Problem 3.40:

MATLAB OUTPUT:

```
K =
   2.5000
Given:

Transfer function:
        50 s^2 + 150 s + 100
---------------------------------------
s^4 + 5.03 s^3 + 40.21 s^2 + 1.5 s + 2.4

Controller (K=2.5):

Transfer function:
2.5 s + 7.5
-----------
  s + 10

Closed Loop System Transfer Function: for K =2.5

Transfer function:
        125 s^3 + 750 s^2 + 1375 s + 750
----------------------------------------------------
s^5 + 15.03 s^4 + 215.5 s^3 + 1154 s^2 + 1392 s + 774

Step_Response_Info =
        RiseTime: 0.1406
    SettlingTime: 4.6958
     SettlingMin: 0.7104
     SettlingMax: 1.0605
       Overshoot: 9.4487
      Undershoot: 0
            Peak: 1.0605
        PeakTime: 2.3013
```

```
K =
   0.1000
Controller (K=0.1):

Transfer function:
0.1 s + 0.3
-----------
  s + 10

Closed Loop System Transfer Function: for K =0.1
```

```
Transfer function:
           5 s^3 + 30 s^2 + 55 s + 30
-----------------------------------------------------
s^5 + 15.03 s^4 + 95.51 s^3 + 433.6 s^2 + 72.4 s + 54

Step_Response_Info =
        RiseTime: 2.8133
    SettlingTime: 53.3769
     SettlingMin: 0.3876
     SettlingMax: 0.8755
       Overshoot: 57.5828
      Undershoot: 0
            Peak: 0.8755
        PeakTime: 7.2801
```

```
K =
     1
Controller (K=1):

Transfer function:
s + 3
------
s + 10

Closed Loop System Transfer Function: for K =1

Transfer function:
           50 s^3 + 300 s^2 + 550 s + 300
-------------------------------------------------------
s^5 + 15.03 s^4 + 140.5 s^3 + 703.6 s^2 + 567.4 s + 324

Step_Response_Info =
        RiseTime: 1.0374
    SettlingTime: 8.7452
     SettlingMin: 0.8362
     SettlingMax: 1.1016
       Overshoot: 18.9777
      Undershoot: 0
            Peak: 1.1016
        PeakTime: 3.0441
```

```
K =
     5
Controller (K=5):

Transfer function:
5 s + 15
--------
```

s + 10

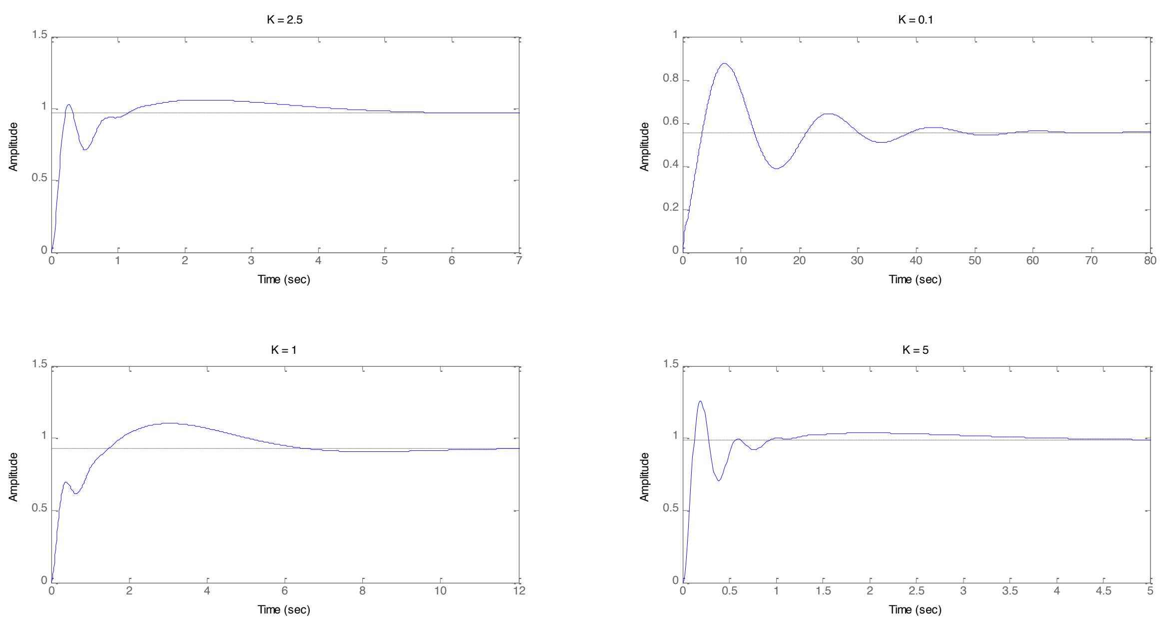
Closed Loop System Transfer Function: for K =5

Transfer function:

```
        250 s^3 + 1500 s^2 + 2750 s + 1500
------------------------------------------------------
s^5 + 15.03 s^4 + 340.5 s^3 + 1904 s^2 + 2767 s + 1524
```

```
Step_Response_Info =
        RiseTime: 0.0844
    SettlingTime: 3.7218
     SettlingMin: 0.7049
     SettlingMax: 1.2573
       Overshoot: 27.7429
      Undershoot: 0
            Peak: 1.2573
        PeakTime: 0.1991
```

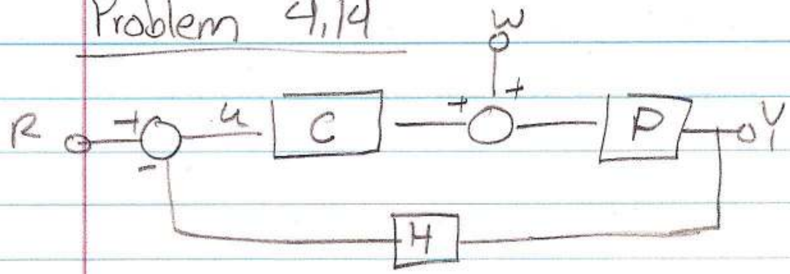
MATLAB PLOTS:



It is not simple easy to meet the design constraints for this system. It was found that K must be in a range approximately from 2.35 to 2.75 so as to meet the constraints. As can be see from the above plots, having a gain that was less than 2.35 or more than 2.75 lead to a system response that was outside of the design specifications.

Hw#3 Solutions

Problem 4.14



$$Y = (Cu + w)P \quad \text{and} \quad u = R - HY$$

$$Y = (CR - CHY + w)P$$

$$Y(1 + CPH) = CRP + Pw$$

$$Y = \frac{CPR + Pw}{1 + CPH}$$ subsitute transfer functions in

$$Y = \frac{\frac{160(s+4)}{s+30} \cdot \frac{1}{s(s+2)} R + \frac{1}{s(s+2)} w}{1 + \frac{160(s+4)}{s+30} \cdot \frac{1}{s(s+2)} \cdot H(s)} \qquad H(s) = 1$$

$$\longrightarrow \quad Y = \frac{160(s+4)R + (s+30)w}{s(s+2)(s+30) + 160(s+4)}$$

Part A

Now $E = R - Y$ so

$$E = R - \frac{160(s+4)}{s(s+2)(s+30) + 160(s+4)} R$$

$$E = \frac{s(s+2)(s+30)}{s(s+2)(s+30) + 160(s+4)}$$

$$e_{ss} = \lim_{s \to 0} sE(s) = 0$$

so our system is Type 1

with $k_v = \frac{4 \cdot 160}{2 \cdot 30} = 10.67$

Part b

$$Y_{ss} = -\lim_{s \to 0} s\,Y(s) = -\frac{s+30}{s(s+2)(s+30)+(s+4)}$$

$$= -\frac{30}{4} = -7.5$$

- This system cannot reject a constant disturbance
  - Type 0

Part c

$$T(s) = \frac{160(s+4)}{s(s+P)(s+30)+160(s+4)}$$

$$S_P^T = \frac{P}{T}\frac{\partial T}{\partial P}$$

$$\frac{\partial T}{\partial P} = \frac{160(s+4)(s)(s+30)}{\left(s(s+30)(s+P)+160(s+4)\right)^2}$$

so $$S_P^T = \frac{2s(s+30)}{s(s+30)(s+2)+160(s+4)}$$

and at $s=0$ sensitivity $=0$

Part d

$$Y = \frac{160(s+4)(s+20)R + (s+30)(s+20)W}{s(s+2)(s+30)(s+20)+20\cdot 160(s+4)}$$

a) $$E = \frac{s(s+2)(s+30)(s+20) - s(160)(s+4)}{s(s+2)(s+30)(s+20)+20\cdot 160\cdot(s+4)}$$

$e_{ss} = \lim_{s \to 0} sE = 0$ so system is still type 1

and $K_v = 22.86$

Part d conti

b)

$$Y_{ss} = -\lim_{s \to 0} s\,Y(s) = -\frac{(s+30)(s+20)}{s(s+2)(s+30)(s+20) + 20 \cdot 160 (s+4)}$$

still type 0

c)

$$T(s) = \frac{160(s+4)(s+20)}{s(s+P)(s+30)(s+20) + 20 \cdot 160 \cdot (s+4)}$$

$$S_P^T = \frac{P}{T}\frac{\partial T}{\partial P}$$

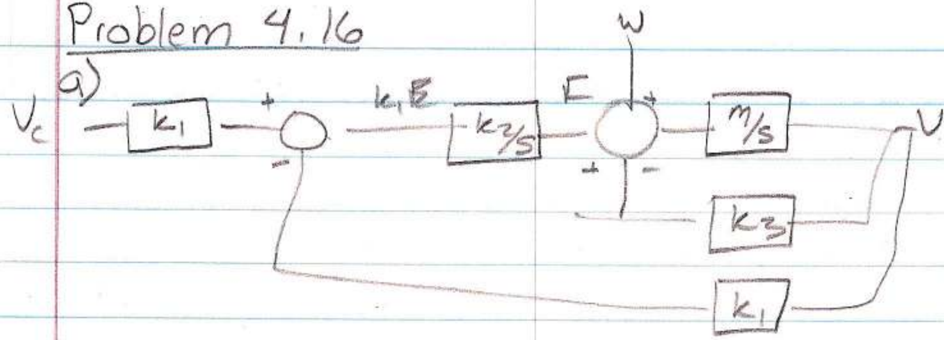
$$\frac{\partial T}{\partial P} = \frac{160(s+4)(s+20)^2 s(s+30)}{(s(s+P)(s+30)(s+20) + 20 \cdot 160 \cdot (s+4))^2}$$

so $$S_P^T = \frac{2s(s+20)(s+30)}{s(s+2)(s+30)(s+20) + 20 \cdot 160 \cdot (s+4)}$$

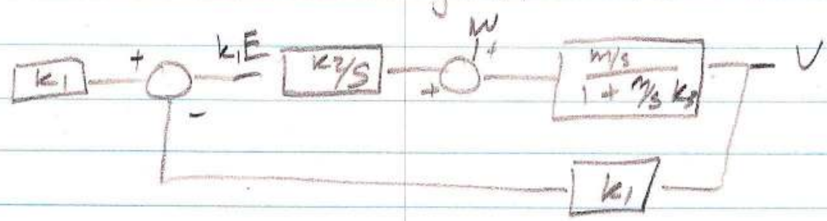
so at $s=0$ sensitivity is 0 like befor

Problem 4.16

a)



Reduce the block diagram to



Reduce again and then find $\frac{V(s)}{W(s)}$

$$\frac{V(s)}{W(s)} = \frac{ms}{s^2 + mk_3 s + mk_1k_2}$$

b) $$V_{ss} = \lim_{s \to 0} s \cdot \frac{V(s)}{W(s)} \cdot \frac{W_0}{s^2}$$

plug in from part a)

$$V_{ss} = \frac{W_0}{k_1 k_2}$$

Problem 4.16 b)

$$V(s) = \frac{ms}{(s^2 + mk_3 s + mk_1 k_2)} \cdot \frac{1}{s^2}$$

Using partial fraction expansion

$$V(s) = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs + D}{s^2 + mk_3 s + mk_1 k_2}$$

$$ms = As(s^2 + mk_3 s + mk_1 k_2) + B(s^2 + mk_3 s + mk_1 k_2) + Cs^3 + Ds^2$$

equating like powers of s to get A and B

s^3 : $0 = A + C$ $\quad C = -\frac{1}{k_1 k_2}$

s^2: $0 = Amk_3 + B + D$ $\quad D = -\frac{mk_3}{k_1 k_2}$

s^1 : $m = Amk_1 k_2 + Bmk_3$ $\quad A = \frac{1}{k_1 k_2}$

s^0 : $0 = B\,mk_1 k_2$ $\quad \Rightarrow B = 0$

$$V(s) = \frac{1/k_1 k_2}{s} + \frac{-1/k_1 k_2\, s - \frac{mk_3}{k_1 k_2}}{s^2 + mk_3 s + mk_1 k_2}$$

$$V(s) = \frac{1}{k_1 k_2} \cdot \frac{1}{s} + \frac{-1}{k_1 k_2} \frac{(s + mk_3)}{(s + \frac{mk_3}{2})^2 + mk_1 k_2 - \frac{m^2 k_3^2}{4}}$$

Now taking the inverse laplace

$v(t) = \frac{1}{k_1 k_2}(t)$ + something that will go to zero as time goes to infinite

so the steady state response equals

$\boxed{v(t) = \frac{1}{k_1 k_2}}$ The steady state response is a <u>step</u>

Part c)

$$E = V_c - V = \left[1 - \left(\frac{k_1 k_2 m}{s(s+mk_3)}\right)\left(\frac{1}{1+\frac{k_1k_2m}{s(s+mk_3)}}\right)\right] V_c$$

$$= \frac{1}{1 + \frac{mk_1k_2}{s(s+mk_3)}} V_c$$

$$e_{ss} = \lim_{s\to 0} \frac{s V_c}{1+G(s)} \qquad \text{where } G_s = \frac{mk_1k_2}{s(s+mk_3)}$$

$$= 0$$

$$k_p = \lim_{s\to 0} G(s) = \infty$$

$$k_v = \lim_{s\to 0} s G(s) = \frac{k_1k_2}{k_3}$$

So system is type 1

Part d)

If the disturbance is a ramp then, using V from part a),

$$e_{ss} = \lim_{s\to 0} sV = \frac{1}{k_1k_2}$$

So the system is Type 1
and $k_v = k_1 k_2$