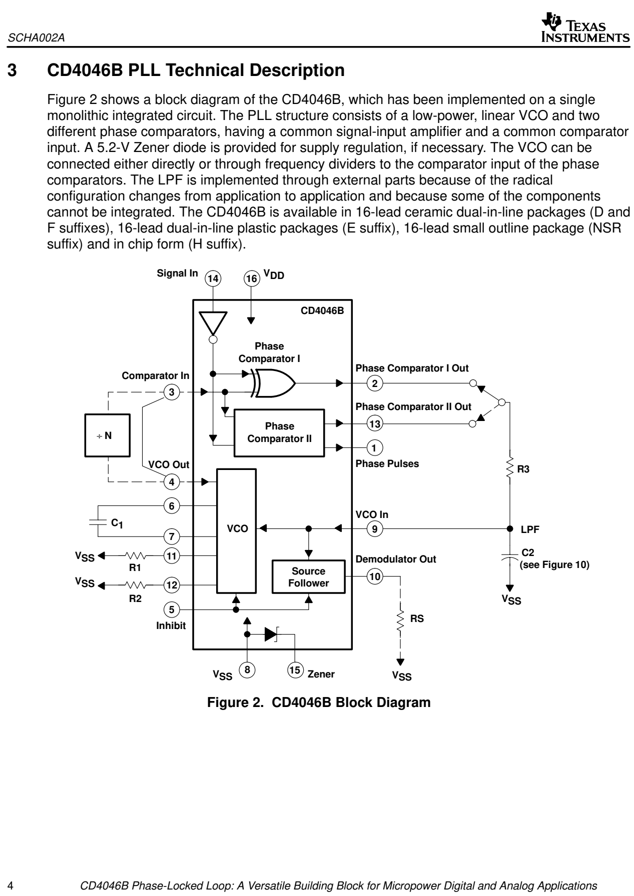
MAE143 B - Linear Control - Spring 2017
Final, June 15th

Instructions:

1. This exam is open book. You may use whatever written materials of your choice, including your notes and the book.
2. You may use a calculator with no communication capabilities
3. You have 170 minutes
4. Don't get stuck! Come back later if you have time
5. Write answers on your "Blue Book"
6. Do not forget to write your name and student id
7. The exam has 4 questions with for a total of 61 points and 5 bonus points

In this exam you will answer questions related to a feedback system known as a *Phase Locked Loop*, PLL for short, as the one described in the TI Application Note in Fig. 1. A PLL can be used in FM and digital radios to *demodulate* signals.



SCHA002A — Texas Instruments

3 CD4046B PLL Technical Description

Figure 2 shows a block diagram of the CD4046B, which has been implemented on a single monolithic integrated circuit. The PLL structure consists of a low-power, linear VCO and two different phase comparators, having a common signal-input amplifier and a common comparator input. A 5.2-V Zener diode is provided for supply regulation, if necessary. The VCO can be connected either directly or through frequency dividers to the comparator input of the phase comparators. The LPF is implemented through external parts because of the radical configuration changes from application to application and because some of the components cannot be integrated. The CD4046B is available in 16-lead ceramic dual-in-line packages (D and F suffixes), 16-lead dual-in-line plastic packages (E suffix), 16-lead small outline package (NSR suffix) and in chip form (H suffix).

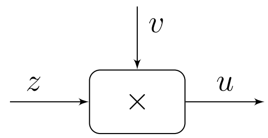
Figure 2. CD4046B Block Diagram

4 CD4046B Phase-Locked Loop: A Versatile Building Block for Micropower Digital and Analog Applications

Figure 1: PLL TI Application Note

Questions:

1. **Phase Comparator.** The product block:



is a simple *phase-comparator*.

(a) (1 point) Show that if

$$z(t) = A\cos(2\pi f_c t + \bar{y}(t) + \phi), \qquad v(t) = -\sin(2\pi f_c t + y(t)), \qquad A > 0$$

then

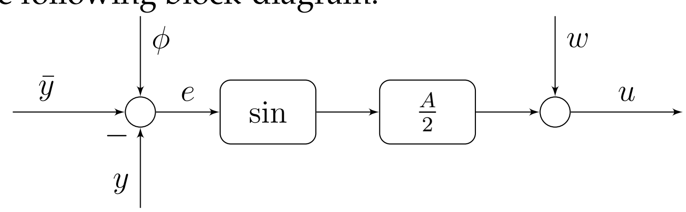
$$u(t) = z(t)v(t) = \frac{A}{2}\sin(\bar{y}(t) + \phi - y(t)) + w(t) \tag{1}$$

where

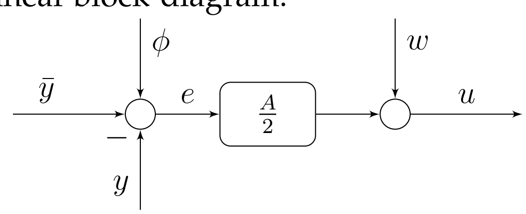
$$w(t) = -\frac{A}{2}\sin(4\pi f_c t + \bar{y}(t) + \phi + y(t))$$

Hint: $2\sin a\cos b = \sin(a+b) + \sin(a-b)$.

(b) (2 points) The relationship between y, $\bar{y}$, ϕ, w and u expressed in (1) can be represented by the following block-diagram:

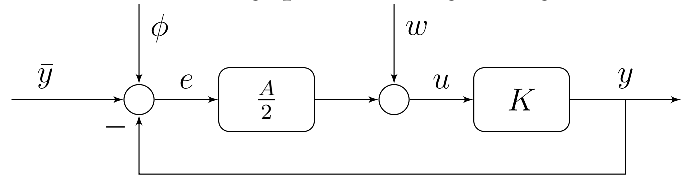


A student claims that if e is small then this block-diagram can be well approximated by the following linear block-diagram:



Justify this approximation.

2. **Controller A.** Answer the following questions regarding the closed-loop diagram:



with the controller

$$K(s) = \frac{K}{s}.$$

The controller $K(s)$ correspond to the combination of the low-pass-filter (LPF) + voltage-controller oscillator (VCO) in Fig. 1.

(a) (1 point) For what values of $K \geq 0$ is the closed-loop asymptotically stable?

(b) (3 points) Sketch the root-locus diagram associated with the loop transfer-function:

$$L(s) = \frac{1}{s}.$$

Explain how this plot can be used to determine the values of $K > 0$ that stabilize the above closed-loop diagram and justify your answer to part (a).

(c) (3 points) Sketch the magnitude and phase of the Bode diagram associated with the loop transfer-function $L(s)$.

(d) (3 points) Sketch the Nyquist plot of the loop transfer-function $L(s)$. Explain how this plot can be used to determine the values of $K > 0$ that stabilize the above closed-loop diagram and justify your answer to part (a).

(e) (4 points) Assume for now that $w = 0$ and show that

$$\bar{y} - y = (1 - H)\bar{y} - H\phi \qquad\qquad e = S(\bar{y} + \phi), \tag{2}$$

where

$$H(s) = \frac{\rho}{s + \rho}, \qquad\qquad S(s) = \frac{s}{s + \rho},$$

and $\rho = KA/2$.

(f) (2 points (bonus)) The goal of this system is to produce an output y which is as close as possible to the original $\bar{y}$ while keeping e small. Show that when the closed-loop is stable, the steady-state response to the inputs

$$\phi(t) = \bar{\phi}, \qquad\qquad \bar{y}(t) = \bar{y}\cos(2\pi f_s t), \qquad\qquad w(t) = 0$$

where $\bar{\phi}$, $\bar{y} > 0$, $\bar{w} > 0$, and $\bar{\psi}$ are constants, is such that:

$$\bar{y}(t) - y_{\text{ss}}(t) = |1 - H(j2\pi f_s)|\, \bar{y}(t - \tau_s) - H(0)\, \phi(t), \tag{3}$$

where $\tau_s = -(2\pi f_s)^{-1}\angle(1 - H(j2\pi f_s))$ and

$$|e_{\text{ss}}(t)| \leq \bar{y}\,|S(j2\pi f_s)| + |S(0)\,\bar{\phi}|. \tag{4}$$

(g) (3 points) Sketch only the Bode magnitude diagrams of the frequency-response of the closed-loop transfer-functions $S(s)$ and $H(s)$.

(h) (3 points) Calculate

$$S(0), \qquad H(0), \qquad |S(j2\pi f_s)|, \qquad \angle S(j2\pi f_s), \qquad S(j\infty), \qquad H(j\infty).$$

Locate those points on your Bode diagrams from part (g).

(i) (3 points) The goal of this system is to achieve $y_{\text{ss}}(t) = \bar{y}(t)$, in which case the system output y has successfully *demodulated* the original signal $\bar{y}$. On the other hand, e should remain small, say $e_{\text{ss}} = 0$, for the linear approximation to hold. Can **Controller A** asymptotically achieve these two goals? Explain.

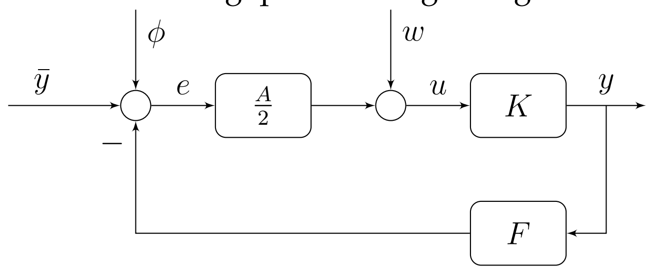
Hint: Use the formulas (3) *and* (4) *and the Bode plot and values from parts (g) and (h).*

(j) (3 points) How can you measure the effect of on the closed-loop performance of a non-zero *disturbance*:

$$w(t) = -\frac{A}{2}\sin(2\pi f_c t + \bar{y}(t) + \phi + y(t))$$

in which $f_c \gg f_s$? Can you modify the controller $K(s)$ in order to completely asymptotically reject the disturbance w? Explain.

3. **Controller B.** Answer the following questions regarding the closed-loop diagram:



with the controller

$$K(s) = K, \qquad\qquad F(s) = \frac{1}{s}.$$

The block $K(s)$ correspond to the low-pass-filter (LPF) and the block $F(s)$ correspond to the voltage-controller oscillator (VCO) in Fig. 1.

(a) (1 point) A student claims that the same loop transfer-function used in Question 2:

$$L(s) = \frac{1}{s}.$$

can be used to determine the values of $K > 0$ that stabilize the above closed-loop diagram. Explain why is he or she correct.

(b) (4 points) Show that if

$$S(s) = \frac{s}{s+\rho}, \qquad H(s) = \rho\, S(s), \qquad \rho = \frac{KA}{2} \tag{5}$$

then the formulas (2) are correct.

(c) (3 points) Sketch only the Bode magnitude diagrams of the frequency-response of the closed-loop transfer-functions $S(s)$ and $H(s)$.

(d) (3 points) Calculate

$$S(0), \qquad H(0), \qquad |S(j2\pi f_s)|, \qquad \angle S(j2\pi f_s), \qquad S(j\infty), \qquad H(j\infty).$$

Locate those points on your Bode diagrams from part (c).

(e) (3 points) The goal of this system is to achieve $y_{\text{ss}}(t) = \bar{y}(t)$, in which case the system output y has successfully *demodulated* the original signal $\bar{y}$. On the other hand, e should remain small, say $e_{\text{ss}} = 0$, for the linear approximation to hold. Can **Controller B** asymptotically achieve these two goals? Explain.

Hint: Use the formulas (3) *and* (4) *and the Bode plot and values from parts (c) and (d).*

(f) (1 point) Compare the performance of **Controller A** with **Controller B**.

4. **Controller C.** Answer the following questions regarding the same closed-loop diagram as in Question 3 with the controller:

$$K(s) = \frac{K(1 + s/a)^2}{1 + (s/\omega_s)^2}, \qquad F(s) = \frac{1}{s}, \qquad \omega_s = 2\pi f_s.$$

(a) (3 points) Sketch the root-locus diagram associated with the loop transfer-function:

$$L(s) = \frac{(1 + s/a)^2}{s + s(s/\omega_s)^2}.$$

Explain how this plot can be used to determine values of a and $K > 0$ that stabilize the above closed-loop diagram.

(b) (3 points) Assume that $a = \omega_s = 2\pi f_s$ and sketch the magnitude and phase of the Bode diagram associated with the loop transfer-function $L(s)$.

(c) (3 points (bonus)) Assume that $a = \omega_s = 2\pi f_s$ and sketch the Nyquist plot of the loop transfer-function $L(s)$. Explain how this plot can be used to determine the values of $K > 0$ that stabilize the above closed-loop diagram. Compare with your answer to part (a).

(d) (4 points) Assume that $a = \omega_s = 2\pi f_s$ and show that if

$$S(s) = \frac{s(s^2 + \omega_s^2)}{p(s)}, \qquad H(s) = \frac{\rho\, s(s + \omega_s)^2}{p(s)}, \qquad \rho = \frac{KA}{2}$$

where $p(s) = s(s^2 + \omega_s^2) + \rho(s + \omega_s)^2$, then the formulas (2) are correct.

(e) (3 points) Assume that $a = \omega_s = 2\pi f_s$ and calculate

$$p(0), \quad p(j2\pi f_s), \quad S(0), \quad H(0), \quad |S(j2\pi f_s)|, \quad \angle S(j2\pi f_s), \quad S(j\infty), \quad H(j\infty).$$

(f) (3 points) The goal of this system is to achieve $y_{\text{ss}}(t) = \bar{y}(t)$, in which case the system output y has successfully *demodulated* the original signal $\bar{y}$. On the other hand, e should remain small, say $e_{\text{ss}} = 0$, for the linear approximation to hold. Can **Controller C** asymptotically achieve these two goals? Explain.

Hint: Use the formulas (3) *and* (4) *and the values from part (e).*

(g) (1 point) Compare the performance of **Controllers A**, **B** and **C**.