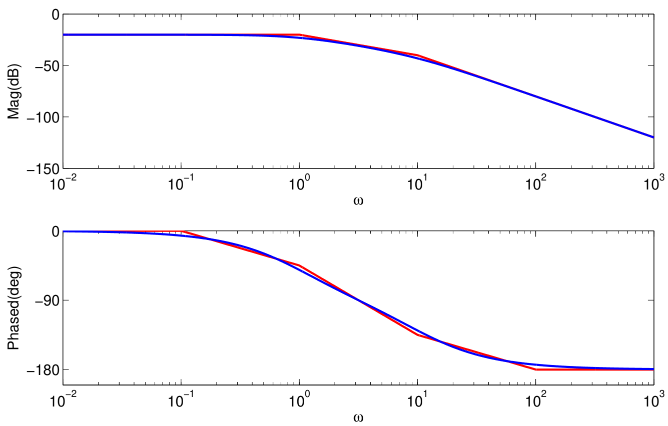
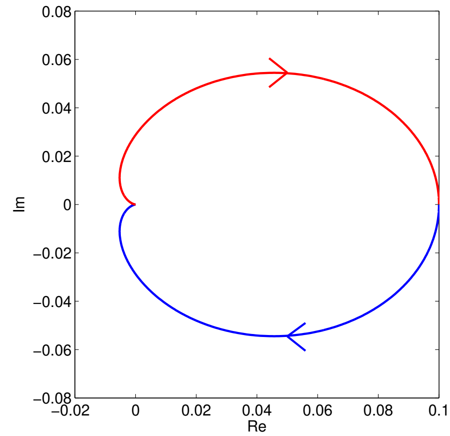
MAE 143B - Homework 9

7.2

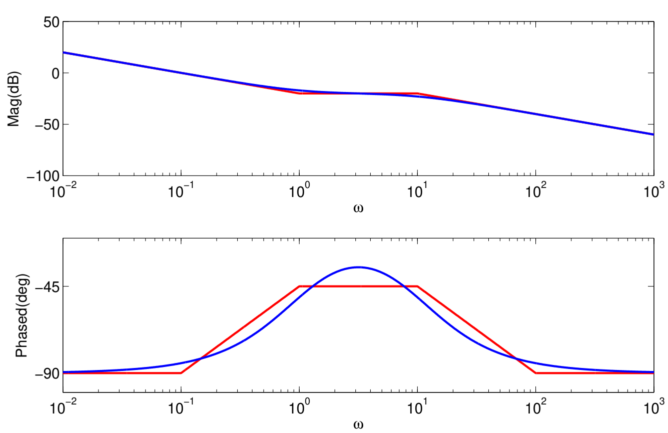


Bode plot; red for straight-line approximation.

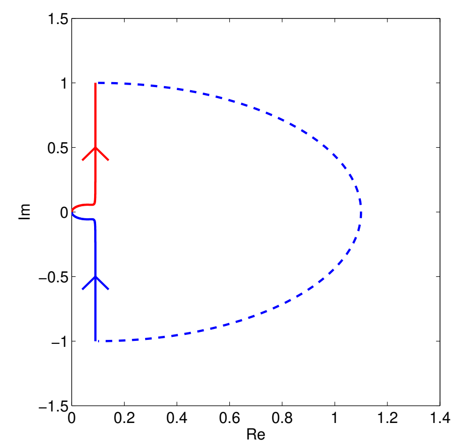


Polar plot; red for negative ω; no encirclements of -1, a.s. under unit feedback.

a) $G(s) = \frac{1}{(s+1)(s+10)}$



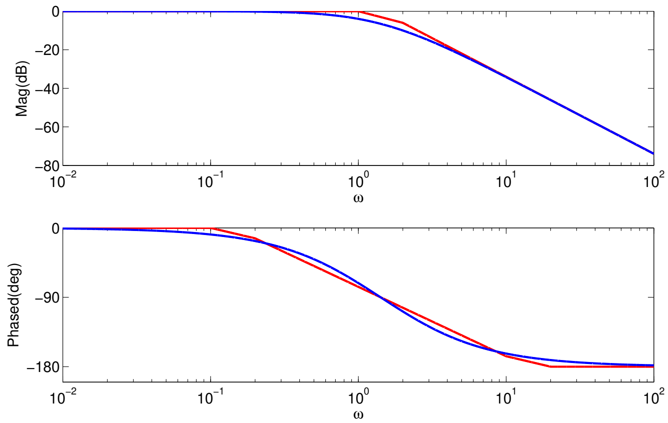
Bode plot; red for straight-line approximation.



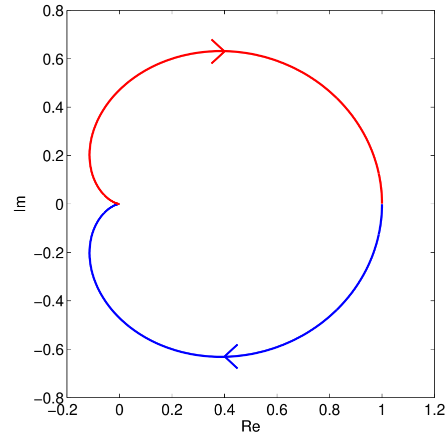
Polar plot; red for negative ω; no encirclements of -1, a.s. under unit feedback; semicircle is traversed at radius $\rho \to \infty$.

h) $G(s) = \frac{s+1}{s(s+10)}$

7.4

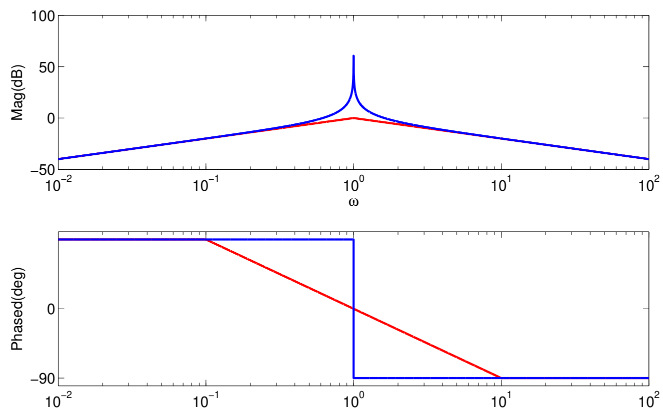


Bode plot; red for straight-line approximation.

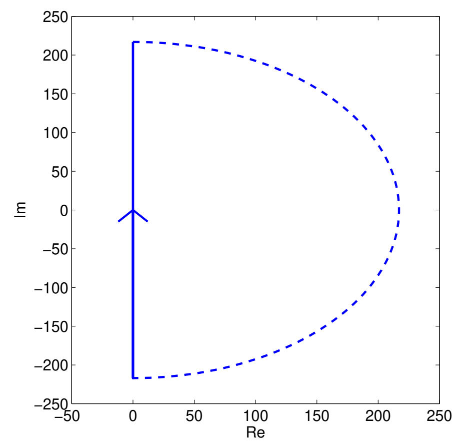


Polar plot; red for negative ω.

a) $G(s) = \frac{2}{(s+1)(s+2)}$



Bode plot; red for straight-line approximation.



Polar plot; semicircle traversed at radius $\rho \to \infty$; same contour for positive/negative ω.

f) $G(s) = \frac{s}{s^2+1}$

7.7

a)

$$G(s) = \frac{s+1}{s+10}$$

d)

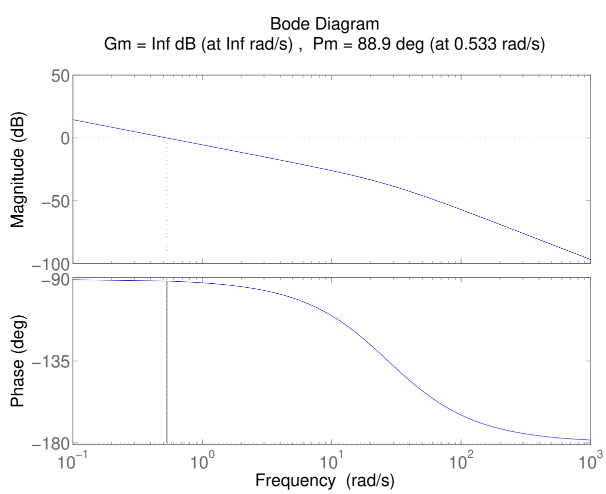
$$G(s) = \frac{s+10}{s^2}$$

h)

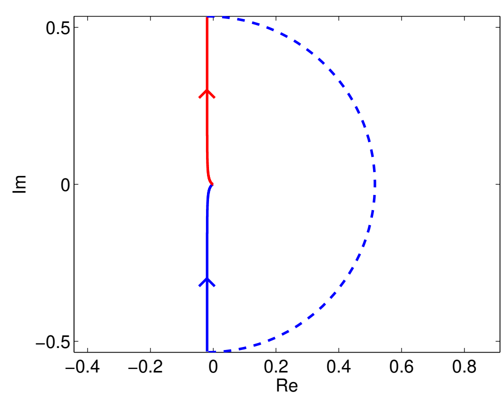
$$G(s) = \frac{s+10}{(s+1)^2}.$$

7.10

As seen in the polar plot below, no point on the negative real axis is encircled. Given that the open-loop transfer function has no poles in the right-half plane, this guarantees asymptotic stability for any $K > 0$. We achieve both asymptotic tracking of a constant reference and rejection of a constant torque disturbance because our controller has a pole at the origin.



Bode plot with gain and phase margins.



Polar plot; red for negative ω; semicircle traversed at radius $\rho \to \infty$.

Bode and polar plots for 7.10

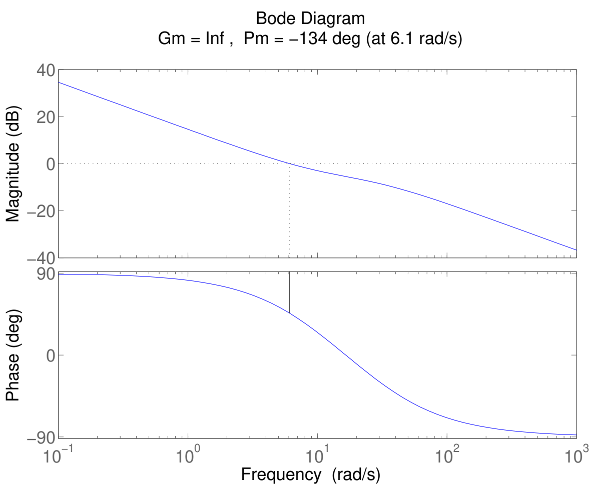
7.11

Note to graders: These solutions explore all possible combinations of positive and negative gains. It is sufficient to give one combination of K_p and K_i that yields asymptotic stability.

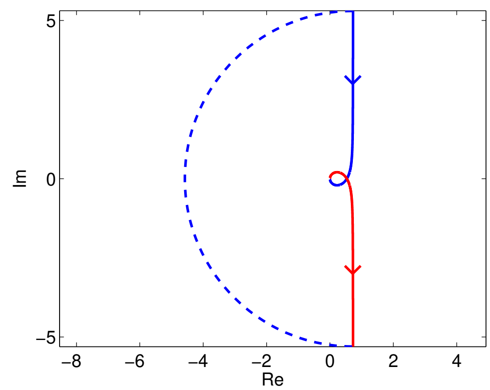
As we have seen in a previous assignment, we can design PI controllers in two steps by first choosing the zero of our controller by fixing the ratio between the two gains K_p and K_i and then choosing K_p using the methods from class. This follows from writing our controller transfer function as

$$K(s) = \frac{K_p(s + K_i/K_p)}{s}.$$

First note that by the zero at the origin, tracking and disturbance rejection are as in 7.10 provided $K_i \neq 0$. Recalling the polar plot in 7.10, notice that we have no chance of stabilizing the system with a zero in the right-half plane and positive K_p, which would cause the circle traversed at radius $\rho \to \infty$ to cover the entire left-half plane and thus result in an encirclement of any given point on the negative real axis. Given that our open-loop transfer function does not have poles in the right-half plane, this means that we cannot stabilize the system with a controller zero in the right-half plane and $K_p > 0$, eliminating all combinations of $K_p > 0$ with $K_i < 0$. For an example with the zero placed at $s = -K_i/K_p = 10$, see the following plots.



Bode plot with gain and phase margins.

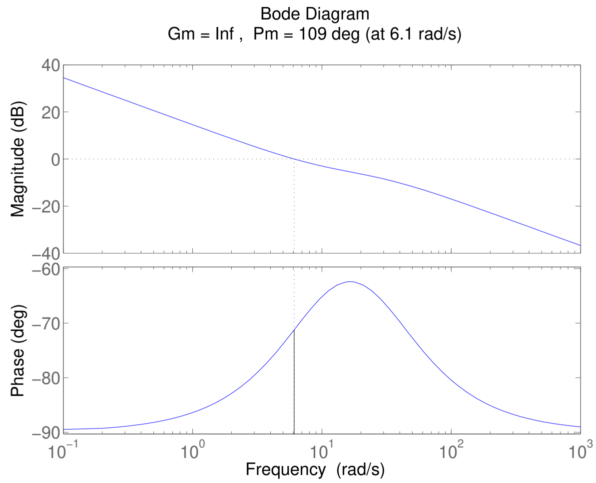


Polar plot; red for negative ω; semicircle traversed at radius $\rho \to \infty$.

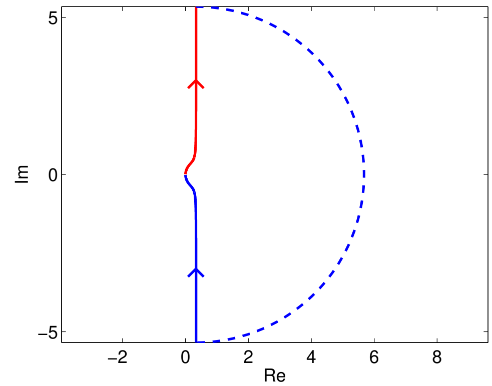
Bode and polar plots for 7.11, zero at $s = -K_i/K_p = 10$.

We can further see from the above plots how $K_p < 0$ does indeed allow for stabilization with the controller zero in the right-half plane, provided $|K_p|$ is sufficiently small. This becomes apparent by noting that negative values of K_p amount to rotating the entire polar plot contour by π around the origin. $|K_p|$ sufficiently small in this case is required to avoid encirclement of the point $-1/K_p$ by the small circle next to the origin. That is, we can stabilize the system by $K_i > 0$ and $K_p < 0$ provided $|K_p|$ is sufficiently small.

This leaves us with placing the zero in the left-half plane. However, again using the insights from our polar plot to 7.10, we can place this left-half plane zero anywhere and choose any positive gain K_p to asymptotically stabilize the system. Given that the zero specifies the ratio of our control gains, this implies that any combination of $K_i > 0$ and $K_p > 0$ yields asymptotic stability (from 7.10 we know that $K_p = 0$ works, too). For an example with the zero placed at $s = -K_i/K_p = -10$, see the following plots.



Bode plot with gain and phase margins.



Polar plot; red for negative ω; semicircle traversed at radius $\rho \to \infty$.

Bode and polar plots for 7.11, zero at $s = -K_i/K_p = -10$.

By the same argument as above (rotation of the polar plot contour by π around the origin), no combination of $K_i < 0$ and $K_p < 0$ can be used to asymptotically stabilize the system.