MAE 143b
Spring 2014
HW#6

4.2c

$$G = \frac{a}{s+a} \quad \text{and} \quad K = k_1 \frac{s+c}{s+b}$$

$$S = \frac{1}{1+GK} = \frac{(s+a)(s+b)}{(s+a)(s+b) + ak_1(s+c)}$$ second order

$$D = \frac{G}{1+GK} = \frac{a(s+b)}{(s+a)(s+b) + ak_1(s+c)}$$ second order

$$H = \frac{GK}{1+GK} = \frac{ak_1(s+c)}{(s+a)(s+b) + ak_1(s+c)}$$ second order

$$Q = \frac{K}{1+GK} = \frac{k_1(s+a)(s+c)}{(s+a)(s+b) + ak_1(s+c)}$$ second order

$\boxed{4.2e}$

$$G = \frac{a_2}{s^2 + a_1 s + a_2} \quad \text{and} \quad K = K_p + K_d s$$

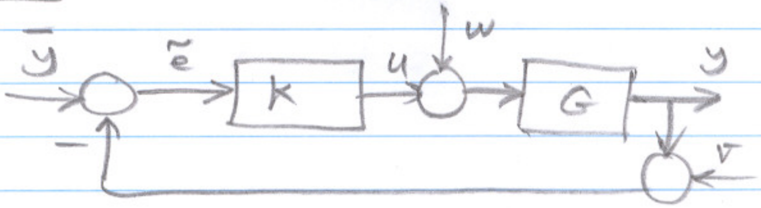
$$S = \frac{1}{1+GK} = \frac{s^2 + a_1 s + a_2}{s^2 + a_1 s + a_2 + (K_p + K_d s) a_2} \quad \text{second order}$$

$$D = \frac{G}{1+GK} = \frac{a_2}{s^2 + a_1 s + a_2 + (K_p + K_d s) a_2} \quad \text{second order}$$

$$H = \frac{GK}{1+GK} = \frac{a_2 (K_p + K_d s)}{s^2 + a_1 s + a_2 + (K_p + K_d s) a_2} \quad \text{second order}$$

$$Q = \frac{K}{1+GK} = \frac{(K_p + K_d s)(s^2 + a_1 s + a_2)}{s^2 + a_1 s + a_2 + (K_p + K_d s) a_2} \quad \text{third order}$$

$\boxed{4.7}$



$w = 0$

$$G = \frac{1}{S+1} \quad \text{and} \quad K = \frac{1}{s}$$

$$\bar{y}(t) = \bar{y} \qquad v(t) = \bar{v} + \cos(wt) \qquad t \geq 0$$

Since, we have cos(wt) function as one of input, thus we need to use below equation to find the steady-state response. Also this can also be used for a constant input considering $w = 0$. Here is the steady-state respons of system considering all inputs.

$$y_{ss}(t) = \bar{y}\, S(0) - \bar{v}\, S(0) + |S(jw)| \cos(wt + \measuredangle S(jw))$$

$S(0) = \underset{w \to 0}{S(jw)}$ and $S(jw)$ is the transfer function.

finding $S(jw)$: $Y(s) = GU(s) = GK\tilde{E}(s) = GK(\bar{Y}(s) - V(s) - Y(s))$

$$\Rightarrow S(s) = \frac{GK}{1+GK} = \frac{1}{S(S+1)+1} \overset{s=jw}{=} \frac{1}{jw + (1-w^2)}$$

$S(0) = 1$

$$|S(jw)| = \frac{1}{\sqrt{w^2 + (1-w^2)^2}} \qquad \measuredangle S(jw) = -\arctan\left(\frac{w}{1-w^2}\right)$$

$$\Rightarrow y_{ss}(t) = \bar{y} - \bar{v} + \frac{1}{\sqrt{w^2 + (1-w^2)^2}} \cos\left(wt - \arctan\left(\frac{w}{1-w^2}\right)\right)$$

$$e_{ss}(t) = \bar{y} - y_{ss}(t) = \bar{v} - \frac{1}{\sqrt{w^2 + (1-w^2)^2}} \cos\left(wt - \arctan\left(\frac{w}{1-w^2}\right)\right)$$

→ The closed-loop <u>does not</u> achieve asymptotic tracking of the reference input ($\bar{y}(t)$) because there non-zero terms in steady-State response.

→ <u>NO</u>, the closed-loop does not achieve asymptotic rejection of the measurement noise input.

→ If $\bar{v} = 0$ and $\omega >> 0 \Longrightarrow$ Tracking is achieved because in this case $y_{ss}(t) = \bar{y}$.

$\boxed{4.13}$

$$J\dot{\omega} + (b_1+b_2)\omega = \tau + gr(m_1 - m_2)$$
$$J = J_1 + J_2 + r^2(m_1+m_2)$$
$$v_1 = r\omega \Rightarrow \omega = \frac{v_1}{r}$$

$r = 1m$

$m_1 = m_2 = 1000\ kg$

$b_1 = b_2 = 120\ kg\ m/s$

$J_1 = J_2 = 200\ kg\ m^2$

$g = 10 m/s^2$

$$\tau = k(\bar{x}_1 - x_1),\ x_1 = \int_0^t v_1(\tau)\,d\tau \Rightarrow \dot{x}_1 = v_1(t) \quad (**)$$

Replacing ω with $v_1 \Rightarrow J\dot{v}_1 + (b_1+b_2)v_1 = \tau \cdot r + gr^2(m_1 - m_2)$

Replacing v_1 with $x_1 \Rightarrow J\ddot{x}_1 + (b_1+b_2)\dot{x}_1 = \tau \cdot r + gr^2(m_1 - m_2)$

Replacing $\tau = k(\bar{x}_1 - x_1) \Rightarrow J\ddot{x}_1 + (b_1+b_2)\dot{x}_1 = k(\bar{x}_1 - x_1)\cdot r + gr^2(m_1 - m_2)$

Taking Laplace from both sides of equation:

$$Js^2X_1(s) + (b_1+b_2)sX_1(s) = k(\bar{X}_1(s) - X_1(s))\cdot r + \frac{gr^2}{s}(m_1 - m_2)$$

$$\Rightarrow X_1(s)\left(Js^2 + (b_1+b_2)s + kr\right) = k\bar{X}_1(s)r + \frac{gr^2}{s}(m_1 - m_2)$$

$$\Rightarrow X_1(s) = \frac{rk\bar{X}_1(s)}{Js^2 + (b_1+b_2)s + kr} + \frac{gr^2(m_1 - m_2)}{s(Js^2 + (b_1+b_2)s + kr)}$$

Roots for denomination:

$$Js^2 + (b_1+b_2)s + Kr = 0 \Rightarrow s = \frac{-(b_1+b_2) \pm \sqrt{(b_1+b_2)^2 - 4JKr}}{2J}$$

Inserting the given values:

$$\Rightarrow s = \frac{-240 \pm \sqrt{(240)^2 - 4(2400) \times K}}{2 \times 2400}$$

In order to make the system internally stable:

$$s = \frac{-240 + \sqrt{57600 - 9600k}}{4800} < 0$$

$$\Rightarrow \boxed{k > 0}$$

$$\left.\begin{array}{l} \lim_{t \to \infty} x_1(t) = \lim_{s \to 0} s X_1(s) \\ \bar{X}_1(s) = \frac{\bar{X}_1}{s} \\ m_1 = m_2 \end{array}\right\} \Rightarrow \lim_{s \to 0} s X_1(s) = \bar{X}_1$$

Yes. As shown, the closed-loop system asymptotically track a constant position reference.

$\boxed{4.14}$

From answer to 4.13, we have;

$$X_1(s) = \frac{rk\bar{X}_1(s)}{Js^2+(b_1+b_2)s+kr} + \frac{gr^2(m_1-m_2)}{s(Js^2+(b_1+b_2)s+kr)}$$

$$Js^2+(b_1+b_2)s+kr=0$$

$$\Rightarrow s = \frac{-(b_1+b_2)\pm\sqrt{(b_1+b_2)^2-4Jkr}}{2J}$$

Inserting the given values:

$$s = \frac{-240\pm\sqrt{57600-4(2200)k}}{2\times2200}$$

$$\Rightarrow s = \frac{-240\pm\sqrt{57600-8800k}}{4400}$$

Again, $k>0$ so the system is internally stable.

$$\left.\begin{aligned} &\lim_{t\to\infty} x_1(t) = \lim_{s\to 0} sX_1(s) \\ &\bar{X}_1(s) = \frac{\bar{x}_1}{s} \\ &m_1 = 1000\,kg \\ &m_2 = 800\,kg \end{aligned}\right\} \Rightarrow \lim_{s\to 0} sX_1(s) = \bar{x}_1 + \frac{2000}{k}$$

No. As shown, the closed-loop system cannot track a constant position.