of K_A you chose in (c), use matlab to simulate the closed-loop response to a unit-step on the reference angle θ_i.