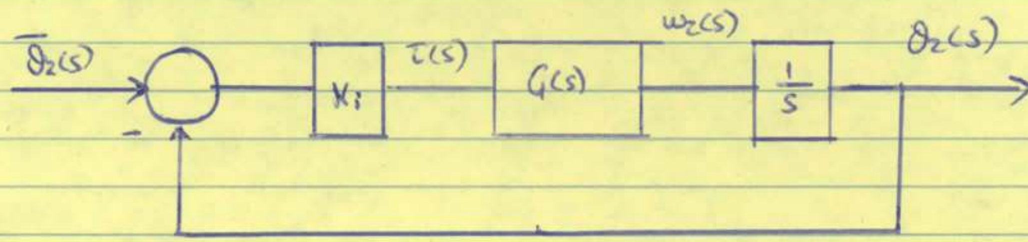
MAE 143B HOMEWORK #4 SOLUTIONS

(4.17) We recall that

$$\frac{w_2(s)}{\tau(s)} = \frac{r_1 r_2/J_r}{s + \frac{b_r}{J_r}} = G(s)$$

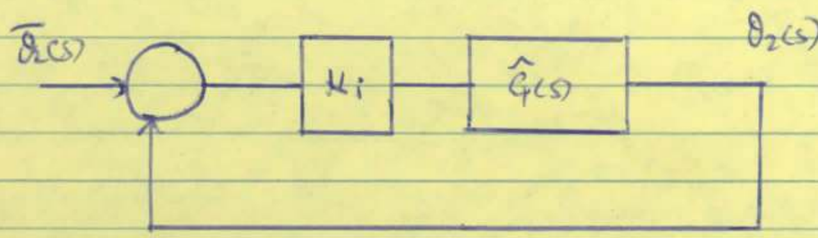
Let us draw the new closed loop block diagram of our closed loop system



Let us redefine G(s) to include the integrator:

$$\tilde{G}(s) = G(s)\cdot\frac{1}{s} = \frac{\theta_2(s)}{\tau(s)}$$

which gives us the block diagram



We observe that this is similar to the system that we examined in (4.16), except now our plant model includes a integrator and the output is $\theta_2(s)$ not $w_2(s)$. For this reason we also need to change the reference signal that we track to $\bar{\theta}_2(s)$.

Let us now compute the values of K_i which would make our closed loop system asymptotically stable. To do this, we look at the poles of our sensitivity transfer function, S.

$$S = \frac{1}{1 + \hat{G}(s)K(s)} = \frac{1}{1 + \frac{r_1 r_2 / J_r}{s\left(s + \frac{b_r}{J_r}\right)} K_i}$$

$$= \frac{s(s + b_r/J_r)}{s\left(s + \frac{b_r}{J_r}\right) + \frac{r_1 r_2 K_i}{J_r}}.$$

So, we get the following characteristic equation:

$$s^2 + \frac{s b_r}{J_r} + \frac{r_1 r_2 K_i}{J_r} = 0$$

To have S asymptotically stable, we require it to have negative roots. We can use the quadratic equation to determine this.

$$s = \frac{-\frac{b_r}{J_r} \pm \sqrt{\left(\frac{b_r}{J_r}\right)^2 - \frac{4 r_1 r_2 K_i}{J_r}}}{2}$$

$$= \frac{-b_r}{2J_r} \pm \frac{1}{2}\sqrt{\left(\frac{b_r}{J_r}\right)^2 - \frac{4 r_1 r_2 K_i}{J_r}}$$

$$= \frac{-b_r \pm \sqrt{b_r^2 - 4 J_r r_1 r_2 K_i}}{2 J_r}$$

We observe for any $K_i > 0$, both roots of S are negative, so we will have asymptotic stability for any $K_i > 0$. This is different from what we saw in (4.16)

Let us now determine if we can asymptotically track a constant reference. We already know that S is asymptotically stable, so we just need to check the poles of $\tilde{G}(s)K(s)$.

$$\tilde{G}(s)K(s) = K_i \cdot \frac{r_1 r_2 / J_r}{s\left(s + \frac{b_r}{J_r}\right)} \qquad (1)$$

So, we see that we have a pole at $s = 0$, so we are able to asymptotically track a constant reference.

We can also test our internal stability condition. We have verified that S is asymptotically stable, so we need to check that $\tilde{G}(s)K(s)$ has no unstable pole/zero cancellations. However, we can see from (1) that we have no pole/zero cancellations at all, so we also have internal stability.

(4.5d)

$$G = \frac{1}{s+1} \qquad N = \frac{1}{s}$$

$$S = \frac{1}{1+GN} = \frac{1}{1+\frac{1}{(s+1)(s)}} = \frac{(s)(s+1)}{s(s+1)+1}$$

$$= \frac{s^2+s}{s^2+s+1}.$$

The poles of S are $s = \dfrac{-1 \pm \sqrt{1-4}}{2}$ which are stable,

so we have asymptotic ~~tracking~~ stability. To check for asymptotic tracking of a constant input, we need $G(s)N(s)$ to have a pole at $s=0$, which we can see immediately, so we also have asymptotic tracking of a constant input.

(4.5e) $\quad G = \dfrac{s}{s+1} \qquad N = \dfrac{1}{s}.$

~~We see immediately that G(s)N(s) has a pole at zero~~, so all We have to verify is asymptotic stability of S.

$$S = \frac{1}{1+GN} = \frac{1}{1+\frac{1}{s+1}} = \frac{s+1}{s+2}$$

which is asymptotically stable. We have a pole-zero cancellation at $s=0$ in $G(s)N(s)$; so we do not satisfy the asymptotic tracking criteria.

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(4.5g)

$$G = \frac{1}{s^2+s+1} \qquad H = \frac{1}{s},$$

$$S = \frac{1}{1+GH} = \frac{1}{1+\frac{1}{s(s^2+s+1)}}$$

$$= \frac{s(s^2+s+1)}{s(s^2+s+1)+1}$$

$$= \frac{s^3+s^2+s}{s^3+s^2+s+1}$$

Roots $s=-1$, $s=0\pm j$.

So we do not have asymptotic stability of S because the real parts of the poles are not all strictly negative. We will not satisfy tracking criteria either - even though $G(s)H(s)$ has a pole at $s=0$.

(4.6) ~~(a)~~ Disturbance rejection requires S to be asymptotically stable and $K(s)$ to have a pole at $s=j\omega$. So for a constant disturbance, we require a pole at $s=0$.

(4.6) d) S is asymptotically stable ✓ (see 4.5)
$K(s)$ has pole at $s=0$ ✓

(e) S is asymptotically stable ✓
$K(s)$ has a pole at $s=0$ ✓

(g) S is asymptotically stable ✗
K has a pole at zero ✓

So (d, e) can reject constant disturbance, g) cannot.

4.9a)

$$G = \frac{1}{s+1} \qquad N = \frac{s+1}{s+2}$$

$$S = \frac{1}{1+GN} = \frac{1}{1 + \frac{1}{\cancel{s+1}} \cdot \frac{\cancel{s+1}}{s+2}}$$

$$= \underline{\frac{s+2}{s+3}} \quad \text{1st order}$$

$$D = \frac{G}{1+GN} = \frac{1}{(s+1)} \cdot \frac{(s+2)}{s+3} = \frac{s+2}{(s+1)(s+3)}$$

2nd order

$$H = \frac{GN}{1+GN} = \frac{s+2}{(s+1)(s+3)} \cdot \frac{(s+1)}{(s+2)} = \frac{1}{s+3}$$

1st order

$$Q = \frac{N}{1+GN} = \frac{s+2}{s+3} \cdot \frac{s+1}{s+2} = \frac{s+1}{s+3} \quad \text{1st order}$$

(d) $$G = \frac{s-1}{s+1} \qquad K = \frac{1}{s-1}$$

$$S = \frac{1}{1+GK} = \frac{1}{1+\frac{s-1}{s+1}\cdot\frac{1}{s-1}} = \frac{s+1}{s+2}$$

1st order

$$D = \frac{s+1}{s+2}\cdot\frac{s-1}{s+1} = \frac{s-1}{s+2}$$ 1st order

$$H = \frac{s-1}{s+2}\cdot\frac{1}{s-1} = \frac{1}{s+2}$$ 1st order

$$Q = \frac{s+1}{s+2}\cdot\frac{1}{s-1} = \frac{(s+1)}{(s+2)(s-1)}$$ 2nd order

(5.13)

$$(J_1 r_2^2 + J_2 r_1^2)\dot{\omega}_1 = r_2^2 \tau$$

$$\omega_2 = \frac{r_1}{r_2}\omega_1$$

$$\Rightarrow \quad (J_1 r_2^2 + J_2 r_1^2)\frac{r_2}{r_1}\dot{\omega}_2 = r_2^2 \tau$$

$$\dot{\omega}_2 = \frac{r_2^2}{J_1 r_2^2 + J_2 r_1^2}\frac{r_1}{r_2}\tau$$

$$\dot{\omega}_2 = \frac{r_1 r_2}{J_1 r_2^2 + J_2 r_1^2}\tau$$

So, let $x_1 = \omega_2$ be our state-space variable.

$$\dot{x}_1 = \frac{r_1 r_2}{J_1 r_2^2 + J_2 r_1^2}\tau$$

Draw a block diagram