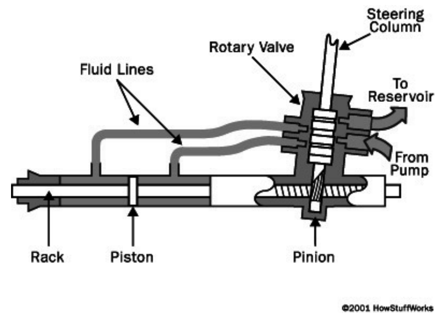
MAE143 B - Linear Control - Spring 2016
Midterm, May 3rd

Instructions:

1. This exam is open book. You may use whatever written materials you choose, including your class notes and textbook. You may use a hand calculator with no communication capabilities
2. You have 70 minutes
3. Most questions show you the answer, and you're asked to prove it. Use the given answers to move on to the next question. Do not get stuck!
4. Write your answers on your "Blue Book"
5. Do not forget to write your name and student number



A hydraulic car power steering *system is composed of a hydraulic cylinder which is coupled to the vehicle's* rack *to assist steering. The hydraulic cylinder is coupled to the* steering column *which is driven by the* steering wheel *by an ingenious* valve *which is essentially a feedback loop. In a* drive-by-wire *system, this mechanical coupling is substituted by a feedback controller similar to the one you will analyze in this exam.*

Questions:

1. **Modeling the Hydraulic Cylinder**

   The behavior of the hydraulic pump + cylinder system can be approximately modeled by the following linearized differential equation model

   $$C\,\dot{p}(t) + B\,p(t) = D\,v(t) - A\,\dot{x}(t)$$

   where p is the pressure on the system, x is the cylinder's piston position, and v is the valve position.

   (a) (2 points) Assume zero initial conditions and show that

   $$P(s) = G_v(s)V(s) + G_x(s)X(s), \qquad G_v(s) = \frac{D/C}{s + B/C}, \qquad G_x(s) = \frac{-sA/C}{s + B/C}.$$

   (b) (2 points) Assume that $x(t) = 0$ and that all constants are positive and use the frequency response method to show that if the valve is kept open at constant $v(t) = \tilde{v}$, $t \geq 0$, then the steady-state pressure is

   $$p^{\text{ss}}(t) = \tilde{p} = \frac{D\,\tilde{v}}{B}.$$

(c) (5 points) Assume that $A = 10\text{cm}^2$, $B = V_0/\beta$, where $V_0 = 0.001\text{m}^3$, and $\beta \approx 1.5\text{MPa}$. Calculate the remaining parameters of the model and use the inverse Laplace transform to calculate and sketch the dynamic response to the input in part (b) knowing that $\tilde{v} = 2.5\text{cm}$, that $\tilde{p} = 10\text{MPa}$ ($\approx 1450\text{psi}$), and that the time-constant of the hydraulic cylinder is 0.03s.

2. **Modeling the Rack**

The rack can be modeled as a mass-spring-damper. A simplified dynamic model for the rack is the differential equation

$$m\,\ddot{x}(t) + b\,\dot{x}(t) + k\,x(t) = A\,p(t) + f_w(t),$$

where x is the position of the rack which is firmly attached to the hydraulic cylinder piston, p is the pressure on the hydraulic system, and f_w represent disturbance forces.

(a) (3 points) Assume $f_w = 0$. Ignore initial conditions and show that

$$X(s) = G(s)V(s), \qquad G(s) = \frac{AD/C}{(s + B/C)(ms^2 + bs + k) + sA^2/C}.$$

(b) (1 point) Assume that $f_w = 0$ and that the system is asymptotically stable and use the frequency response method to show that if the valve is kept open at constant $v(t) = \tilde{v}$, $t \geq 0$ then the steady-state rod displacement is

$$x^{\text{ss}}(t) = \tilde{x} = \frac{A\,\tilde{p}}{k},$$

where $\tilde{p}$ is as in Question 1(c).

3. **Closing the Loop**

Let θ denote the angle of the *steering wheel* and consider the power-steering valve command

$$v(t) = L(r\,\theta - x), \tag{1}$$

where $r > 0$ is equivalent to the radius of a standard rack-and-pinion steering column.

(a) (2 points) Explain what requirements need to be imposed on the feedback gain L for the feedback loop to be internally stable. You do not need to explicitly calculate L!

(b) (2 points) Assume that all constants are positive and that the closed-loop is asymptotically stable. Explain why the closed-loop system does not achieve asymptotic tracking of a constant reference $\theta(t) = \tilde{\theta}$, $t \geq 0$ and show that the expected steady-state error when $f_w = 0$ satisfies

$$|e^{\text{ss}}(t)| \leq \frac{Bk\,r\,\tilde{\theta}}{Bk + LAD}$$

Answer without calculating the closed-loop transfer-function!

(c) (2 points) Modify the feedback law (1) so that the closed-loop can potentially achieve asymptotic tracking of a constant reference $\theta(t) = \tilde{\theta}$, $t \geq 0$. Explain the requirements.

(d) (3 points) Draw a simulation diagram showing all components and all signals of the closed-loop system using only gains, summers and integrators.

(e) (2 points (bonus)) Does the closed-loop system achieve asymptotic rejection of a constant disturbance $f_w(t) = \tilde{f}_w$, $t \geq 0$?