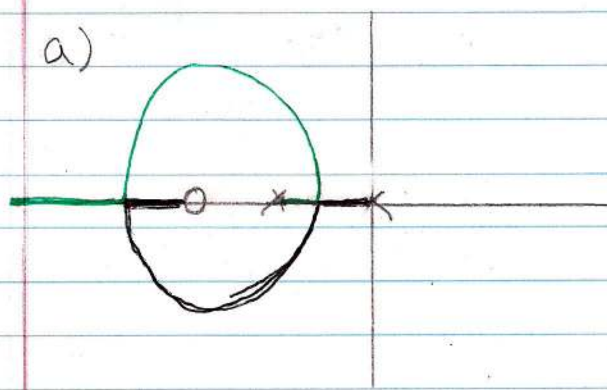
Hw # 6 solutions

Problem 5.2 Assume all have positive gain

a)



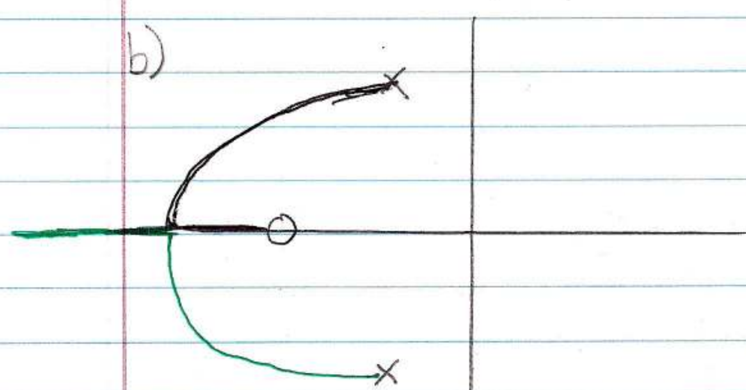
Use the rules for drawing root locus:

so $\phi = \frac{180 + 360 \cdot (l-1)}{2-1}$ $l=1$

Asy. $= 180$

$n = 2$ so two branches of root locus

b)

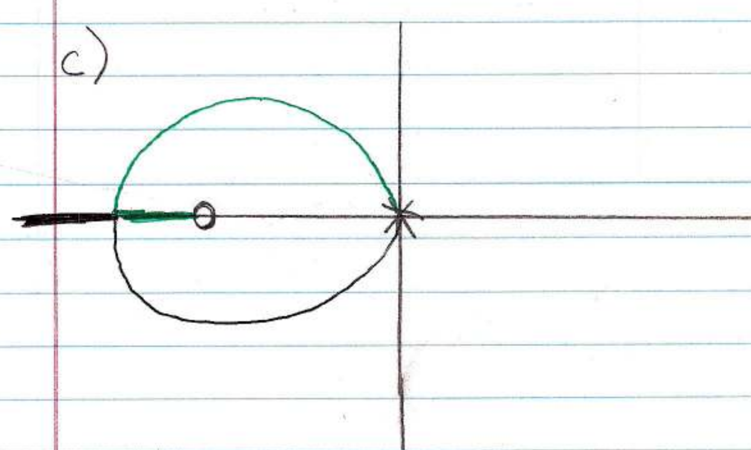


$\phi = \frac{180 + 360(l-1)}{2-1}$ $l=1$

$= 180$

$n = 2$ so two branches

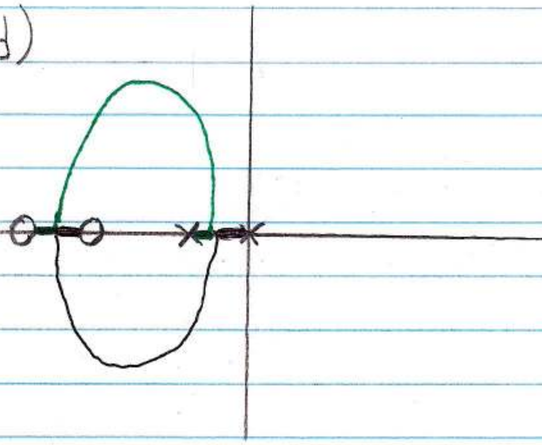
c)



$\phi = 180$

$n = 2$ so two branches

d)

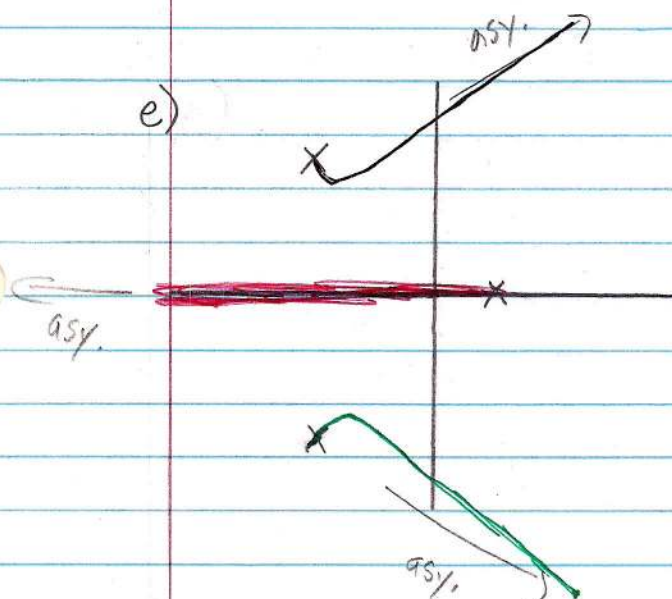


$$\phi = \frac{180 - 360(\ell-1)}{0} \quad \ell = 0$$

= no asymptotes

n => 2 branches

e)

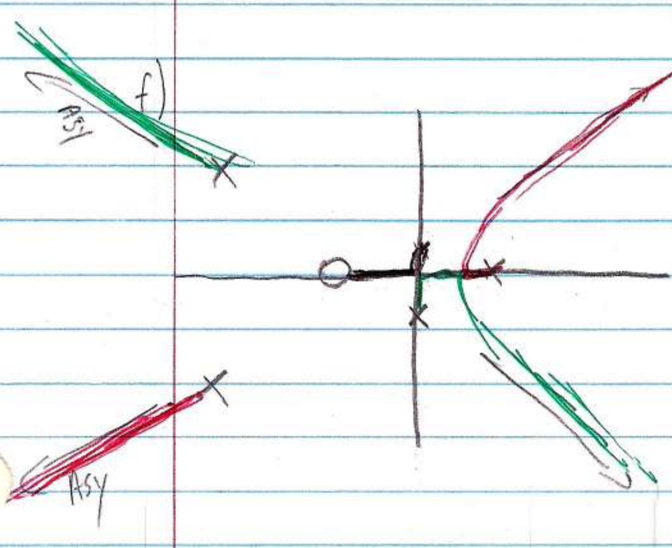


$$\phi = \frac{180 - 360(\ell-1)}{3} \quad \ell = 1,2,3$$

$$\phi = 60, -60, -180$$

n => 3 branches

f)



$$\phi = \frac{180-360(\ell-1)}{4} \quad \ell = 1,2,3,4$$

$$\phi = 45, -45, -135, -225$$

n = 5 => 5 branches

Note that I don't have enough colors

Problem 5.32 / online writeup

a)
$$G(s) = \frac{\Theta}{V_a} = \frac{K_t}{R_a(J_m + J_L)s^2}$$

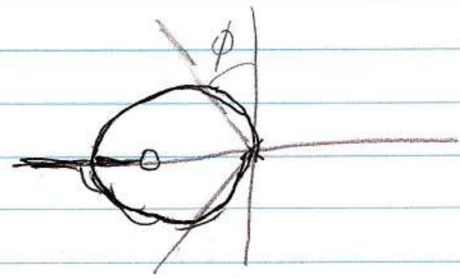
$$F(s) = \frac{w}{s+2w} \qquad \text{where } w_c = \frac{1}{RC}$$

b) Look at Word Doc.
c) Look at Word Doc.

d) A gain of 0.195 gives you ξ of 0.7

A number of ways to determine that.
$\phi = \sin^{-1}(\xi) = \sin^{-1}(0.7) = 44.427°$

Now locate where root locus crosses ϕ



Roots at $K_a = 0.195$
-9.0408
$-0.47958 \pm 0.9211i$

e) Look at Word Doc

Problem 5.35)

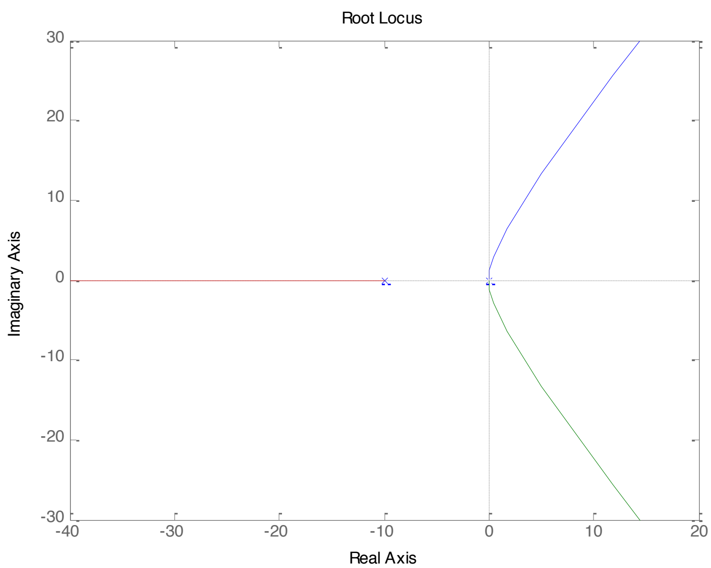
Part a)

$$G(s) = \frac{Kt}{Ra(Jm - Jl)s^2}$$

$$F(s) = \frac{w}{s + 2w} \qquad where \quad w = \frac{1}{RC}$$

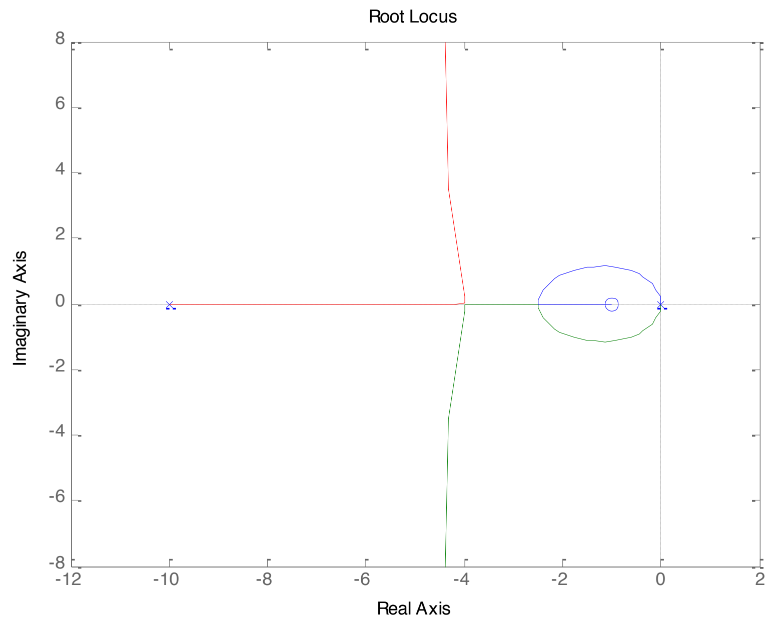
Part b)

From looking at the open loop root locus you can see that no matter what Ka you choose your system will always be unstable except for when gain = 0 where your system is marginally stable.



Part c)

$0 < Ka < inf$



Part d)

A gain of 0.4 gives you a damping coefficient of 0.7. There are a number of ways to determine this value gain. One way: $\phi = \sin^{-1}(\zeta) = \sin^{-1}(0.7) = 44.427$. Now locate where the root locus crosses your ϕ line.

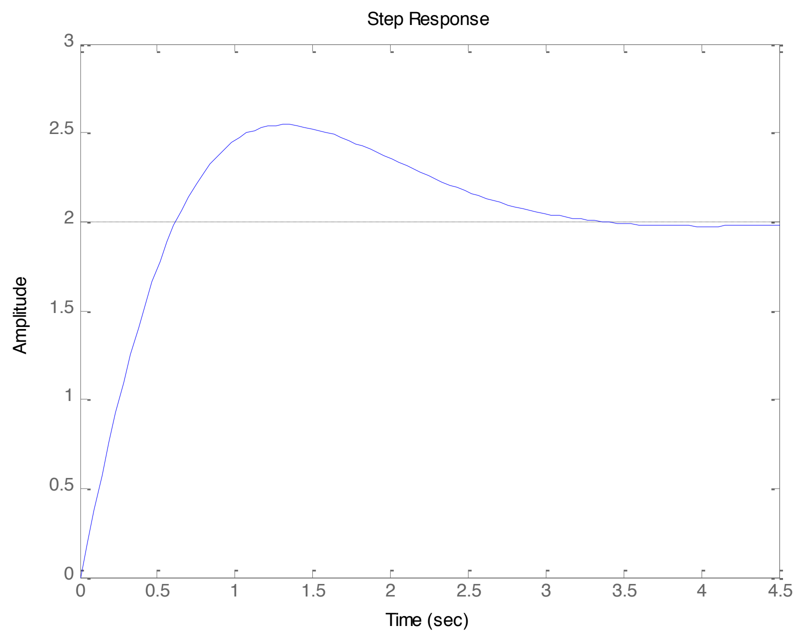
The zeroes, poles and gain of the closed loop system for Ka = 0.195 are:

Z =
 -10
 -1

P =
 -7.7531
 -1.1234 + 1.1478i
 -1.1234 - 1.1478i

K =
 4

Part e)



Matlab Code for Problem 5.35

```
clc;clear;format compact
% part b)
Kt = 0.1; Ra = 10; R = 1000; C = 200*10^-6;
Ka = 1;
G = Kt/(Ra*(10^-3))* tf(1,[1 0 0])
F = 1/(R*C)*tf(1,[1 2*1/(R*C)])
OpenLoopSys = Ka*G*F;
figure(1)
rlocus(OpenLoopSys);

%part c)
Kt = 0.1; Ra = 10; R = 1000; C = 200*10^-6;
Ka = 1;
K = Ka*tf([1 1],1)
G = Kt/(Ra*(10^-3))* tf(1,[1 0 0])
F = 1/(R*C)*tf(1,[1 2*1/(R*C)])
OpenLoopSys = K*G*F;
figure(2)
rlocus(OpenLoopSys);

% part(d)
Kt = 0.1; Ra = 10; R = 1000; C = 200*10^-6;
Ka = .4;
K = Ka*tf([1 1],1)
G = Kt/(Ra*(10^-3))* tf(1,[1 0 0])
F = 1/(R*C)*tf(1,[1 2*1/(R*C)])
OpenLoopSys = K*G*F;
```

Problem 7.2 Parts d) and e) on Word Doc.

a) Eqns. of Motion are

$$m_1\ddot{y}_1 = u - k(y_1 - y_2) - b(\dot{y}_1 - \dot{y}_2)$$
$$m_2\ddot{y}_2 = -k(y_2 - y_1) - b(\dot{y}_2 - \dot{y}_1)$$

b) Plugging in values

$$2000(10^{-6}\ddot{z}_1) = \frac{1}{1000}v - 10^{-6}(3.2\times10^6)(z_1 - z_2) - 10^{-6}(4.6\times10^3)(\dot{z}_1 - \dot{z}_2)$$
$$1000(10^{-6}\ddot{z}_2) = -10^{-6}(3.2\times10^6)(z_2 - z_1) - 10^{-6}(4.6\times10^3)(\dot{z}_2 - \dot{z}_1)$$

so after cleaning up the equations

$$\ddot{z}_1 = -(1.6\times10^3)(z_1 - z_2) - (2.3)(\dot{z}_1 - \dot{z}_2) + \tfrac{1}{2}v$$
$$\ddot{z}_2 = -(3.2\times10^3)(z_2 - z_1) - 4.6(\dot{z}_2 - \dot{z})$$

c)
$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1.6\times10^3 & -2.3 & 1.6\times10^3 & 2.3 \\ 0 & 0 & 0 & 1 \\ 3.2\times10^3 & 4.6 & -3.2\times10^3 & -4.6 \end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{2}\\ 0\\ 0\end{bmatrix}v$$

$$y = \begin{bmatrix}0 & 0 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\end{bmatrix} + 0$$

```
figure(3)
rlocus(OpenLoopSys);
H=feedback(K*G,F,-1)
[Z,P,K] = zpkdata(H,'v')

% Part (e)
figure(4)
step(H)
```

Problem 7.2 c) and d)

MATLAB CODE:

```
F =[   0    1    0   0;
    -1600 -2.3 1600 2.3;
      0   0    0    1;
    3200 4.6 -3200 -4.6];
G =[0; 1/2; 0; 0];
H =[ 0 0 1 0];
J =[0];

sysGPB = ss(F,G,H,J);
impulse(sysGPB,2);

Poles = eig(F)
Zeroes =  tzero(F,G,H,J)
```

MATLAB OUTPUT:

a =

	x1	x2	x3	x4
x1	0	1	0	0
x2	-1600	-2.3	1600	2.3
x3	0	0	0	1
x4	3200	4.6	-3200	-4.6

b =

	u1
x1	0
x2	0.5
x3	0
x4	0

c =

	x1	x2	x3	x4
y1	0	0	1	0

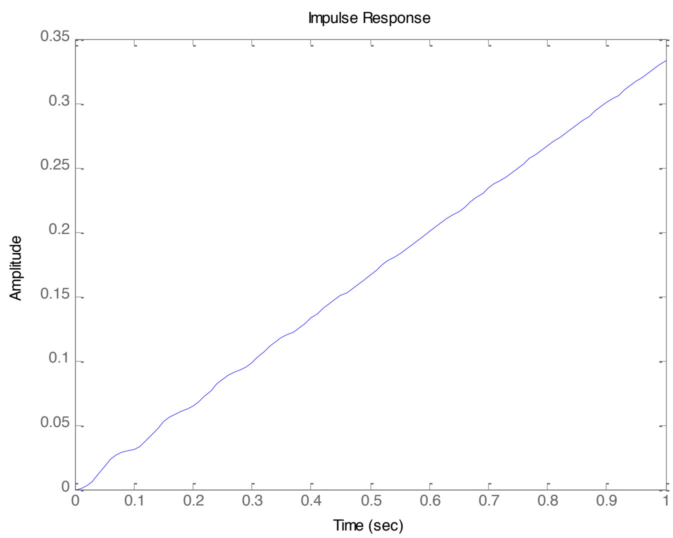
d =

	u1
y1	0

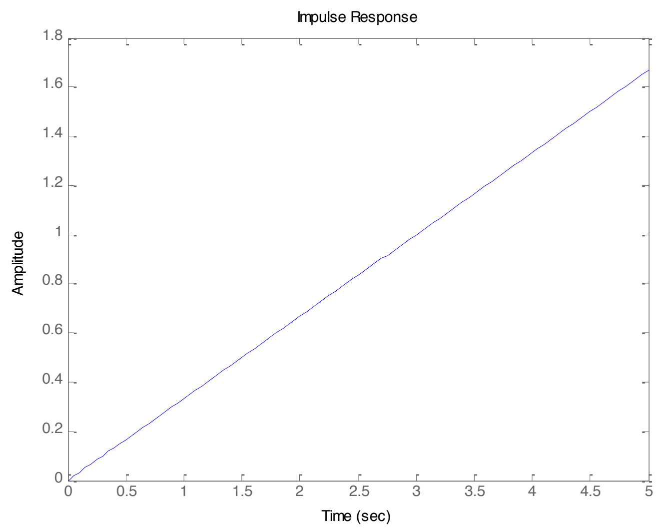
Continuous-time model.
Poles =
 -3.4500 +69.1961i
 -3.4500 -69.1961i
 -0.0000 + 0.0000i
 -0.0000 - 0.0000i
Zeroes =
 -695.6522

Depending on what you choose you could get a number of different plots.

Impulse response to 1 sec:



Impulse response to 5 sec:



Impulse response for no time being specified when using matlab command: