Hw #8 MAE 143b

Prob. 6.2

a) $|G(s)| = |G(j\omega)| = \sqrt{Re[G(j\omega)]^2 + Im[G(j\omega)]^2}$

$$\phi = \tan^{-1}\left[\frac{Im[G(j\omega)]}{Re[G(j\omega)]}\right] = \angle G(j\omega)$$

so $G(s) = \dfrac{1}{s+10}$

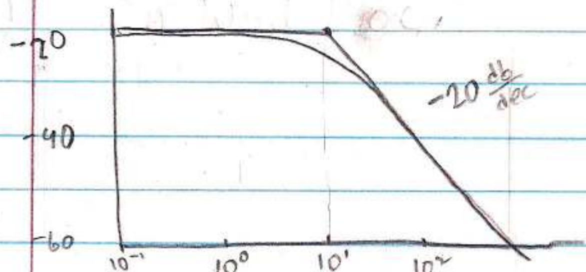
$$|G(j\omega)| = \frac{1}{j\omega+10} = \frac{10 - j\omega}{100+\omega^2}$$

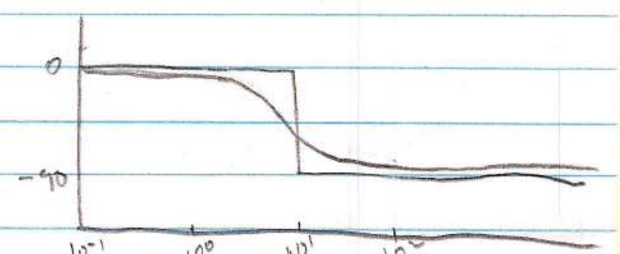
$$= \frac{1}{\sqrt{100+\omega^2}}$$

$$\angle G(j\omega) = \tan^{-1}\frac{\omega}{10} =$$

ω	$\lvert G(j\omega)\rvert$	$\angle G(j\omega)$
1	0.0995	-5.71
2	0.0981	-11.3
5	0.0894	-26.6
10	0.0707	-45.0
20	0.0447	-63.4
50	0.0196	-78.7
100	0.00995	84.3

b



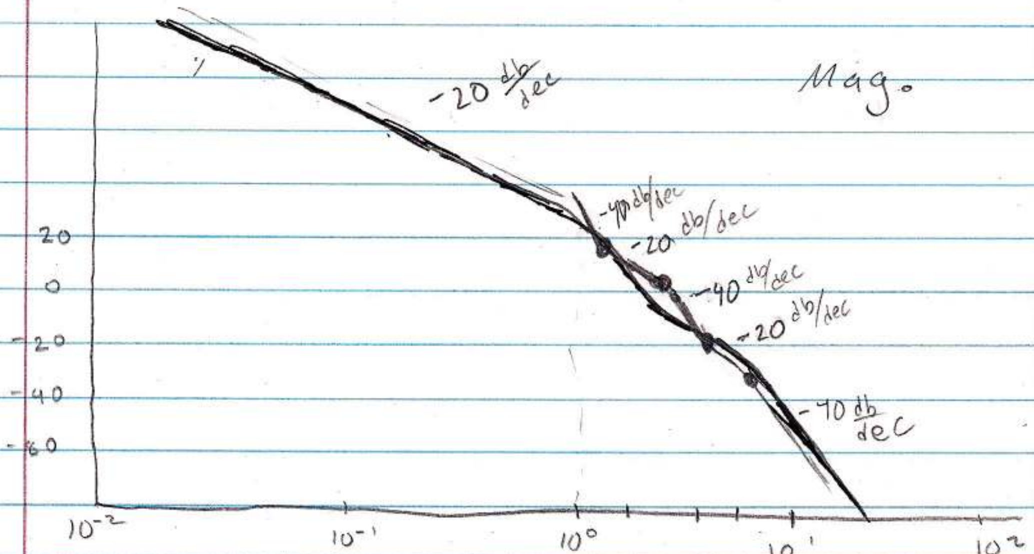


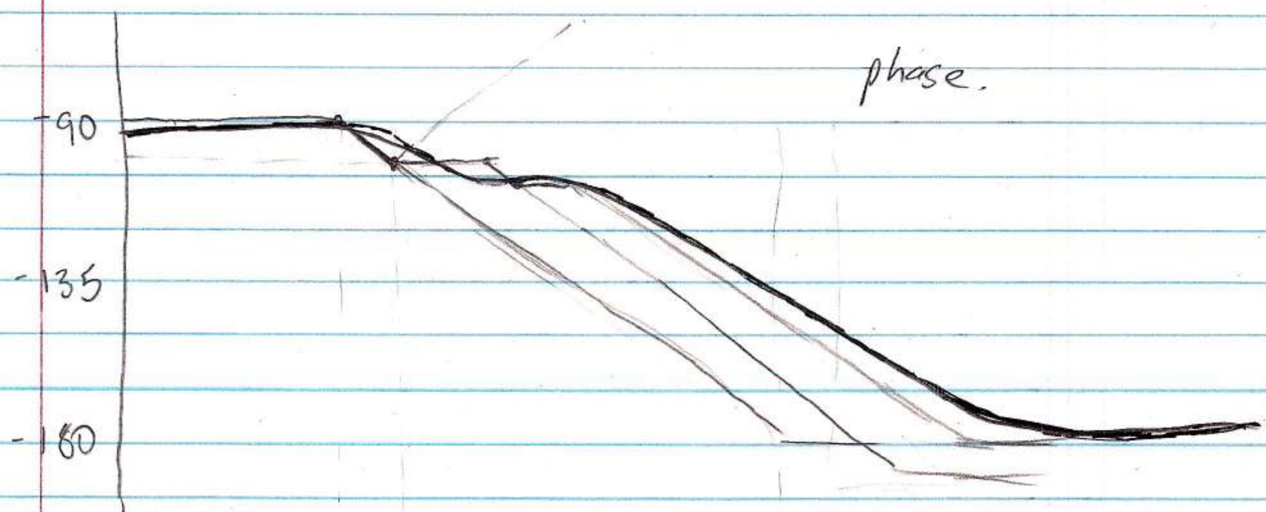
Prob (6.4c)

$$L(s) = \frac{(s+2)(s+6)}{s(s+1)(s+5)(s+10)} = \frac{2 \cdot 6}{5 \cdot 10} \frac{(s/2+1)(s/6+1)}{s(s+1)(s/5+1)(\frac{s}{10}+1)}$$

$$= 0.24 \frac{(s/2+1)(s/6+1)}{s(s+1)(s/5+1)(\frac{s}{10}+1)}$$

gain $= 20\log(0.24)$
$= -12.4$





Pole at 1 => $W_0 = .1 \to 10$

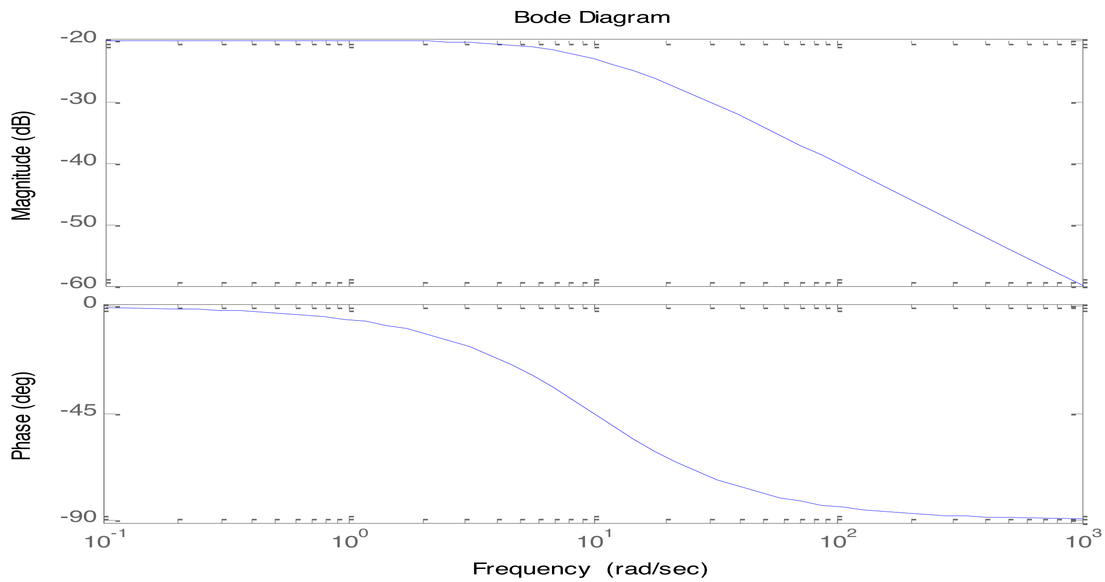
Zero at 2 => $W_0 = .2 \to 20$

Pole at 5 -> $W_0 = .5 \to 50$

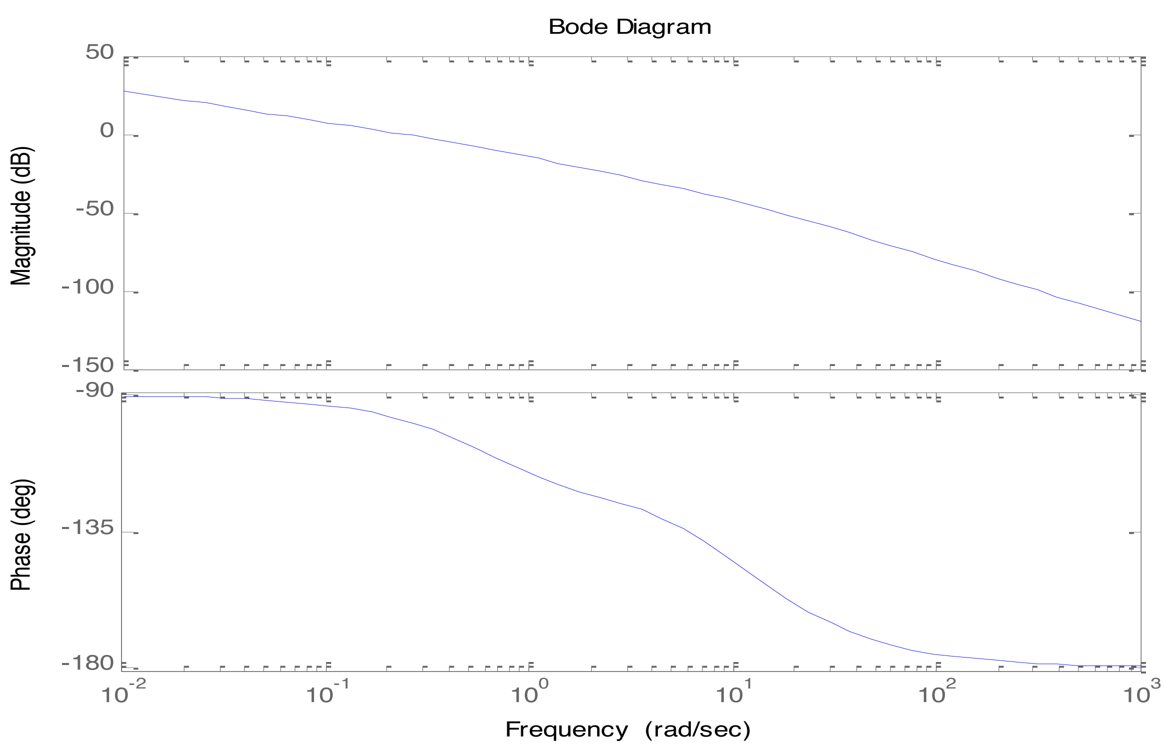
Zero at 6 => $W_0 = .6 \to 60$

Pole at 10 => $W_0 = 1 \to 100$

Problem 6.2:



Problem 6.4c)



Prob. 6.5 a)

$$L(s) = \frac{1}{s^2+3s+10} = \frac{1}{10}\cdot\frac{1}{\frac{s^2}{10}+\frac{3s}{10}+1} \approx \frac{1}{10}\,\frac{1}{\left(\frac{s}{w_0}\right)^2+2\zeta\left(\frac{s}{w_0}\right)+1}$$

$0 < \zeta < 1$

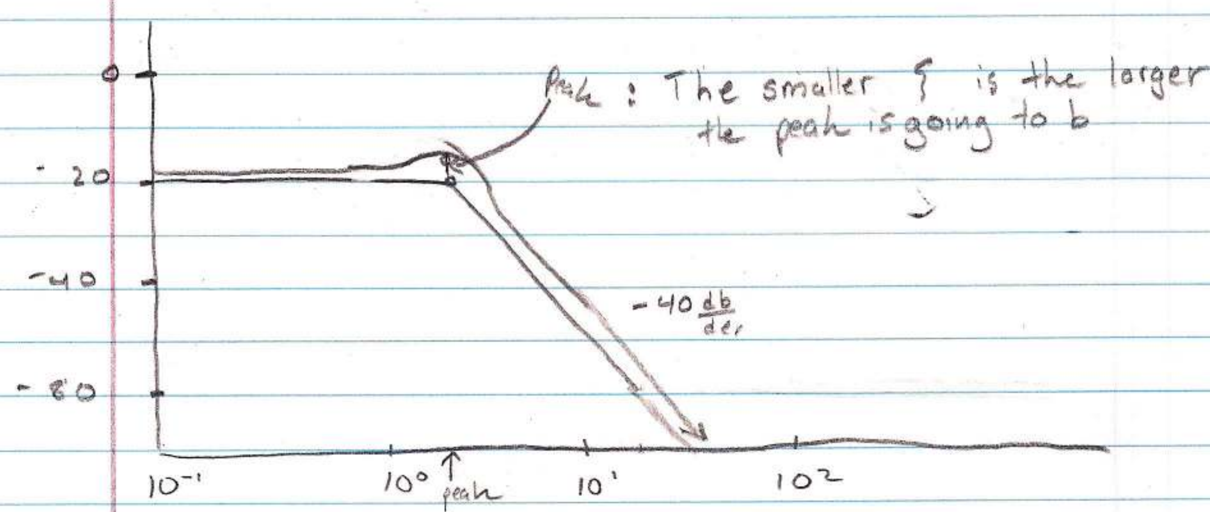
$w_0 = \sqrt{10} = 3.162$

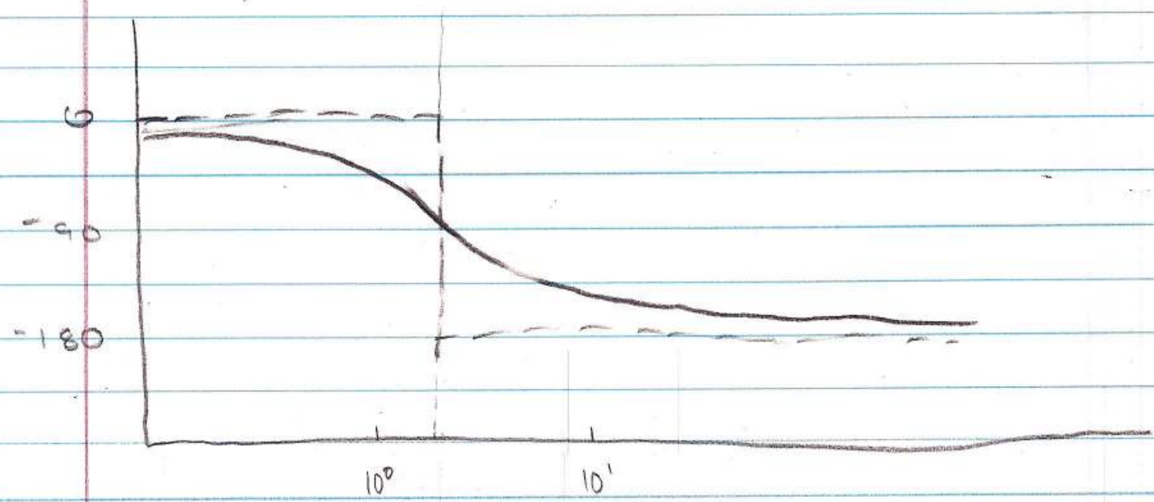
$\zeta = \frac{3\cdot 3.162}{10\cdot 2} = 0.474$

Peak Height $= -20\log\left(2\zeta\sqrt{1-\zeta^2}\right) = 1.565$ dB

at $w_r = w_0\sqrt{1-2\zeta^2} = 2.346$ rad/s

gain $= 20\log(1/10) = -20$

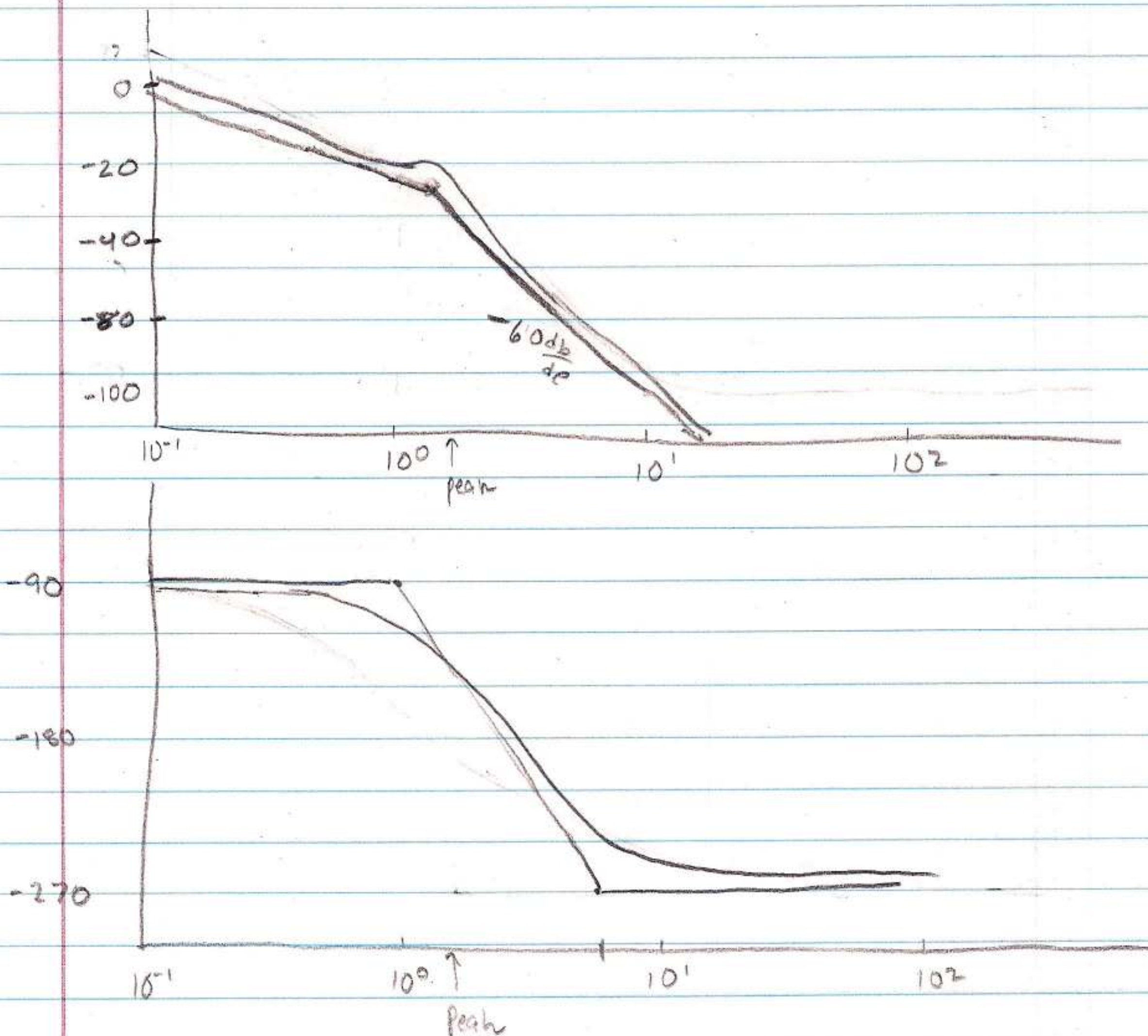




Prob 6.5 b)

$$L = \frac{1}{s(s^2+3s+10)}$$ gain = -20

Same as part a) but with a pole at zero

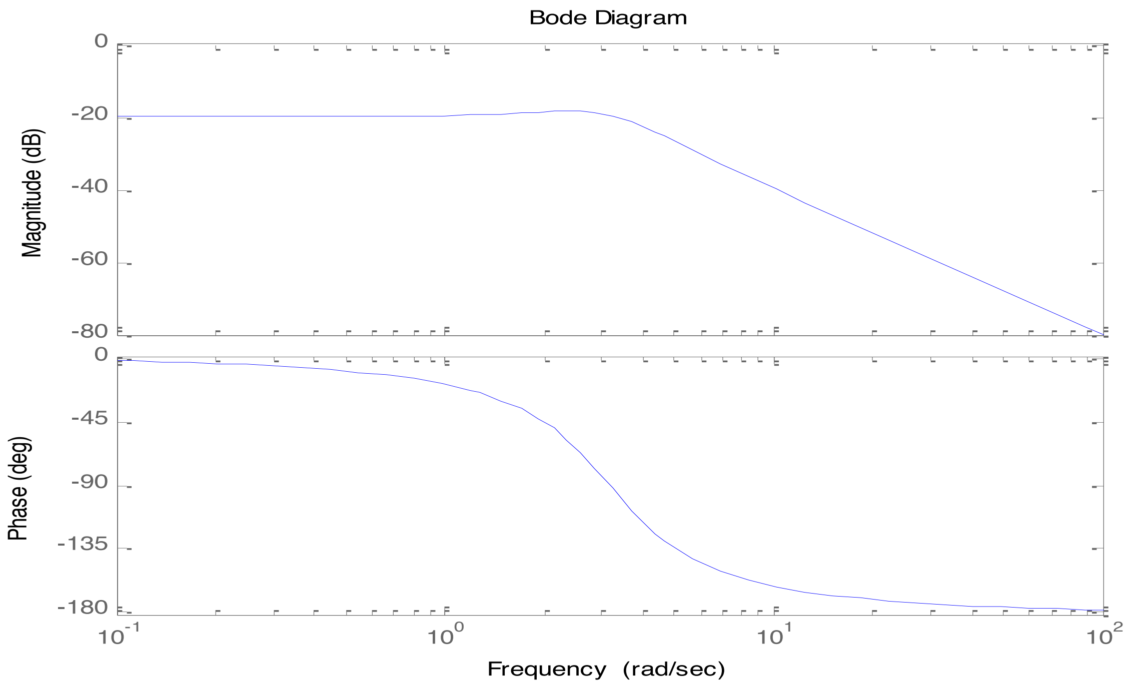


phase for complex poles: $\omega_o = 3.162$ $\zeta = 0.474$

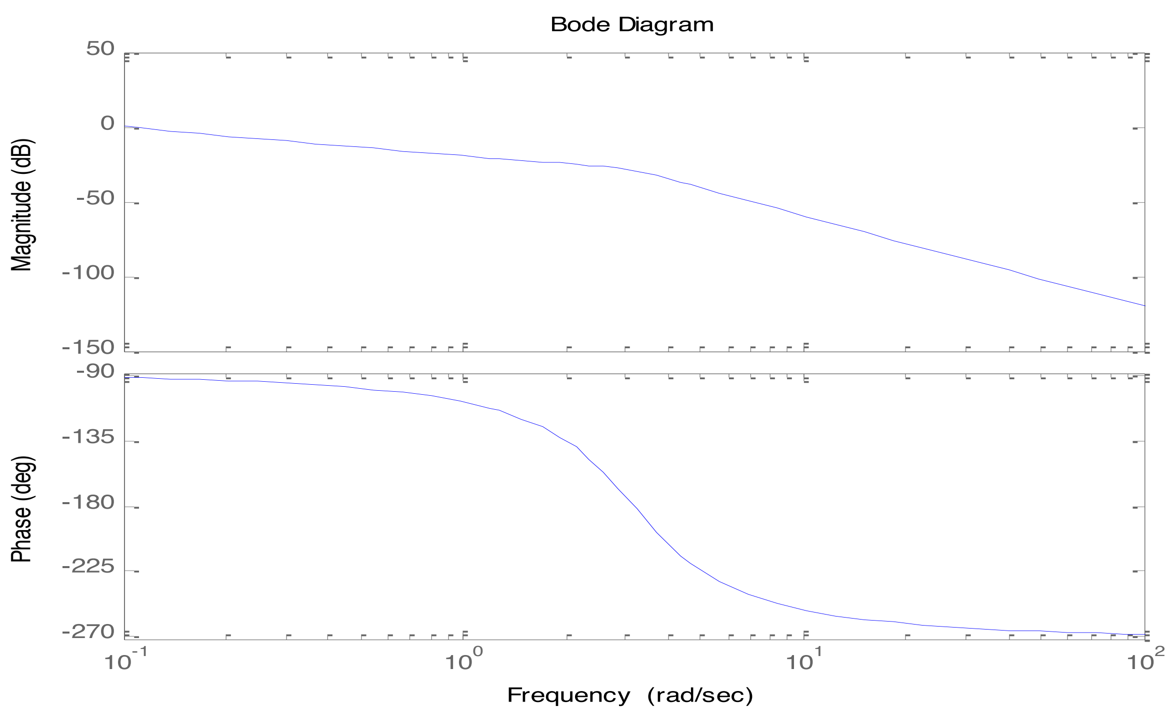
$\frac{\omega_o}{5^\zeta} \longrightarrow \omega_o 5^\zeta$

$1.4745 \longrightarrow 6.78$

Problem 6.5a)



Problem 6.5b)



Problem 6.5 e)

$L(s) = \frac{(s^2+1)}{s(s^2+4)} = \frac{1}{4}\frac{(s^2+1)}{s(\frac{s^2}{4}+1)}$ gain = -12.04

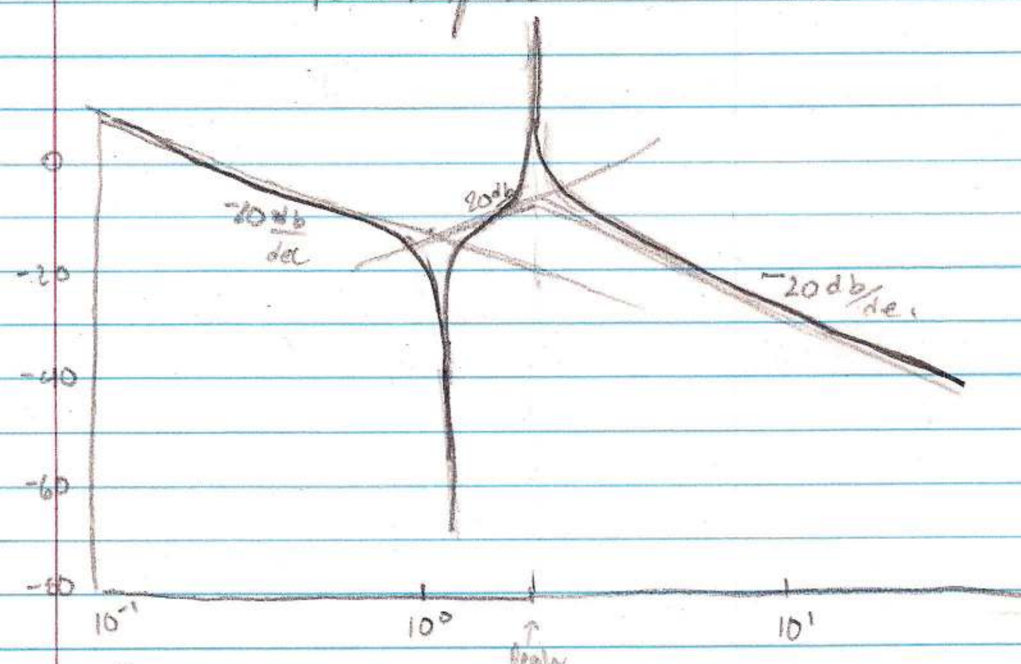
compare the complex poles and zeroes to $(\frac{s}{w_0})^2 + 2\zeta(\frac{s}{w_0})+1$

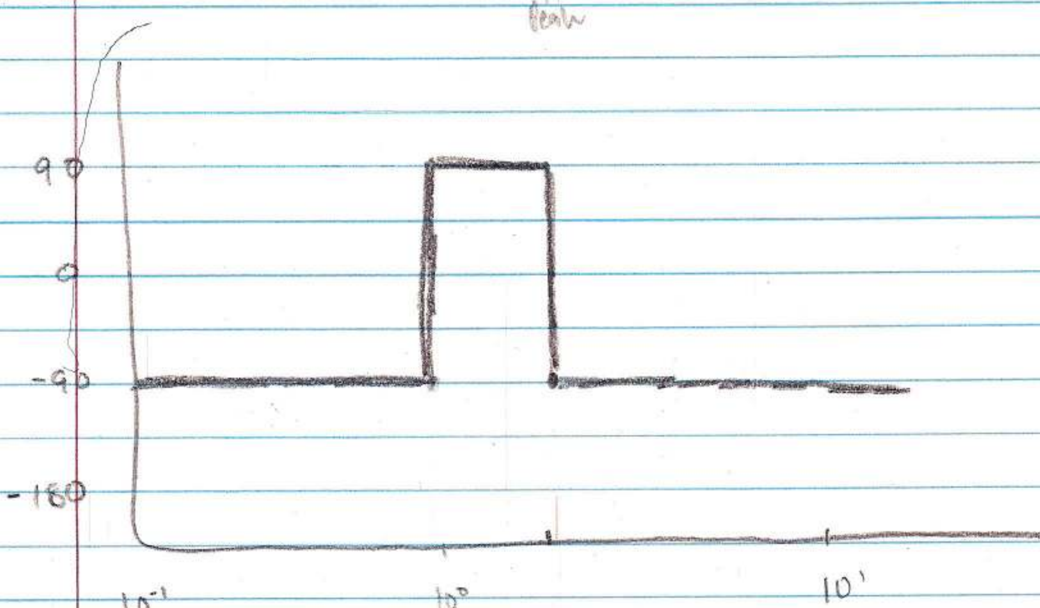
For (s^2+1) $w_0 = 1$ $\zeta = 0$

Peak Response at 1 rad/s is ∞

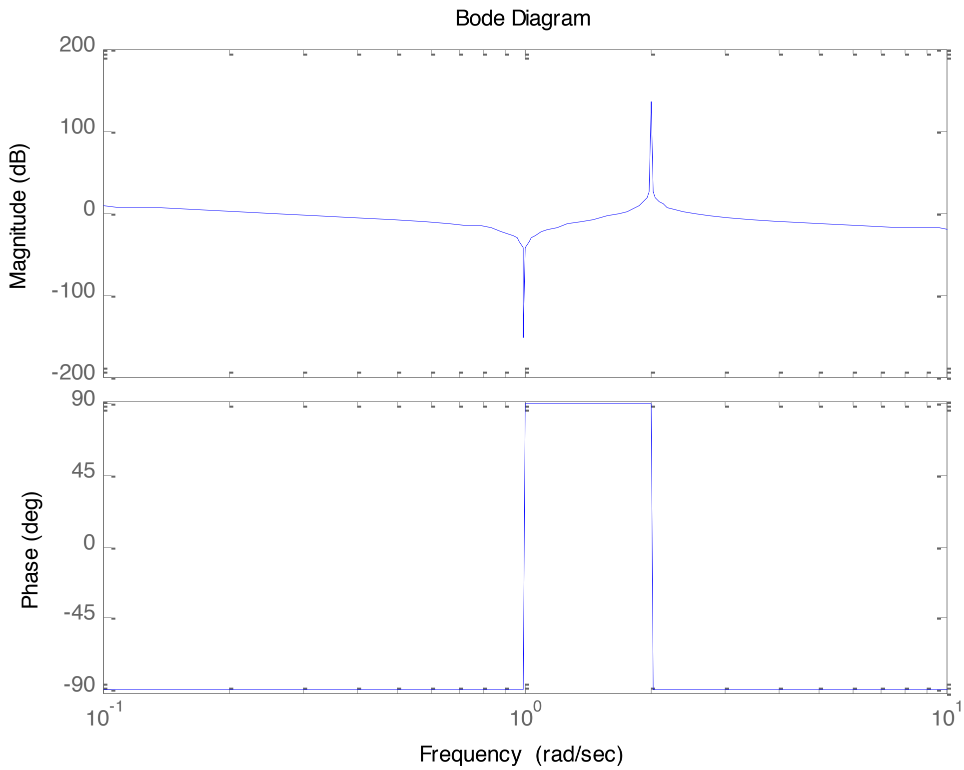
For (s^2+4) $w_0 = 2$ $\zeta = 0$

Peak Response at 2 is ∞





Problem 6.5e)



Matlab Code for all problems:

```
clc;clear
% Problem 6.2
L = tf([1],[1 10])
figure(1)
bode(L)

% Problem 6.4c
figure(2)
L = zpk([-2 -6],[0 -1 -5 -10],[1] )
bode(L);

% Problem 6.5a
L = tf([1],[1 3 10])
figure(3)
bode(L)

% Problem 6.5b
L = tf([1],[1 3 10 0])
figure(4)
bode(L)
```

```
% Problem 6.5e
L = tf([1 0 1],[1 0 4 0])
figure(5)
bode(L)
```

Prob 6.9

From looking at the figure or going by what the professor gave in the email you get that the bode plot is of:

$$G(s) = \frac{s+10}{s}$$

System Response to a unit step response

$$Y(s) = G(s)\,U(s) = \frac{s+10}{s} \cdot \frac{1}{s} = \frac{s+10}{s^2}$$

$$Y(s) = \frac{1}{s} + \frac{10}{s^2} \longrightarrow$$

Using partial fraction expansion find

$$y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$= 1(t) + 10(t) \qquad \text{for } t \geq 0$$