Hw #1 Solutions

Problem 3.2 (c)

given: $e^{-t} + 2e^{-2t} + te^{-3t}$

Find the Laplace transform

Solution: $f(t) = e^{-t} + 2e^{-2t} + te^{-3t}$

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{e^{-t}\} + \mathcal{L}\{2e^{-2t}\} + \mathcal{L}\{te^{-3t}\}$$

Using the look-up tables on the front cover of Franklin:

$$\mathcal{L}\{f(t)\} = \boxed{F(s) = \frac{1}{s+1} + \frac{2}{s+2} + \frac{1}{(s+3)^2}}$$

Problem 3.3 (a)

given: $f(t) = 3\cos(6t)$

Find the Laplace transform

Solution:

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{3\cos(6t)\}$$

Using look-up table:

$$\mathcal{L}\{f(t)\} = \boxed{F(s) = 3\frac{s}{s^2+36}}$$

Problem 3.3 (b)

given: $\sin 2t + 2\cos 2t + e^{-t}\sin 2t$

Find the Laplace Transform

Solution:

$$\mathcal{L}\{f(t)\} = \mathcal{L}\{\sin 2t\} + \mathcal{L}\{2\cos 2t\} + \mathcal{L}\{e^{-t}\sin 2t\}$$

Using look-up table

$$\boxed{F(s) = \frac{2}{s^2+4} + \frac{2s}{s^2+4} + \frac{2}{(s+1)^2+4}}$$

Problem 3.7 (a)

given: $F(s) = \frac{2}{s(s+2)}$

Find time function using partial-fraction expansion.

Solution:

$F(s) = \frac{2}{s(s+2)}$ use partial fraction expansion

$F(s) = \frac{C_1}{s} + \frac{C_2}{s+2}$ Now find C_1, C_2

$C_1 = \frac{2}{s+2}$ where $s = 0$

$C_1 = 1$

$C_2 = \frac{2}{s}$ where $s = -2$

$C_2 = -1$

conti ...

Conti Problem 3.7 (a)

Plug in C_1, C_2 into $F(s)$

$$F(s) = \frac{1}{s} - \frac{1}{s+2}$$ Now using look up tables

$$\mathcal{L}^{-1}\{F(s)\} = \boxed{f(t) = 1(t) - e^{-2t}1(t)}$$

Problem 3.7 (b)

given: $F(s) = \dfrac{10}{s(s+1)(s+10)}$

Find time function using partial fraction expansion

Solution:

$$F(s) = \frac{C_1}{s} + \frac{C_2}{(s+1)} + \frac{C_3}{(s+10)}$$ Now find C_1, C_2, C_3

Multiply every thing by s

$$C_1 = \frac{10}{(s+1)(s+10)} \quad \text{where } s = 0$$

$$C_1 = 1$$

Multiply $F(s)$ by $(s+1)$

$$C_2 = \frac{10}{s(s+10)} \quad \text{where } s = -1$$

$$C_2 = -\frac{10}{9}$$

Multiply $F(s)$ by $(s+10)$

$$C_3 = \frac{10}{s(s+1)} \quad \text{where } s = -10$$

$$C_3 = 1/9$$

Conti...

Conti Problem 3.7(b)

Knowing what C_1, C_2, C_3 are, plug them into $F(s)$

$$F(s) = \frac{1}{s} - \frac{10}{9}\cdot\frac{1}{(s+1)} + \frac{1}{9}\cdot\frac{1}{s+10}$$ Use Look-up Tables

$$\mathcal{L}^{-1}\{F(s)\} = \boxed{f(t) = 1(t) - \frac{10}{9}e^{-t}1(t) + \frac{1}{9}e^{-10t}1(t)}$$

Problem 3.7 (c)

given: $F(s) = \dfrac{3s+2}{s^2+4s+20}$

Find time function using partial-fraction expansion.

Solution:

we need to rewrite $F(s)$ before we can use partial fraction expansions

$$F(s) = 3\,\frac{(s+2) - \frac{4}{3}}{(s+2)^2+4^2}$$

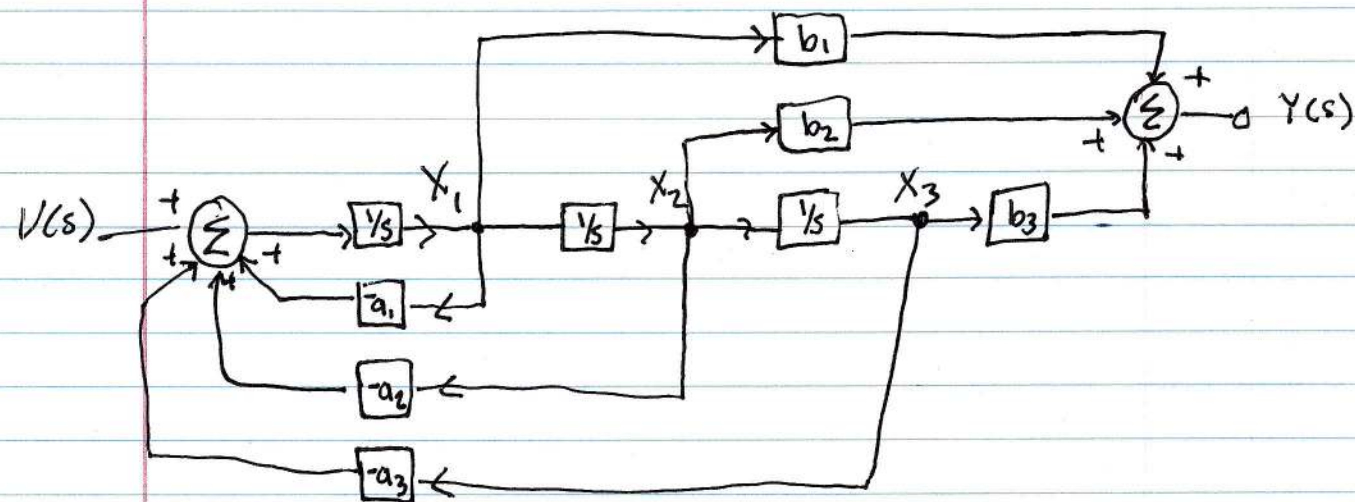
$$= \frac{3(s+2)}{(s+2)^2+4^2} - \frac{4}{(s+2)^2+4^2}$$

from here you can just use the look up tables

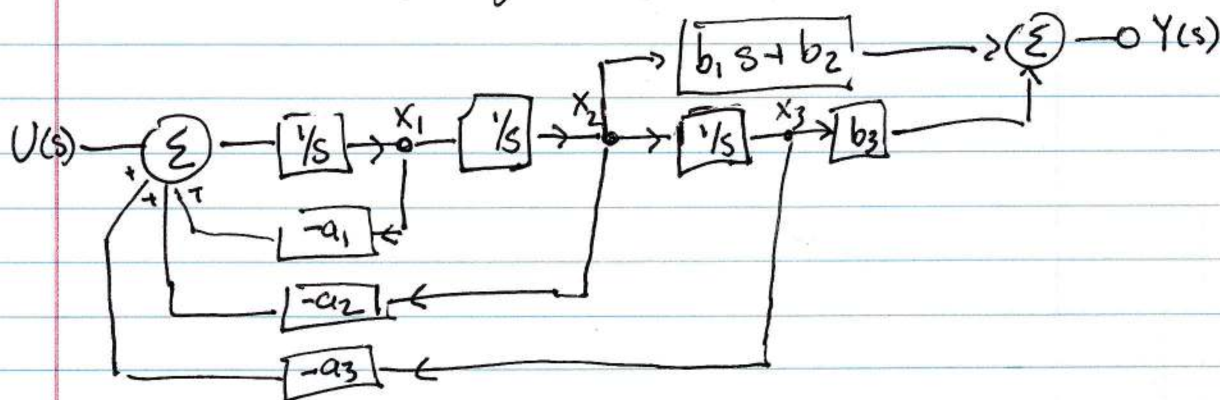
$$\mathcal{L}^{-1}\{F(s)\} = f(t) = 3e^{-2t}\cos 4t - e^{-2t}\sin 4t$$

Problem 3.19

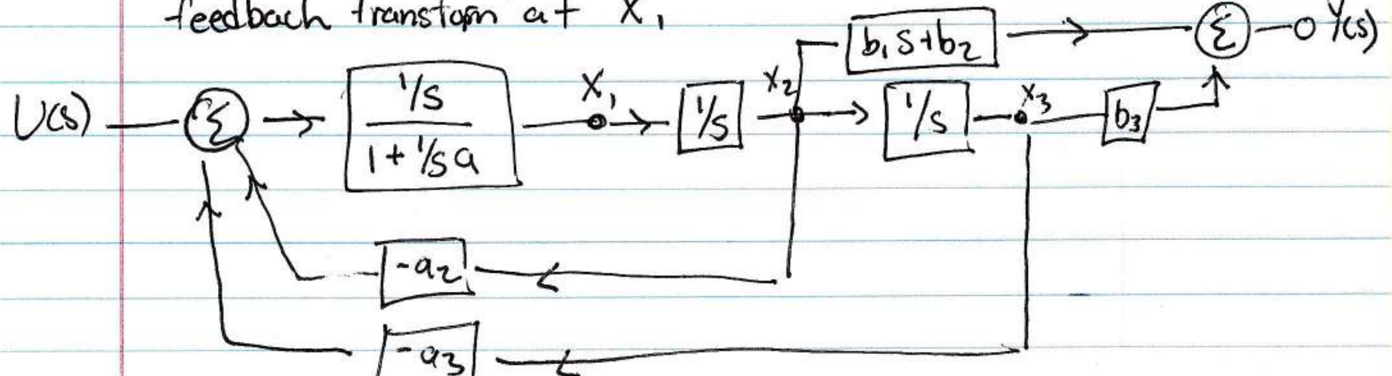
Compute the transfer function for this system



• Do some block-diagram algebra at Pitch-off point X_1

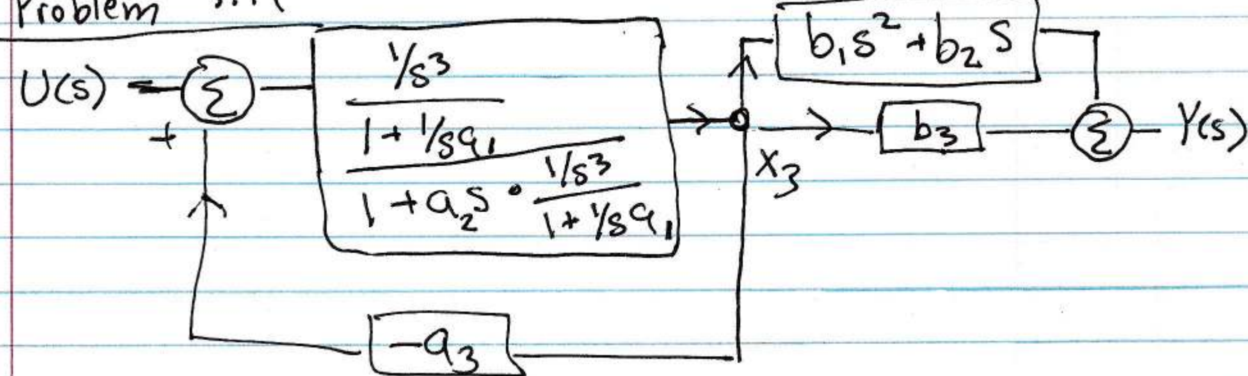


• Do ~~another pitch-off point transform and then~~ a feedback transform at X_1

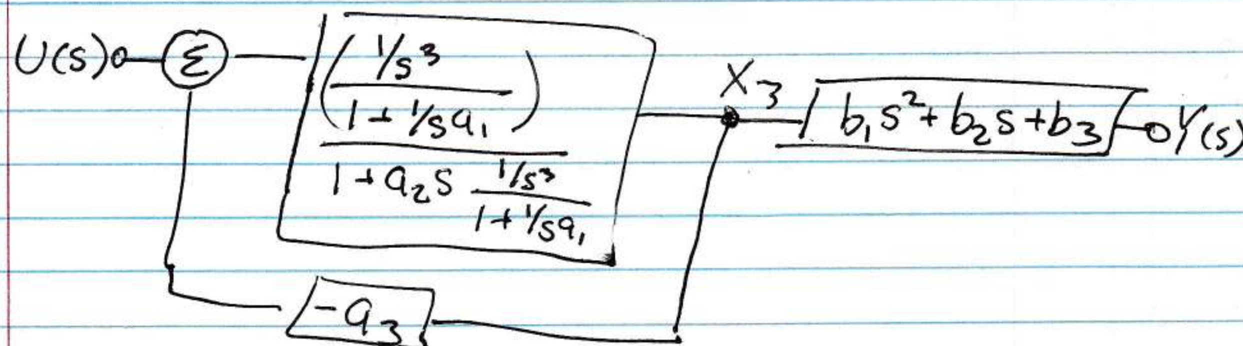


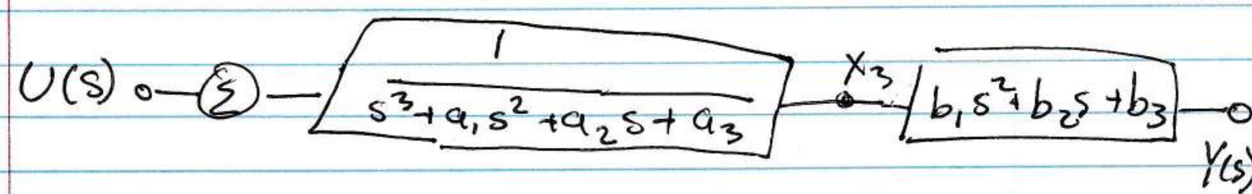
• Now do a similar pitch-off point transform and feedback transform at X_2

Conti Problem 3.19



Finally one more feedback transform on the Left of X_3 and one transform on the right of X_3. Then combine transfer functions.



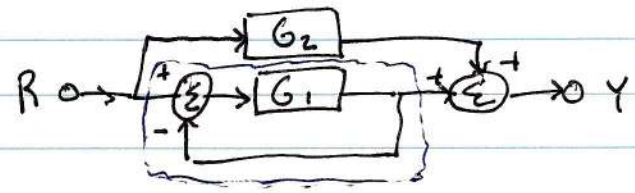


So the transfer function then equals

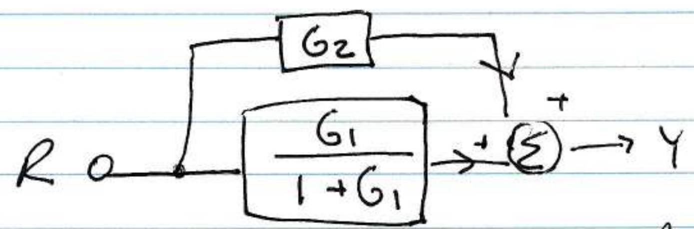
$$\frac{\text{Output}}{\text{Input}} = \boxed{\frac{Y}{U} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}}$$

Problem 3.20(a)

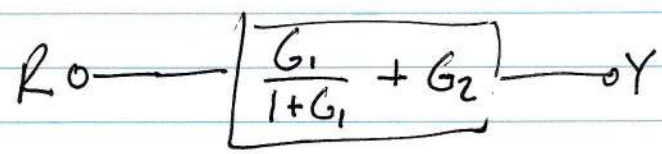
Find the transfer function of



First do a negative ~~transform~~ feedback transform of the feedback that is boxed in blue.



Now the two boxes are in parallel so

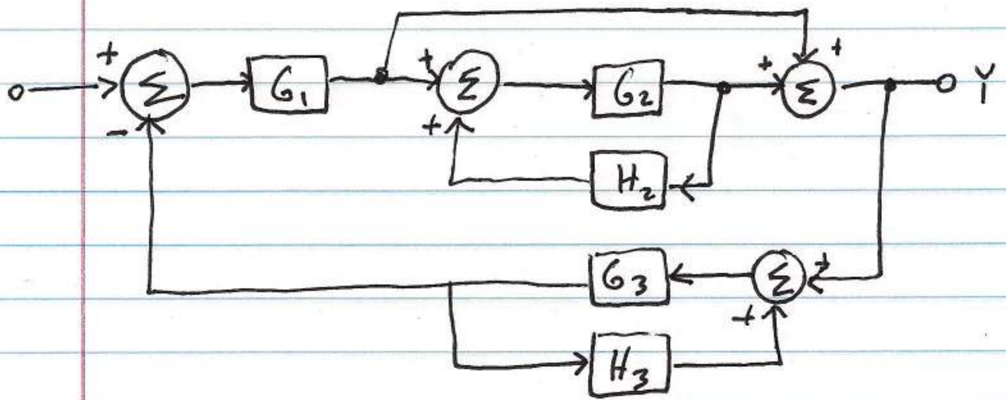


So the transfer function is

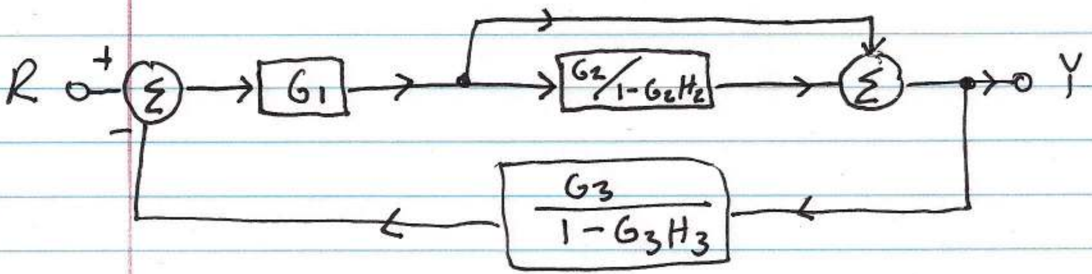
$$\frac{Y}{R} = \frac{G_1}{1+G_1} + G_2$$

Problem 3.21 (a)

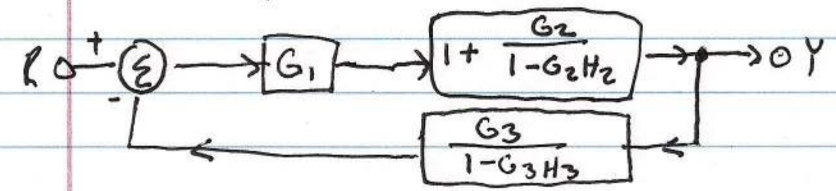
Find the transfer function of:



To start do two feedback transformations.



do a parallel transform



Now you have two blocks in series and a feedback loop

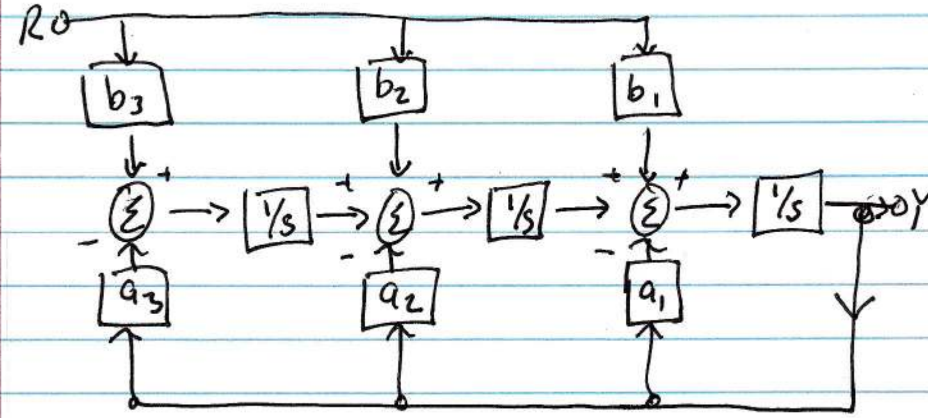
$$R \longrightarrow \boxed{\frac{G_1 + \frac{G_1 G_2}{1 - G_2 H_2}}{1 + \left(G_1 + \frac{G_1 G_2}{1 - G_2 H_2}\right)\left(\frac{G_3}{1 - G_3 H_3}\right)}} \longrightarrow Y$$

$$\boxed{\frac{Y}{R} = \frac{G_1 + \frac{G_1 G_2}{1 - G_2 H_2}}{1 + \left(G_1 + \frac{G_1 G_2}{1 - G_2 H_2}\right)\left(\frac{G_3}{1 - G_3 H_3}\right)}}$$

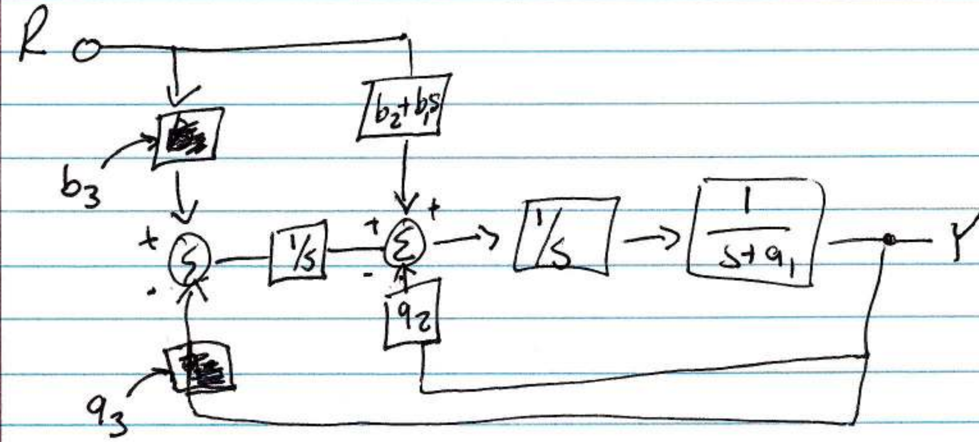
Your Transfer Function

Problem 3.21 (b)

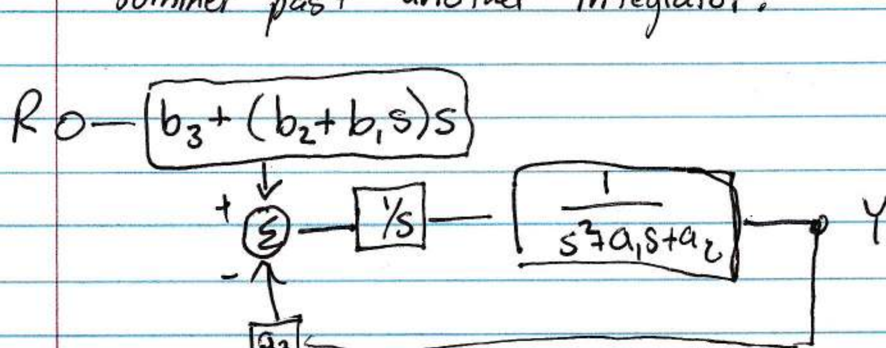
Find the transfer function of



So we can apply feedback on the right and we can also move the summer on the right past the middle integrator. ~~to get~~



Now repeat the feedback transform and moving the summer past another integrator.



conti...

Conti. Problem 3.21 (b)

Do another feed back transform

$$R \circ\!\!- \boxed{b_3 + b_1 s^2 + b_2 s} \longrightarrow \boxed{\frac{1}{s^3 + a_1 s^2 + a_2 s + a_3}} \longrightarrow\!\circ\, Y$$

$$R \longrightarrow \boxed{\frac{b_3 + b_2 s + b_1 s^2}{s^3 + a_1 s^2 + a_2 s + a_3}} \longrightarrow Y$$

So the transfer function is:

$$\boxed{\frac{Y}{R} = \frac{b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3}}$$

Compare 3.19 and 3.21 (b):

Similar:

- The system transfer functions are the same.
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Difference:

- The input/output relationships of the individual transfer functions.
- The number of and location of the summations.
- You have negative feedback in 3.21 (b) and positive feedback in 3.19. Note: that in 3.19 the $\boxed{a}$ blocks are negative which makes it negative feedback.