Problems

P4.1. Calculate

$$S = \frac{1}{1+GK}, \qquad D = \frac{G}{1+GK}$$
$$H = \frac{GK}{1+GK}, \qquad Q = \frac{K}{1+GK}$$

for the system and controller transfer-functions:

$$G(s) = \frac{a}{s+a}, \qquad K(s) = K.$$

What is the order of each transfer-function?

P4.2. Repeat **P4.1** for the following system and controller transfer-functions:

a) $G = \dfrac{a}{s+a}, \qquad K = \dfrac{K_p}{s};$

b) $G = \dfrac{a}{s+a}, \qquad K = K_p + \dfrac{K_i}{s};$

c) $G = \dfrac{a}{s+a}, \qquad K = K_l \dfrac{s+c}{s+b};$

d) $G = \dfrac{a_2}{s^2 + a_1 s + a_2}, \qquad K = K_p;$

e) $G = \dfrac{a_2}{s^2 + a_1 s + a_2}, \qquad K = K_p + K_d s;$

f) $G = \dfrac{a_2}{s^2 + a_1 s + a_2}, \qquad K = K_p + \dfrac{K_i}{s}.$

P4.3. Consider the standard feedback connection in Fig. 4.16. Is the closed-loop system asymptotically stable if the system and controller transfer-functions are:

$$G = \frac{1}{s}, \qquad K = 1,$$

Does the closed-loop system achieve asymptotic tracking of a constant input $y(t) = \bar{y}$, $t \geq 0$?

P4.4. Repeat **P4.3** for the following combinations of system and controller transfer-functions:

a) $G = \dfrac{1}{s}, \qquad K = \dfrac{1}{s+1};$

b) $G = \dfrac{1}{s}, \qquad K = \dfrac{1}{s};$

c) $G = \dfrac{1}{s+1}, \qquad K = 1;$

d) $G = \dfrac{1}{s+1}, \qquad K = \dfrac{1}{s};$

e) $G = \dfrac{s}{s+1}, \qquad K = \dfrac{1}{s};$

f) $G = \dfrac{1}{s^2+s+1}, \qquad K = 1;$

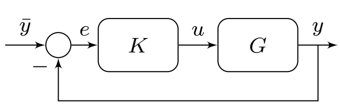
g) $G = \dfrac{1}{s^2+s+1}, \qquad K = \dfrac{1}{s};$

h) $G = \dfrac{1}{s^2+s+1}, \qquad K = \dfrac{s+1}{s};$

i) $G = \dfrac{1}{s^2+s+1}, \qquad K = \dfrac{1}{s+1};$

j) $G = \dfrac{1}{s^2+s+1}, \qquad K = \dfrac{1}{s(s+1)};$

Fig. 4.16: Diagram for **P4.3**



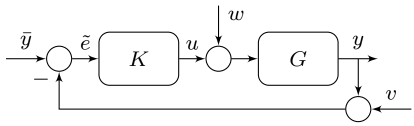


Fig. 4.17: Diagram for **P4.5**

P4.5. Consider the closed-loop connection in Fig. 4.17 with $v = 0$ and the combinations of system and controller transfer-functions in **P4.3** and **P4.4**. Does the closed-loop system achieve asymptotic rejection of a constant disturbance $w(t) = \bar{w}$, $t \geq 0$?

P4.6. Consider the closed-loop connection in Fig. 4.17 with $v = 0$ and the system and controller transfer-functions:

$$G = \frac{1}{s}, \qquad K = \frac{(s+1)^2}{s^2+1}.$$

Show that the closed-loop system asymptotically tracks a constant reference input $y(t) = \bar{y}$, $t \geq 0$, and asymptotically rejects an input disturbance $w(t) = \bar{w}\cos(t)$, $t \geq 0$.

P4.7. Consider the closed-loop connection in Fig. 4.17 with $w = 0$ and the system and controller transfer-functions:

$$G = \frac{1}{s+1}, \qquad K = \frac{1}{s}.$$

Calculate the steady-state component of the output, y, and the tracking error, $e = \bar{y} - y$, in response to:

$$\bar{y}(t) = \bar{y}, \quad v(t) = \bar{v} + \cos(\omega t), \quad t \geq 0,$$

Does the closed-loop achieve asymptotic tracking of the reference input, $\bar{y}(t)$? Does the closed-loop achieve asymptotic rejection of the measurement noise input, $v(t)$? What happens if ω is very large and $\bar{v}$ is zero?

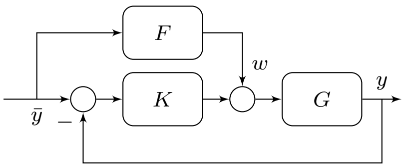


Fig. 4.18: Diagram for **P4.9**

P4.8. Repeat **P4.1** for the following system and controller transfer-functions:

a) $G = \dfrac{1}{s+1}$, $\quad K = \dfrac{s+1}{s+2}$;

b) $G = \dfrac{1}{s-1}$, $\quad K = \dfrac{s-1}{s+1}$;

c) $G = \dfrac{1}{s(s+1)}$, $\quad K = \dfrac{s}{s+1}$;

d) $G = \dfrac{s-1}{s+1}$, $\quad K = \dfrac{1}{s-1}$;

Is the closed-loop system internally stable? Explain your reasoning.

P4.9. The controller in Fig. 4.18 is known as a two degrees-of-freedom controller. The transfer-function K is the feedback part of the controller and the transfer-function F is the *feedforward* part of the controller. Show that

$$y = (H + FD)\bar{y},$$

where

$$H = \frac{GK}{1+GK}, \qquad D = \frac{G}{1+GK}.$$

How does the choice of the feedforward term F affect closed-loop stability? Name one advantage and one disadvantage of this scheme if you are free to pick any suitable feedforward, F, and feedback, K, transfer-functions.

P4.10. With respect to **P4.9** and the block diagram in Fig. 4.18 show that if

$$F = G^{-1}$$

then $y = \bar{y}$ regardless of K. Compare this with the open-loop solution discussed in § 1. Is it possible to chose $F = G^{-1}$ when: a) G is not asymptotically stable? b) G has a zero on the right-hand side of the complex plane? c) G has a delay in the form $G(s) = e^{-s\tau}\tilde{G}(s)$, $\tau > 0$?

P4.11. Let

$$G(s) = \frac{s+1}{s-1}, \qquad K(s) = K.$$

Select K and design F in the block diagram in Fig. 4.18 so that $y = \bar{y}$. If $\bar{y}$ is a unit step, what is the corresponding signal w?

P4.12. You have shown in **P2.7** that the ordinary differential equation

$$(J_1 r_1^2 + J_2 r_2^2)\,\dot{\omega}_1 = r_2^2\,\tau,$$
$$\omega_2 = (\,r_1/r_2\,)\,\omega_1$$

is a simplified description of the motion of a rotating machine driven by a belt without slip as in Fig. 2.18, where ω_1 is the angular velocity of the driving shaft and ω_2 is the machine's angular velocity. Let $r_1 = 0.05$m, $r_2 = 0.25$m, $m_1 = 1$kg, $m_2 = 10$kg, $b_1 = 0.125$kg m/s, $b_2 = 6.25$kg m/s, $J_i = m_i r_i^2/2$, $i = 1, 2$. Design a feedback controller:

$$\tau = K(\bar{\theta}_2 - \theta_2), \quad \theta_2 = \int_0^t \omega_2(\tau)\,d\tau,$$

and select K such that the closed-loop system is internally stable. Can the closed-loop system asymptotically track a constant angular reference $\bar{\theta}_2(t) = \bar{\theta}_2$, $t \geq 0$? Explain.

P4.13. You have shown in **P2.10** that the ordinary differential equation

$$J\,\dot{\omega} + (b_1 + b_2)\,\omega = \tau + g\,r(m_1 - m_2),$$
$$J = J_1 + J_2 + r^2(m_1 + m_2),$$
$$v_1 = r\,\omega,$$

is a simplified description of the motion of the elevator in Fig. 2.19, where ω is the angular velocity of the driving shaft and v_1 is the elevator's load linear velocity. Let $r = 1$m, $m_1 = m_2 = 1000$kg, $b_1 = b_2 = 120$kg m/s, $J_1 = J_2 = 200$kg m^2, and $g = 10$m/s^2. Design a feedback controller:

$$\tau = K(\bar{x}_1 - x_1), \quad x_1 = \int_0^t v_1(\tau)\,d\tau,$$

and select K such that the closed-loop system is internally stable. Can the closed-loop system asymptotically track a constant position reference $x_1(t) = \bar{x}_1$, $t \geq 0$? Explain.

P4.14. Repeat **P4.13** with $m_2 = 800$kg.

P4.15. Design a feedback controller so that the closed-loop elevator system in **P4.13** can asymptotically track a constant reference $\bar{x}_1(t) = \bar{x}_1$, $t \geq 0$ when $m_2 = 800$kg.