Hw4 solution

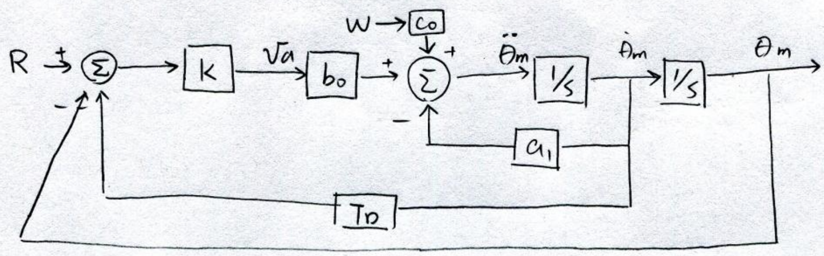
4.22

4.22

a) $\ddot{\theta}_m + a_1 \dot{\theta}_m = b_0 V_a + c_0 w$

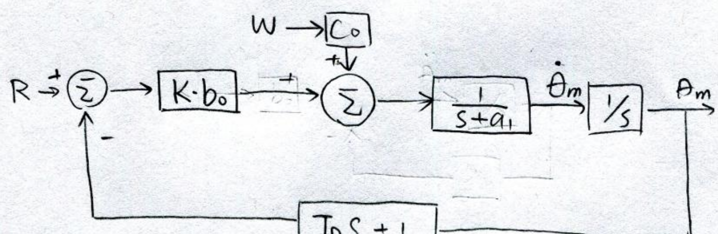
$= b_0 K(e - T_D \dot{\theta}_m) + c_0 w$

①

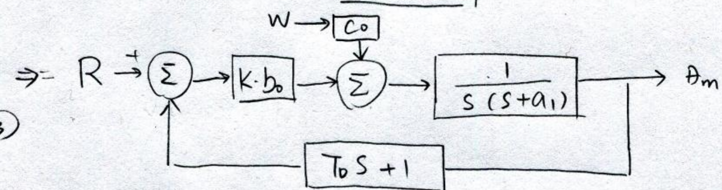


The block diagram can be simplified as following:

②



③



All three block diagrams can describe the ODE model. State components of $\dot{\theta}_m$ and θ_m are presented in ①, ②

b) $a_1 = 65$. $b_0 = 200$. $c_0 = 10$, $w = 0$ $V_a = 100$

$\ddot{\theta}_m + 65\dot{\theta}_m = 20000$

Particular sol $\dot{\theta}_p = c_1 t + c_2$

$\ddot{\theta}_p = c_1$

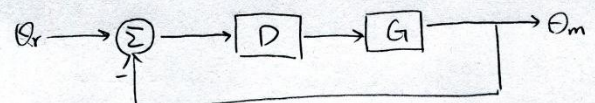
$\Rightarrow c_1 + 65(c_1 t + c_2) = 20000$

$c_1 = 0 \qquad c_2 = 307.7$

Steady state angular speed $\dot{\theta}_{ss} = 307.7$ (rad/s)

$= 2938$ (rpm)

c)



$D = K_P + K_D s$, $\quad G = \dfrac{1}{s(s+a_1)}$

Transferfunction from θ_r to θ_m :

$$T(s) = \frac{DG}{1+DG} = \frac{K_D s + K_P}{s^2 + (a_1 + K_D)s + K_P}$$

This is a 2nd order transfer function.

5% settling time $= T_s \le 0.05$

$$e^{-3\omega_n \cdot T_s} = 0.05 \qquad -3\omega_n T_s = \ln 0.05 \simeq -3$$

$$\zeta\omega_n = \frac{3}{T_s} \geq 60$$

$$a_1 + K_D = 2\zeta\omega_n \geq 120 \qquad \boxed{\text{Choose } K_D = 55}$$

$$M_p \simeq e^{-\pi\zeta/\sqrt{1-\zeta^2}}, \qquad \zeta = \frac{\ln M_p}{(\ln M_p^2 + \pi^2)^{1/2}}$$

For $M_p \le 17\%$, $\quad \zeta \geq 0.5 \qquad \boxed{\text{Set } \zeta = 0.5.}$

$$2\zeta\omega_n = a_1 + K_D, \qquad \omega_n = \frac{a_1 + K_D}{2\zeta} = 120$$

$$\boxed{K_P = \omega_n^2 = 14400}$$

Use Matlab to verify and adjust K_P & K_D:

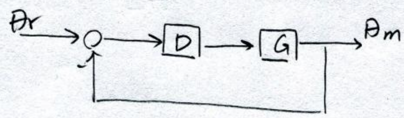
(C)

Matlab Code:

```
KD=65;KP=14400;a1=65;
num=[KD KP];
den=[1 a1+KD KP];
T1=tf(num,den);
s=stepinfo(T1,'SettlingTimeThreshold',0.05);
```

Set Kp=14400 KD=55 => Settling time=0.04s Overshoot=18.5%
Because the overshoot is a little higher than required, let's increase the damping to lower it:
Set Kp=14400 KD=65 => Settling time= 0.04s Overshoot = 16%
This is a workable design.

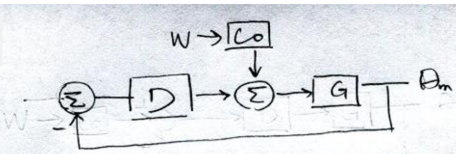
(d)



$$E(s) = \theta_r - \theta_m = \theta_r(s) \cdot \frac{1}{1+DG} = \theta_r(s) \cdot \frac{s(s+a_1)}{s^2 + (a_1 + K_D)s + K_P}$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot E(s) = \underline{0} \qquad \left[\theta_r(s) = \frac{1}{s}\right]$$

(e)



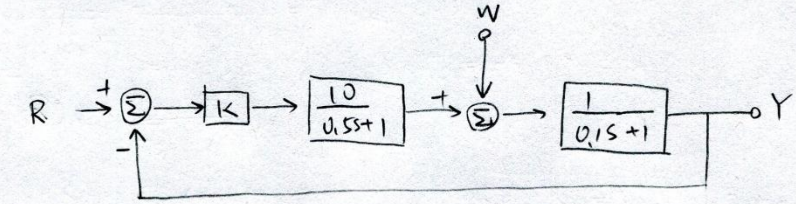
$$E(s) = C_o \cdot W \cdot \frac{G}{1+DG} = W(s) \cdot \frac{C_o}{s^2 + (a_1 + K_D)s + K_P}$$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} s \cdot E(s) = \underline{\frac{C_o}{K_P}} \qquad \left[W(s) = \frac{1}{s}\right]$$

4.26

4.26.

(a)



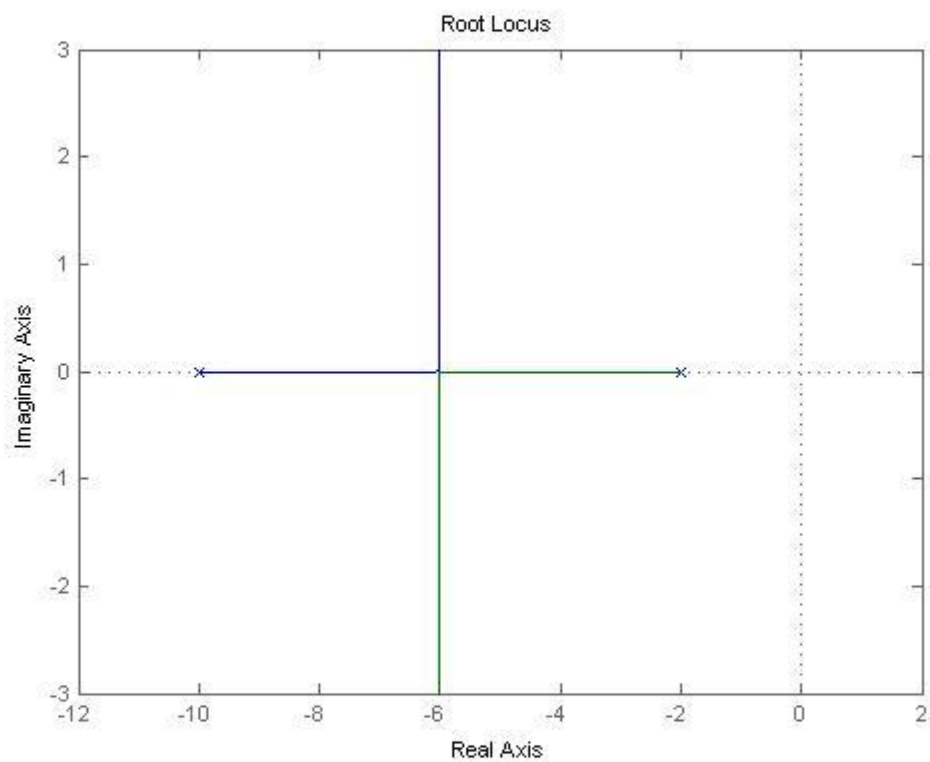
For R = 0.

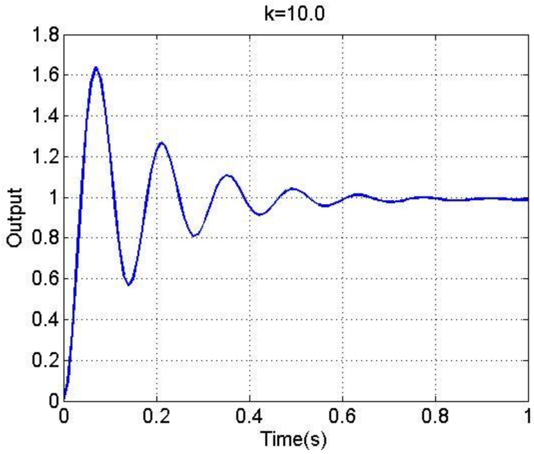
$$Y(s) = W(s)\cdot \frac{1/(0.1s+1)}{1+K\cdot\frac{10}{0.5s+1}\cdot\frac{1}{0.1s+1}} = \frac{0.5s+1}{(0.5s+1)(0.1s+1)+10K}\cdot W(s)$$

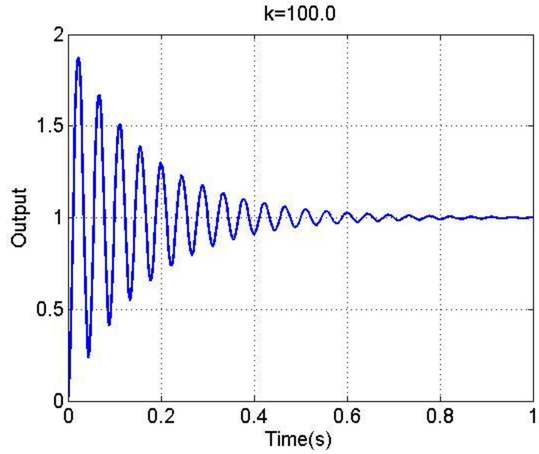
$$e_{ss} = \lim_{t\to 0} y(t) = \lim_{s\to 0} s\cdot Y(s) = \frac{1}{1+10K} \qquad \left[W(s)=\frac{1}{s}\right]$$

$$\frac{1}{1+10K} \le 0.01 \quad \Rightarrow \quad K \geqslant 9.9$$

(b)



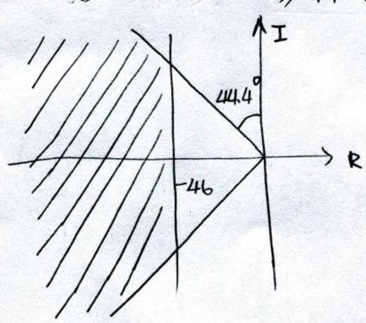




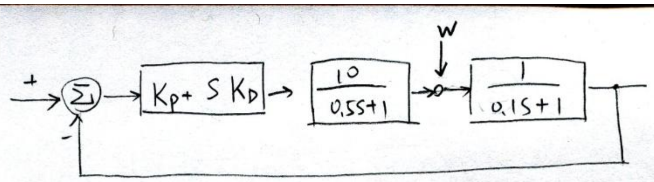
c) 1 % settling time (t_s) $= \dfrac{4.6}{\sigma} < 0.1 \quad \Rightarrow \sigma > 46$

$M_p \simeq e^{-\pi\zeta/\sqrt{1-\zeta^2}} < 5\% \quad \Rightarrow \zeta \geq 0.7$

$\theta = \sin^{-1}\zeta \geq 44.4°$



d)



$$T(s) = \frac{10 K_D S + 10 K_P}{(0.5S+1)(0.1S+1) + 10(K_D S + K_P)}$$

$$= \frac{200 K_D S + 200 K_P}{S^2 + (12 + 200 K_D) S + (20 + 200 K_P)}$$

$\sigma \geq 46 \quad \Rightarrow \quad 6 + 100 K_D \geq 46 \qquad K_D \geq 0.4 \qquad$ choose $K_D = 0.4$

$\zeta \geq 0.7 \quad \Rightarrow \quad \zeta = \dfrac{\sigma}{\omega_n} = \dfrac{6 + 100 K_D}{\sqrt{20 + 200 K_P}} \geq 0.7$, $K_P \leq 21.5$ choose $K_P = 21.5$

Use Matlab to verify the coefficient.

Matlab Code:

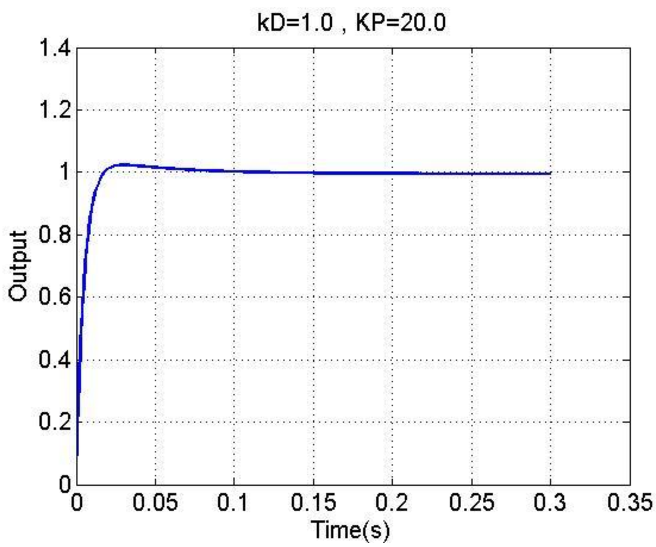
```
KD=0.4; KP=21.5;
T=tf([200*KD 200*KP],[1 12+200*KD 20+200*KP]);
s=stepinfo(T,'SettlingTimeThreshold',0.01);
```

Set KP=21.5 KD=0.4 => Settling time=0.08s Overshoot=16.1%

Settling time satisfies the requirement but overshoot is too high.

Set KP=21.5 KD=1.0 => Settling time=0.085s Overshoot=3.2%

This set of KD and KP satisfy the specifications.



(e)

$$E(S) = W(S) \cdot \frac{\frac{1}{0.1S+1}}{1+(K_P + SK_D)\cdot\frac{10}{0.5S+1}\cdot\frac{1}{0.1S+1}} = \frac{1}{S}\cdot\frac{0.5S+1}{(0.5S+1)(0.1S+1)+10(K_D S+K_P)}$$

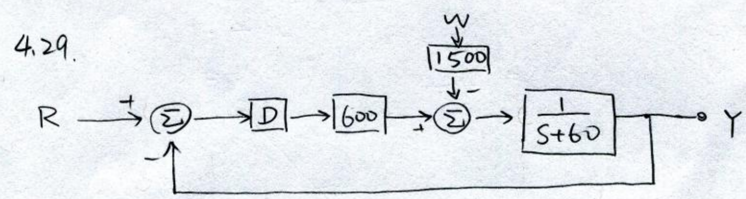
$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\cdot E(s) = \frac{1}{10K_P}$$

For $K_P = 21.5$, $e_{ss} \simeq 0.0046$

In order to eliminate steady-state error due to a step disturbance, we can add one integrator to our controller.

A PID controller with $K_I \neq 0$ can fulfill the task.

4.29.



$D(s) = -\left(K_p + \frac{K_I}{s}\right)$

a, Transfer function from w to Y:

$$T_1(s) = \frac{Y(s)}{W(s)} = \frac{1500\,s}{s^2 + (60 - 600K_p)s - 600K_I}$$

b. $P_1 + P_2 = -120$; $60 - 600K_p = 120$; $K_p = -0.1$

$P_1 P_2 = 7200$; $-600K_I = 7200$; $K_I = -12$

4.30 a, $R(s) = \frac{1}{s}$

$$E(s) = R - Y = R(s)\cdot\frac{1}{1 + D\cdot 600\cdot\frac{1}{s+60}} = \frac{(s+60)}{s(s+60) - 600(K_p s + K_I)}$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\cdot E(s) = 0$$

b) $R(s) = \frac{1}{s^2}$

$$E(s) = \frac{1}{s}\cdot\frac{s+60}{s(s+60) - 600(K_p s + K_I)}$$

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\cdot E(s) = \frac{-1}{10K_I}$$

c, $R(s) = 0$, $W(s) = \frac{1}{s}$

$$E_R(s) = \frac{1}{s}\cdot T_1(s) = \frac{1500}{s^2 + (60 - 600K_p)s - 600K_I}$$

$$e_{ss} = \lim_{s\to 0} s\cdot E(s) = 0$$

d, $R(s) = 0$ $W(s) = \frac{1}{s^2}$

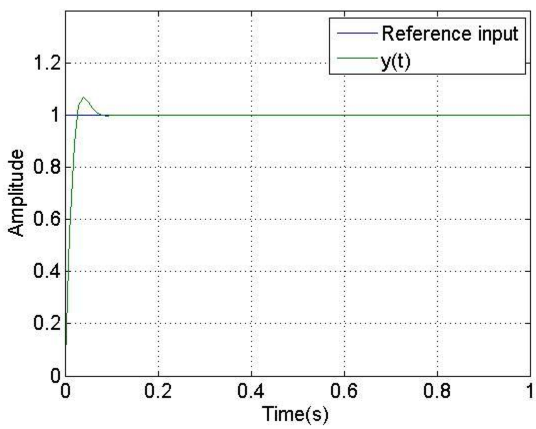
$$e_{ss} = \lim_{s\to 0} E_R(s) = \frac{-5}{2K_I}$$

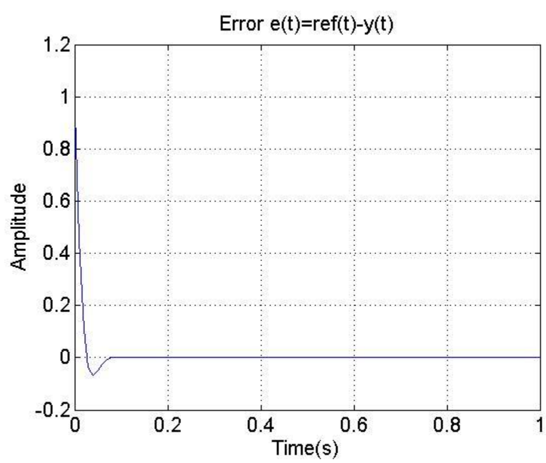
4.30

(e)

Matlab Code:

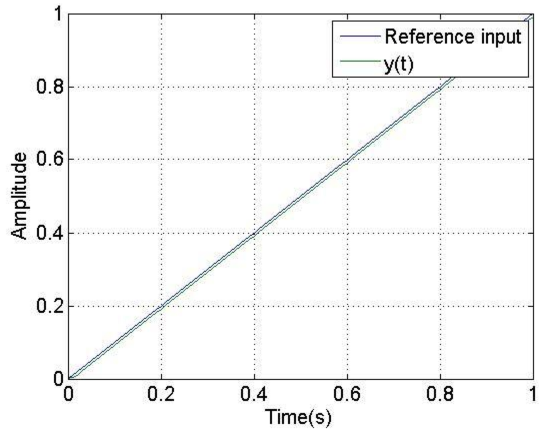
```
KP=-0.1;KI=-12;
G=tf(1,[1 60]);
D=tf(600*[-KP -KI],[1 0]);
Ta=feedback(series(G,D),1);
Tb=series(Ta,tf(1,[1 0]));
Tc=feedback(G,D)*1500;
Td=series(Tc,tf(1,[1 0]));

t=0:0.01:1;
y=step(Ta,t);
ref=ones(1,101);
plot(t,ref,t,y);
plot(t,1-y);
```

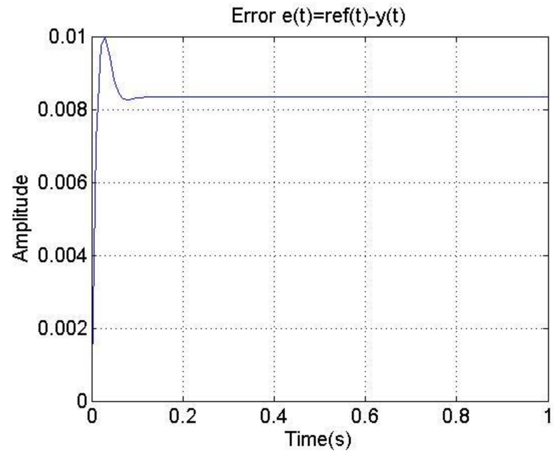




Steady state error goes to 0 as predicted.

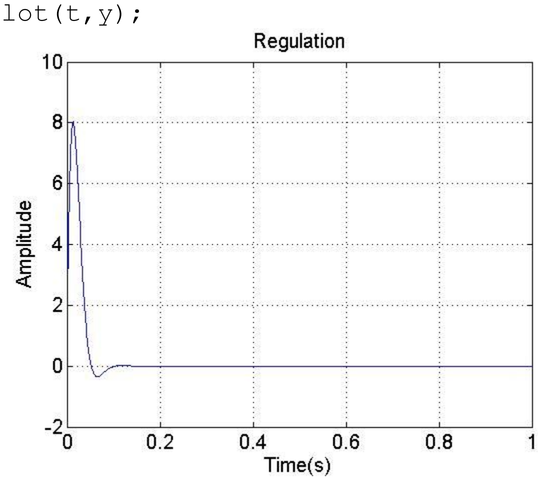
```
t=0:0.01:1;
y=step(Tb,t);
plot(t,t,t,y);
plot(t,t'-y);
```





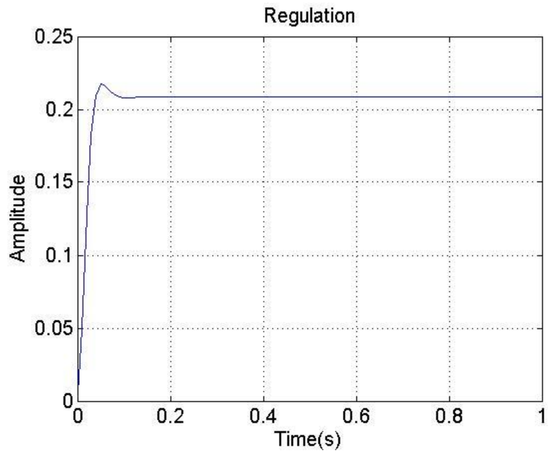
K_I=-12, steady state error goes to -1/(10*K_I)=0.083

```
t=0:0.01:1;
y=step(Tc,t);
plot(t,y);
```



Steady state error goes to 0 as predicted.

```
t=0:0.01:1;
y=step(Td,t);
plot(t,y);
```



K_I=-12, steady state error goes to -5/(2*K_I)=0.2083