HW #2
MAE143B
Spring 2014

$\boxed{2.4}$

$m\mathring{v} + bv = mg$

$m = 70\,kg$

free $\leftarrow$ $\tilde{v}_f = 200\,km/h = 55.56\,\frac{m}{s}$

parachute $\leftarrow$ $\tilde{v}_p = 20\,km/h = 5.56\,\frac{m}{s}$

$g = 10\,m/s^2$

constant speed $\Rightarrow$ $\cancel{m\mathring{\tilde{v}}_f} + b\tilde{v}_f = mg \Rightarrow b = \frac{mg}{\tilde{v}_f} = \frac{70 \cdot 10}{55.56} = 12.6\,kg/s$

$m\mathring{\tilde{v}}_p + b\tilde{v}_p = mg \Rightarrow b = \frac{mg}{\tilde{v}_p} = \frac{70 \cdot 10}{5.56} = 125.9\,kg/s$

$m\mathring{v} + b\tilde{v} = mg \Rightarrow \mathring{v} + \frac{b}{m}\tilde{v} - g = 0$ (*)

The complete solution to equation (*) takes a form of;

$v(t) = v_p(t) + c\,v_H(t) = \frac{mg}{b} + ce^{-\frac{b}{m}t}$

$v_p(t) = \tilde{v}$

Free-fall case:

$\frac{b}{m} = \frac{12.6}{70} = 0.18\,\frac{1}{s}$

$\Rightarrow v(t) = 55.56 + ce^{-0.18t}$

At $t=0 \Rightarrow v(0) = 0 \Rightarrow c = -55.56$

$\Rightarrow v(t) = 55.56\,(1 - e^{-0.18t})$

time constant $= \tau = -\frac{1}{\lambda} = 5.56$

Parachute case:

$$\frac{b}{m} = \frac{125.9}{70} = 1.8\ \frac{1}{s}$$

$$\Rightarrow v(t) = 5.56 + Ce^{-1.8t}$$

$$\text{At } t = 0 \Rightarrow v(0) = 55.56 \Rightarrow c = 50$$

$$\Rightarrow v(t) = 5.56 + 50e^{-1.8t} \qquad (**)$$

$$\tau = -\frac{1}{\lambda} = 0.56s$$

$$v = 29\ \frac{km}{h} = 8.056\ \frac{m}{s}$$

using (**) and inserting $v(t) = 8.056 \Rightarrow$

$$8.056 = 5.56 + 50e^{-1.8t} \Rightarrow t = 1.66\ s$$

$$\text{height} = \int_0^{1.66} v(t)\,dt = \int_0^{1.66} (5.56 + 50e^{-1.8t})\,dt = 35.6m$$

$$\text{distance1} = \int_0^{60} 55.56(1 - e^{-0.18t})\,dt = 3025m$$

$$\text{distance2} = 4000 - 3025 = 925m$$

$$925 = \int_0^{t_2} (5.56 + 50e^{-1.8t})\,dt \Rightarrow t_2 = 180s$$

$$\Rightarrow t_{total} = 60 + 180 = 240s$$

$\boxed{2.11}$

$\tau = 0$

$r = 1m$

$m_1 = m_2 = 1000\ kg$

$b_1 = b_2 = 120\ kg\ m/s$

$J_1 = J_2 = 200\ kg\ m^2$

$g = 10\ m/s^2$

$v_1(0) = 0, 1\ m/s$, or $-1\ m/s$

$$J\dot{\omega} + (b_1 + b_2)\omega = \tau + gr(m_1 - m_2)$$

$$J = J_1 + J_2 + r^2(m_1 + m_2)$$

$$v_1 = r\omega$$

$$v_2 = -r\omega$$

$$v_1 = r\omega \Rightarrow \omega = \frac{v_1}{r}, \quad \dot{\omega} = \frac{\dot{v}_1}{r}$$

$$\Rightarrow \dot{v}_1 + \frac{(b_1 + b_2)}{J} v_1 = \tau \frac{r}{J} + g\frac{r^2}{J}(m_1 - m_2)$$

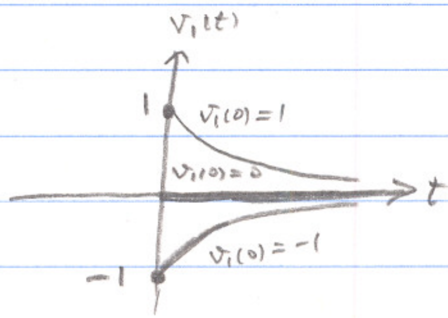
$$\lambda = -\frac{b_1 + b_2}{J},$$

$$v_1(t) = \tilde{v}_1(1 - e^{\lambda t}) + v_1(0)e^{\lambda t}$$

$$\lambda = \frac{-(b_1 + b_2)}{J_1 + J_2 + r^2(m_1 + m_2)} = -0.1$$

$$m_1 = m_2 \Rightarrow \tilde{v}_1 = 0$$

$$\Rightarrow v_1(t) = v_1(0)e^{-0.1t}$$



$\boxed{2-14}$

$$\frac{J}{r}\mathring{v}_1 + \frac{(b_1+b_2)}{r} v_1 = k(\bar{v}_1 - v_1)$$

$$\Rightarrow \mathring{v}_1 + \frac{b_1+b_2+rk}{J} v_1 = \frac{r}{J} k \bar{v}_1 \quad (*)$$

$$\lambda = -\frac{b_1+b_2+rk}{J} = -\frac{240+k}{2400}$$

$$\tau = -\frac{1}{\lambda} = 1 \Rightarrow \frac{2400}{240+k} = 1 \Rightarrow k = 2160$$

From $(*)$ $\Rightarrow \tilde{v}_1 = \frac{rk}{b_1+b_2+rk}\bar{v}_1 = \frac{k}{240+k}\bar{v}_1$

$$E(t) = \bar{v}_1(t) - \tilde{v}_1(t) = \left(1 - \frac{2160}{2400}\right)\bar{v}_1$$

$$\Rightarrow \lim_{t\to\infty} E(t) = 0.1\,\bar{v}_1$$

$\boxed{2-15}$

$$\dot{\bar{v}}_1 + \frac{b_1+b_2+rk}{J} v_1 = \frac{r}{J} k \bar{v}_1 + \frac{gr^2}{J}(m_1 - m_2)$$

$$\tilde{v}_1 = \frac{rk\bar{v}_1 + gr^2(m_1-m_2)}{b_1+b_2+rk}, \quad \lambda = -\frac{b_1+b_2+rk}{J}$$

$$J = J_1 + J_2 + r^2(m_1+m_2)$$

$$\Rightarrow J = 200+200+1(1000+800) = 2200$$

$$\tau = -\frac{1}{\lambda} \Rightarrow \tau = \frac{2200}{240+k} = 1 \Rightarrow k = 1960$$

$$\lambda = -\frac{240+k}{2200}$$

$$E(t) = \bar{v}_1 - \tilde{v}_1$$

$$\tilde{v}_1 = \frac{1960\bar{v}_1 + 10(1000-800)}{240+1960}$$

$$\Rightarrow E(t) = \frac{1960\bar{v}_1 + 2000}{2200}$$

$$\lim_{t\to\infty} E(t) = 0.11\bar{v}_1 + 0.91$$