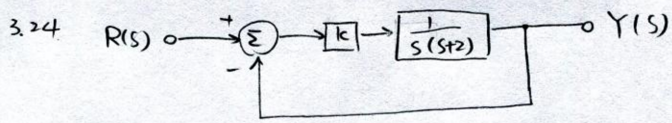
3.24



Transferfunction from R(S) to Y(S) is:

$$T(S) = \frac{K \cdot \frac{1}{s(s+2)}}{1 + K \cdot \frac{1}{s(s+2)}} = \frac{K}{S^2 + 2S + K}$$

overshoot in response to a unit step: $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$, $0 \le \zeta \le 1$

where ζ is damping ratio

For $T(S)$, $= \dfrac{K}{S^2+2S+K} = \dfrac{\omega_n^2}{S^2 + 2\zeta\omega_n S + \omega_n^2}$ $\Rightarrow$ $\omega_n = \sqrt{K}$, $\zeta = 1/\sqrt{K}$

Method 1

By checking Fig 3.23 on Page 118

For $M_p < 10\%$, $\zeta > 0.6$, $\boxed{K = \frac{1}{\zeta^2} < 2.78}$ ☆

Method 2

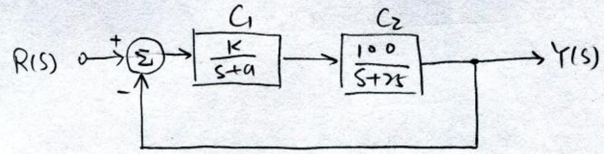
$M_p = 10\% = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$, $\ln(0.1) = -\pi\zeta/\sqrt{1-\zeta^2}$

$\Rightarrow [\ln(0.1)]^2 = \dfrac{\pi^2\zeta^2}{1-\zeta^2}$ $\quad \zeta^2(\pi^2 + [\ln(0.1)]^2) > [\ln(0.1)]^2$

$\Rightarrow \zeta > 0.59$

$\boxed{K = \frac{1}{\zeta^2} < 2.86}$ ☆

3.25.



$$T(S) = \frac{C_1 C_2}{1 + C_1 C_2} = \frac{K \cdot 100}{S^2 + (25+a)S + 25a + 100k} = \frac{K \cdot 100}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Design criteria: response to a unit-step input with:

① Overshoot $(M_p) < 25\%$

② 1 % settling time $(t_s) < 0.1$ s

By checking Fig 3.23 $M_p(\zeta)$, the criteria of $M_p < 25\%$ can be satisfied by setting: $\underline{\zeta > 0.4}$ —(1)

1% Settling time

$$t_s = \frac{4.6}{\sigma} < 0.1 \quad \Rightarrow \quad \underline{\sigma > 46} \quad \text{—(2)}$$

Obsering the transfer function we can find σ is a function of 'a' while ζ is a function of 'a' and 'k'

It will be more efficient to first determine the value of 'a' by eq.(2) then look for value of 'k' by eq(1)

Determine the value of a:

$\sigma > 46 \Rightarrow \frac{25+a}{2} > 46. \quad a > 67$. choose $a = 100$

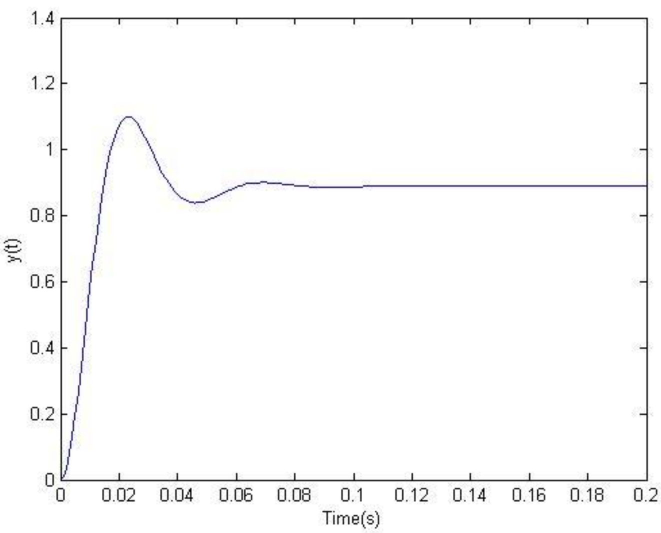
$\zeta > 0.4 \Rightarrow \zeta = \frac{25+a}{2\omega_n} = \frac{25+a}{2 \cdot \sqrt{25a+100k}} > 0.4$

$$K < \frac{a^2 + 34a + 625}{64}$$

For $a = 100$ $\quad K < 219$ $\qquad$ choose $k = 200$

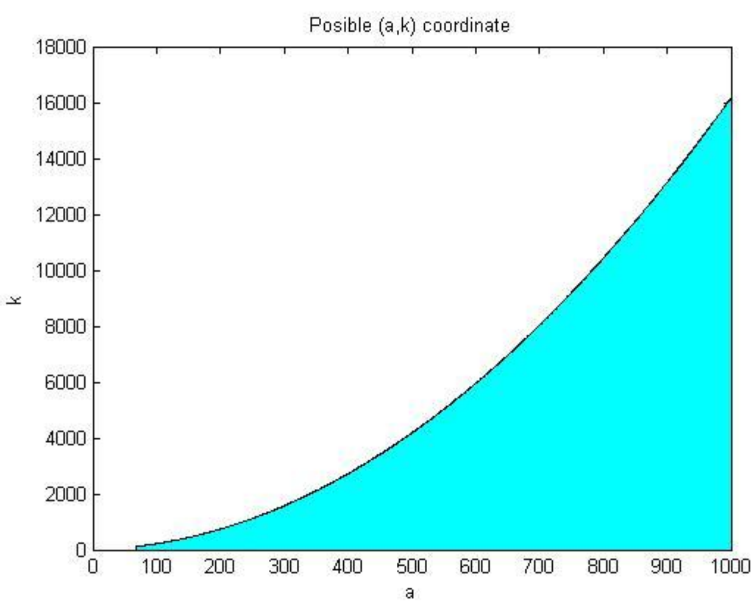
3.25

With a=100 and k=200

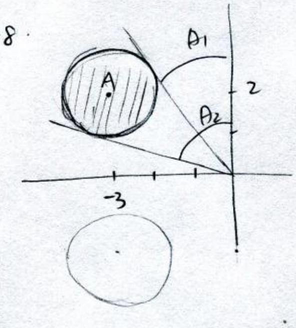


Corresponding Ts=0.0736 and Mp=0.2369

The area in the next plot indicate possible choices of (a,k) which can satisfy designing criteria.



3.28



(a) simple estimation of ω_n & ζ

For point A : $\omega_n = \sqrt{3^2+2^2} = \sqrt{13}$

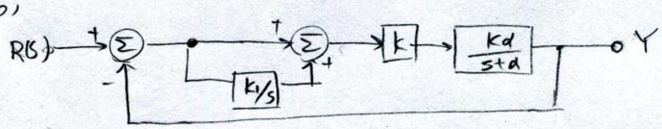
$\Rightarrow$ The range is a circle radius 1 center at A.

$\Rightarrow \quad \sqrt{13}-1 < \omega_n < \sqrt{13}+1$, $\underline{2.6 < \omega_n < 4.6}$

The range of ζ : $\sin\theta_1 < \zeta < \sin\theta_2$

$\Rightarrow \quad \underline{0.64 < \zeta < 0.95}$

(b)



$$T(s) = \frac{(1+k_1/s)\cdot k \cdot \left(\frac{K\alpha}{s+\alpha}\right)}{1+\left(1+\frac{k_1}{s}\right)\cdot k \cdot \left(\frac{K\alpha}{s+\alpha}\right)} \qquad \text{where } \begin{matrix} K\alpha = 2 \\ \alpha = 2 \end{matrix}$$

$$= \frac{2k\,(s+k_1)}{s^2+s\,(2+2k)+2k\cdot k_1}$$

set poles $= -3 \pm 2i$. $\Rightarrow$ $2+2k = -\alpha_1-\alpha_2 = 6$, $\underline{k=2}$

$\Rightarrow$ $2k\cdot k_1 = \alpha_1\alpha_2 = 13$, $\underline{k_1 = \frac{13}{4}}$

(c)

$$T(s) = \frac{k\,K\alpha\,(s+k_1)}{s^2+s\,(\alpha+k\,K_\alpha)+kk_1K_\alpha}$$

set poles of $T(s) = P_1, P_2$

$\alpha + K\cdot K_\alpha = -P_1 - P_2$. $\qquad K = \dfrac{-P_1-P_2-\alpha}{K\alpha}$

$K\cdot K_1 K_\alpha = P_1 P_2$ $\qquad K_1 = \dfrac{P_1P_2}{K\cdot K\alpha} = \dfrac{P_1P_2}{-P_1-P_2-\alpha}$

So no matter what the values of $K\alpha$ and.

The controller provides enough flexibility to place the poles.

3.30 $\quad J_m \ddot{\theta}_m + \left(b + \frac{K_t K_e}{R_a}\right)\dot{\theta}_m = \frac{K_t}{R_a} V_a$

Plug in numbers: $J_m = 0.01\ kg\cdot m^2$. $b = 0.001\ N\cdot m\cdot sec$

$k_e = 0.02\ V\text{-}sec \quad k_t = 0.02\ N\cdot m/A \quad R_a = 10\Omega$

$$0.01\ddot{\theta}_m + 0.00104\,\dot{\theta}_m = 0.002\,V_a$$

$$\ddot{\theta}_m + 0.104\,\dot{\theta}_m = 0.2\,V_a \quad \text{—— (1)}$$

a). Transfer function between V_a & $\dot{\theta}_m$

Set $\mathcal{L}(\dot{\theta}_m) = Y_1(s)$. $\mathcal{L}(V_a) = U_1(s)$ ignoring initial conditions

$$(1) \Rightarrow \quad sY_1(s) + 0.104\,Y_1(s) = 0.2\,U_1(s)$$

$$T_1(s) = \frac{Y_1(s)}{U_1(s)} = \underline{\frac{0.2}{s+0.104}}\ _{**}$$

b) $V_a = 10V\cdot u(t) \Rightarrow \mathcal{L}(V_a) = \frac{10}{s}$

$$Y_1(s) = U_1(s)\cdot T_1(s) = \frac{2}{s(s+0.104)}$$

To find steady-state response, we can use final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\cdot Y(s)$$

$$\lim_{t\to\infty} \dot{\theta}_m(t) = \lim_{s\to 0} s\cdot Y_1(s) = \lim_{s\to 0}\frac{2}{s+0.104} = \underline{19.23}_{**}\ (\text{rad}/\text{sec})$$

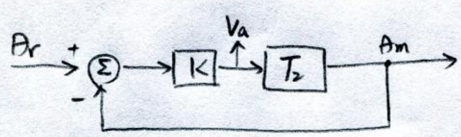
(c) Transfer function between V_a & θ_m

Set $\mathcal{L}(\theta_m) = Y_2(s)$ $\quad \mathcal{L}(V_a) = U_2(s)$ ignoring initial conditions.

$$(1) \Rightarrow \quad s^2 Y_2(s) + 0.104\,s\,Y_2(s) = 0.2\,U_2(s)$$

$$T_2(s) = \frac{Y_2(s)}{U_2(s)} = \underline{\frac{0.2}{s^2+0.104s}}\ _{*}$$

(d)



$$T_3(s) = \frac{K T_2}{1+K T_2} = \underline{\frac{0.2K}{s^2+0.104s+0.2K}}\ _{*}$$

(e) $M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}}$,

For $M_p < 20\%$, $\zeta > 0.45$,

From $T_3(s)$, $s^2 + 0.104s + 0.2K = s^2 + 2\zeta\omega_n s + \omega_n^2$

$$\zeta = \frac{0.104}{2\omega_n} = \frac{0.104}{2\cdot\sqrt{0.2K}} > 0.45$$

$\Rightarrow K < 0.0067$ *

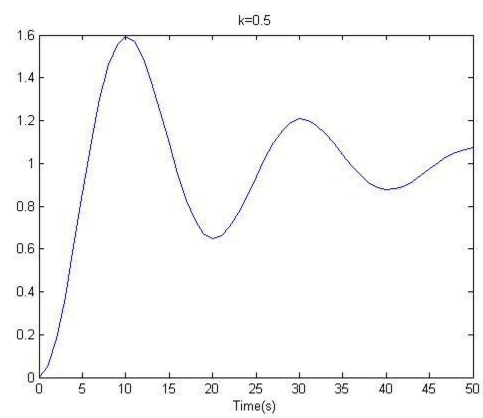
(f) Rise time (t_r) $< 4s$

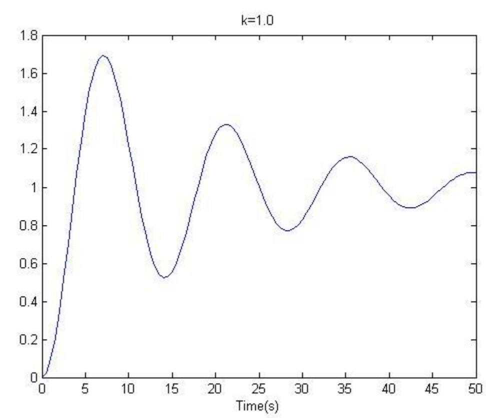
$$t_r \cong \frac{1.8}{\omega_n} \Rightarrow \frac{1.8}{\sqrt{0.2K}} < 4$$

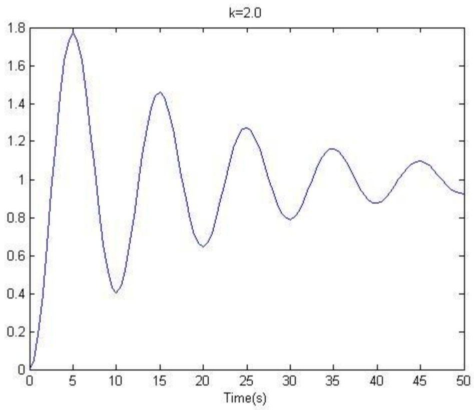
$\Rightarrow K > 1.0125$ *

(g)

k	0.5	1	2
Overshoot (%)	59	69	77
Rise time (s)	5.7	4.02	2.8







Yes, they are consistent with the calculation in (e) and (f). Because k values used here are all greater than 0.0067, overshoot is always greater than 20%. Rise time decrease as k values increase. When k values greater than 1.01, rise time can be smaller than 4 sec.