MAE143 B - Linear Control - Spring 2015
Midterm, May 5th

Instructions:

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities
2. You have 70 minutes
3. Most questions show you the answer, and you're asked to prove it. Use the given answers to move on to the next question. Do not get stuck!
4. Write your answers on your "Blue Book"
5. Do not forget to write your name and student number

Questions:

1. **System Modeling**

   As seen in class, the first-order ordinary differential equation

   $$m\,\dot{y}(t) + b\,y(t) = p\,u(t),$$

   can model the velocity response of a car traveling in a straight line where y is the car's velocity and the control input u is the pedal excursion. When there is a delay of $\tau \geq 0$ seconds between the car response and the control input this equation becomes

   $$m\,\dot{y}(t) + b\,y(t) = p\,u(t-\tau), \qquad \tau \geq 0.$$

   (a) (2 points) Ignoring initial conditions, use the Laplace transform to show that

   $$Y(s) = G(s)\,U(s), \qquad\qquad G(s) = \frac{p}{m}\frac{e^{-s\tau}}{s + \frac{b}{m}}.$$

   (b) (2 points) Use the inverse Laplace transform to show that:

   $$y(t) = \frac{p}{b}\left(1 - e^{-\frac{b}{m}(t-\tau)}\right), \qquad t \geq \tau$$

   when $u(t) = 1$, $t \geq 0$, is a unit step.

   (c) (2 points) Sketch $y(t)$ from (b) when $p = b = m = \tau = 1$.

   (d) (1 point) Use the fact that $e^x \approx 1 + x$ when x is small to show that

   $$G(s) \approx G_\tau(s) = \frac{p}{m}\frac{1 - \tau\,s}{s + \frac{b}{m}}.$$

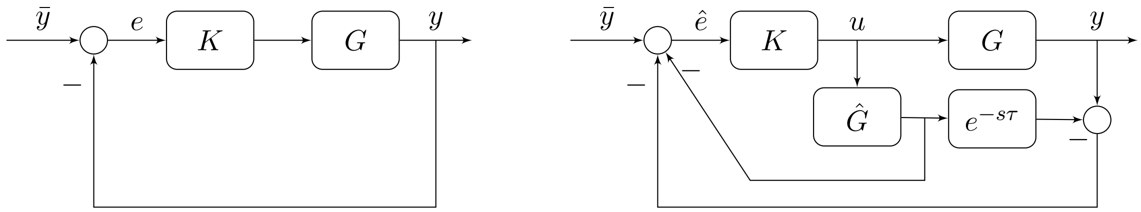
   (e) (2 points) Let $Y(s) \approx Y_\tau(s) = G_\tau(s)\,U(s)$ and use the inverse Laplace transform to show that:

   $$y_\tau(t) = \frac{p}{b}\left[1 - \left(1 + \tau\frac{b}{m}\right)e^{-\frac{b}{m}t}\right], \qquad t \geq 0,$$

   when $u(t) = 1$, $t \geq 0$, is a unit step.

   (f) (2 points) Sketch $y_\tau(t)$ from (e) when $p = b = m = \tau = 1$.

   (g) (1 point) Comment on the similarities and differences between $y(t)$ and $y_\tau(t)$.



(a) Problem 2 (b) Problem 3

Figure 1: Figures for Problems 2 and 3

2. **Proportional-Integral Control** $\boxed{\text{ASSUME: } p = b = m = 1.}$

Consider the standard feedback diagram shown in Fig. 1a where the controller has transfer-function:

$$K(s) = K_p + \frac{K_i}{s},$$

and let the system G be represented by the approximate transfer-function $G_\tau(s)$ from Problem 1.(d).

(a) (3 points) Calculate the transfer-function from $\bar{y}$ to e.

(b) (3 points) Let $K_i = K_p$ and calculate the range of positive values of K_p for which the closed-loop is asymptotically stable as a function of $\tau > 0$. What if $\tau = 0$?

(c) (3 points) Is it possible for the closed-loop to achieve asymptotic tracking of a constant reference signal $\bar{y}$? Explain why or why not.

(d) (3 points) The choice $K_i = K_p$ is very special. Explain what it does and why it does not affect internal stability in this case.

(e) (2 points (bonus)) Let G be the transfer-function $G(s)$ from Problem 1.(a) and repeat part (a). Explain why answering part (b) in this case is much more involved. What about part (c)?

3. **Smith Predictor**

Consider the feedback diagram shown in Fig. 1b with $\tau \geq 0$.

(a) (4 points) Show that the transfer-function from $\bar{y}$ to $\hat{e}$ is

$$\hat{S} = \frac{1}{1 + G\,K + (1 - e^{-s\tau})\,\hat{G}\,K}.$$

and that $\hat{S} = (1 + \hat{G}K)^{-1}$ when $G = e^{-s\tau}\hat{G}$.

(b) (2 points) Let the system G be represented by the transfer-function $G(s)$ from Problem 1.(a). Assume that $p = b = m = 1$ and use (a) to calculate the transfer-function from $\bar{y}$ to $\hat{e}$.

(c) (2 points) Show that

$$u = \hat{K}\,(\bar{y} - y), \qquad \hat{K} = \frac{K}{1 + (1 - e^{-s\tau})\,\hat{G}\,K}.$$

(d) (2 points) Draw a block diagram representing $\hat{K}$ in terms of the blocks $e^{-s\tau}$, K, and $\hat{G}$. Redraw the closed-loop diagram in Fig. 1b in terms of $\hat{K}$ for a bonus point. *Hint: think feedback!*

(e) (2 points) When $G = e^{-s\tau}\hat{G}$, explain why the controller in Fig. 1b can only be used if $\hat{G}$ is asymptotically stable. *Hint: think pole-zero cancellation!*

(f) (1 point (bonus)) A friend noticed that when $G = e^{-s\tau}\hat{G}$ the same closed-loop transfer-function from (a) can be obtained with the "way simpler" controller

$$K(s) = e^{s\tau}\,K, \quad \tau \geq 0.$$

Name one problem with this idea.