Problems

P7.1. Draw the straight-line approximation and sketch the Bode magnitude and phase diagrams for the following transfer functions:

a) $G = \dfrac{1}{(s+1)(s+10)}$;

b) $G = \dfrac{s+0.1}{(s+1)(s+10)}$;

c) $G = \dfrac{s+10}{s^2+0.1s+1}$;

d) $G = \dfrac{s-0.1}{(s-1)(s+10)}$;

e) $G = \dfrac{s+10}{s^2-0.1s+1}$;

f) $G = \dfrac{s}{(s+1)(s+10)}$;

g) $G = \dfrac{s+10}{(s+1)^2}$;

h) $G = \dfrac{s+1}{s(s+10)}$;

i) $G = \dfrac{s+1}{s^2(s+10)}$;

j) $G = \dfrac{s+10}{(s^2+0.1s+1)^2}$;

Compare your sketch with the one produced by MATLAB's bode function.

P7.2. Draw the polar plot associated with the rational transfer-functions in **P7.1**. Use Nyquist's stability criterion to decide whether the are stable under negative unit feedback. If not, is there a gain for which the closed-loop system can be made asymptotically stable?

P7.3. Find a minimum-phase rational transfer-function that matches the Bode magnitude diagrams in Figs. 7.31 and 7.32. The straight-line approximations are plotted as thin lines.

P7.4. Calculate a rational transfer-function that matches the Bode magnitude diagrams in Figs. 7.31 and 7.32 and the corresponding phase diagrams in in Figs. 7.33 and Fig. 7.34.

P7.5. Draw the polar plot associated with the Bode diagrams in Figs. 7.31, 7.32, 7.33, and 7.34. Use the Nyquist stability criterion to decide whether the corresponding rational transfer-functions are stable under negative unit feedback. If not, is there a gain for which the closed-loop system can be made asymptotically stable?

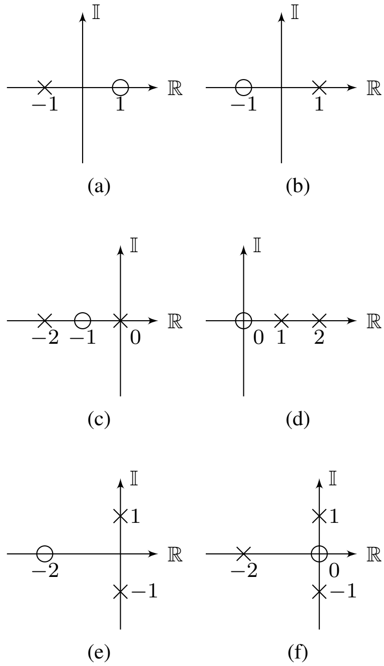

Fig. 7.30: Block diagrams for **P7.6**

P7.6. Draw the Bode plot and the polar plot associated with the pole-zero diagrams in Fig. 7.30 assuming that the transfer functions have unit gain at $\omega = 0$. Use the Nyquist stability criterion to decide whether the corresponding rational transfer-functions are stable under negative unit feedback. If not, is there a gain for which the closed-loop system can be made asymptotically stable?

P7.7. You have designed in **P6.4** a mass-spring-damper suspension system for a car with $1/4$ mass equal to 640kg to have a natural frequency $f_n = 10$Hz and damping ratio $\zeta = 0.08$. Draw the Bode magnitude and phase diagrams corresponding to your design. Interpret the results of **P6.5** and **P6.6** in light of the frequency response.

P7.8. You have designed in **P6.8** a mass-spring-damper suspension system for a car with $1/4$ mass equal to 600kg, tire stiffness equal to $k_u = 200000$N/m, and negligible tire damping coefficient $b_u = 0$, to have a natural frequency $f_n = 10$Hz and damping ratio $\zeta = 0.08$. Draw the

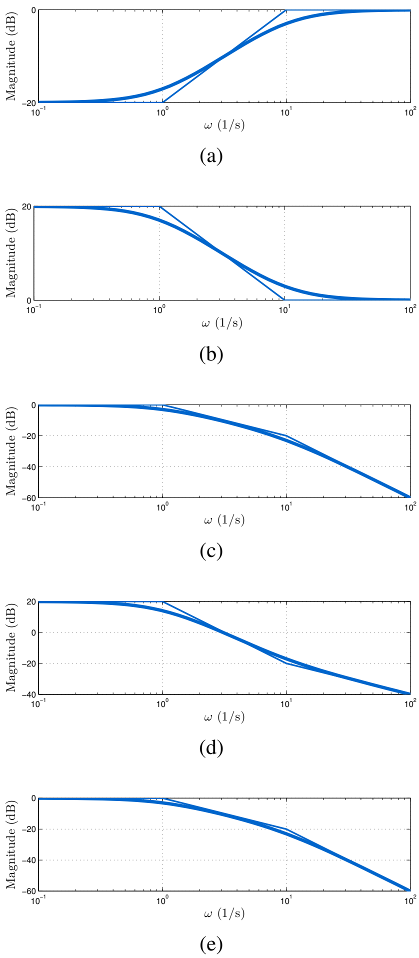

Fig. 7.31: Diagrams for **P7.3-P7.5**

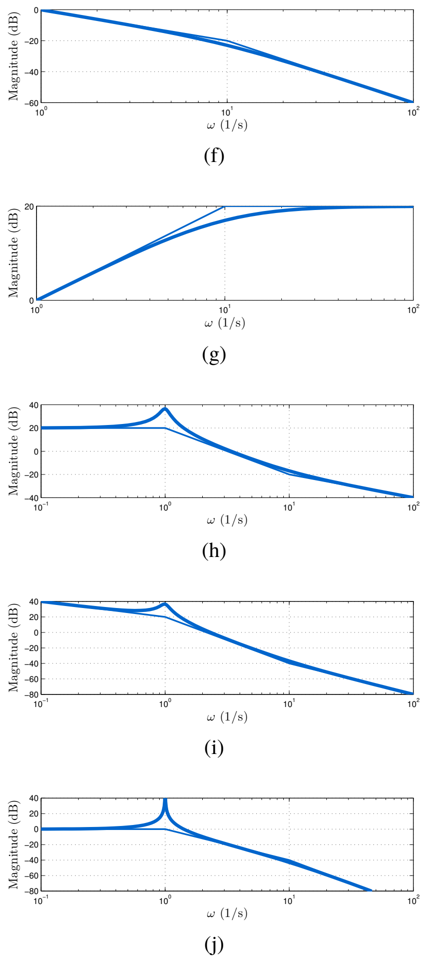

Fig. 7.32: Diagrams for **P7.4-P7.5**

Bode magnitude and phase diagrams corresponding to your design. Interpret the results of **P6.9** and **P6.10** in light of the frequency response.

P7.9. You have shown in **P2.7** and **P2.9** that the ordinary differential equation

$$J_r\,\dot{\omega}_1 + b_r\,\omega_1 = r_2^2\,\tau, \quad \omega_2 = (\,r_1/r_2\,)\,\omega_1,$$

is a simplified description of the motion of a rotating machine driven by a belt without slip as in Fig. 2.18, where

$$J_r = J_1 r_2^2 + J_2 r_1^2, \quad b_r = b_1 r_2^2 + b_2 r_1^2,$$

ω_1 is the angular velocity of the driving shaft and ω_2 is the machine's angular velocity. Let $r_1 = 0.05$m, $r_2 = 0.25$m, $m_1 = 1$kg, $m_2 = 10$kg, $b_1 = 0.125$kg m^2/s, $b_2 = 6.25$kg m^2/s, $J_i = m_i r_i^2/2$, $i = 1, 2$. Use Bode plots and the Nyquist diagram to design an I-controller:

$$\tau = K \int_0^t e(\sigma)\,d\sigma, \qquad e = \bar{\omega}_2 - \omega_2$$

and select K so that the closed-loop system is asymptotically stable. Calculate the corresponding gain and phase margin. Is the closed-loop capable of asymptotically tracking a constant reference input $\bar{\omega}_2$?

P7.10. The belt system in **P7.9** is connected to a piston that applies a periodic torque that can

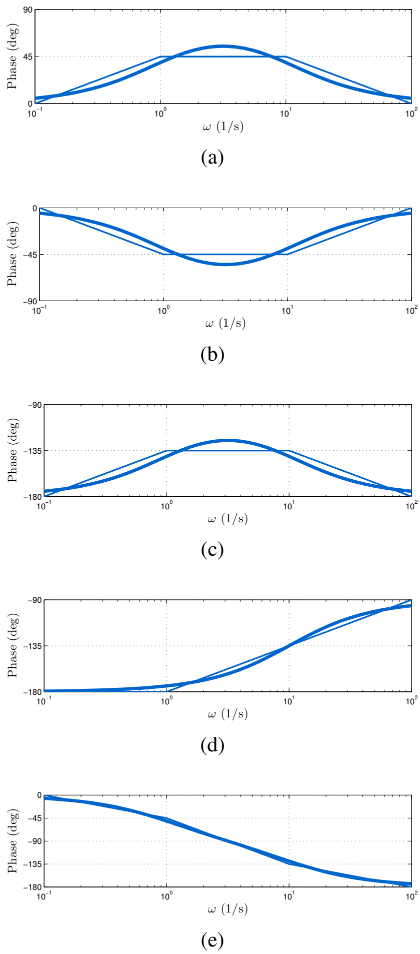

Fig. 7.33: Diagrams for **P7.4-P7.5**

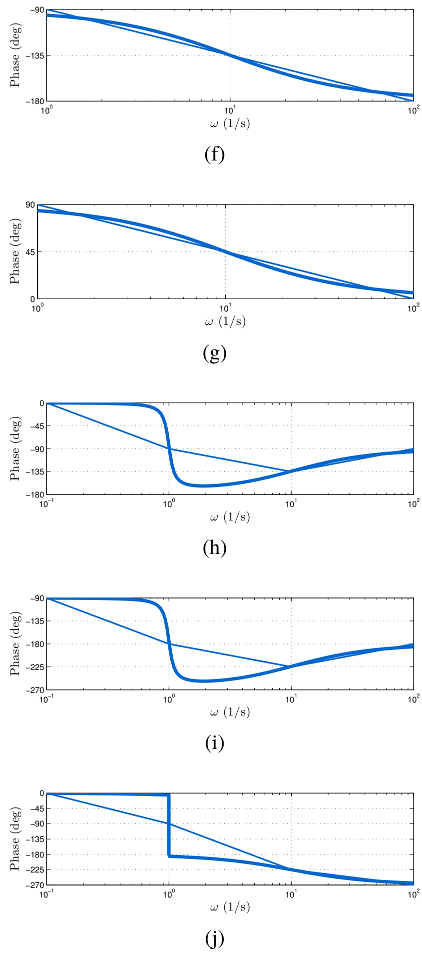

Fig. 7.34: Diagrams for **P7.4-P7.5**

be approximated by $\tau_2(t) = h\cos(\sigma t)$, where the angular frequency σ is equal to the angular velocity ω_2. The modified equations including this additional torque is given by:

$$J_r\,\dot\omega_1 + b_r\,\omega_1 = r_2(r_2\,\tau_1 + r_1\tau_2).$$

Use Bode plots and the Nyquist diagram to design a controller so that the closed-loop system is capable of asymptotically tracking a constant reference input $\bar\omega_2(t) = \bar\omega_2 = 4\pi$, $t \geq 0$, and asymptotically rejecting the torque perturbation $\tau_2(t) = h\cos(\sigma t)$ when $\sigma = \bar\omega_2$. Calculate the corresponding gain and phase margins.

P7.11. You have shown in **P2.10** that the ordinary differential equation

$$J\,\dot\omega + (b_1 + b_2)\,\omega = \tau + g\,r(m_1 - m_2),$$
$$J = J_1 + J_2 + r^2(m_1 + m_2),$$
$$v_1 = r\,\omega,$$

is a simplified description of the motion of the elevator in Fig. 2.19, where ω is the angular velocity of the driving shaft and v_1 is the elevator's load linear velocity. Let $r = 1$m, $m_1 = m_2 = 1000$kg, $b_1 = b_2 = 120$kg m^2/s, $J_1 = J_2 = 200$kg m^2, and $g = 10$m/s^2. Use Bode plots and the Nyquist diagram to design a dynamic controller $\tau = K(\bar{x}_1 - x_1)$ that can asymptotically track a constant position reference $\bar{x}_1(t) = \bar{x}_1$, $t \geq 0$, where $x_1 = \int_0^t v_1(\tau)\, d\tau$ is the elevator's load linear position. Calculate the corresponding gain and phase margins.

P7.12. Repeat **P7.11** with $m_2 = 800$kg.

P7.13. You have shown in **P6.22** that

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 \\ 3\Omega^2 & 0 & 2\Omega \\ 0 & -2\Omega & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m} \end{bmatrix} u,$$

$$\tilde{y} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x,$$

where

$$x = \begin{pmatrix} r - R \\ \dot{r} \\ R\omega - R\Omega \end{pmatrix}, \qquad \tilde{y} = y - R,$$

are a simplified description of a satellite orbiting earth linearized about the equilibrium point where $\Omega^2 R^3 = GM$. Use MATLAB to calculate the transfer-function from the tangential thrust u to the output $\tilde{y}$ and design a controller using Bode plots and the Nyquist diagram that can stabilize the radial distance of the satellite in closed-loop. Calculate the corresponding gain and phase margins.