

MAE143 B - Linear Control - Spring 2017

Final, June 15th

Instructions:

1. This exam is open book. You may use whatever written materials of your choice, including your notes and the book.
2. You may use a calculator with no communication capabilities
3. You have 170 minutes
4. Don't get stuck! Come back later if you have time
5. Write answers on your "Blue Book"
6. Do not forget to write your name and student id
7. The exam has 4 questions with for a total of 61 points and 5 bonus points

In this exam you will answer questions related to a feedback system known as a *Phase Locked Loop*, PLL for short, as the one described in the TI Application Note in Fig. 1. A PLL can be used in FM and digital radios to *demodulate* signals.

SCHA002A



3 CD4046B PLL Technical Description

Figure 2 shows a block diagram of the CD4046B, which has been implemented on a single monolithic integrated circuit. The PLL structure consists of a low-power, linear VCO and two different phase comparators, having a common signal-input amplifier and a common comparator input. A 5.2-V Zener diode is provided for supply regulation, if necessary. The VCO can be connected either directly or through frequency dividers to the comparator input of the phase comparators. The LPF is implemented through external parts because of the radical configuration changes from application to application and because some of the components cannot be integrated. The CD4046B is available in 16-lead ceramic dual-in-line packages (D and F suffixes), 16-lead dual-in-line plastic packages (E suffix), 16-lead small outline package (NSR suffix) and in chip form (H suffix).

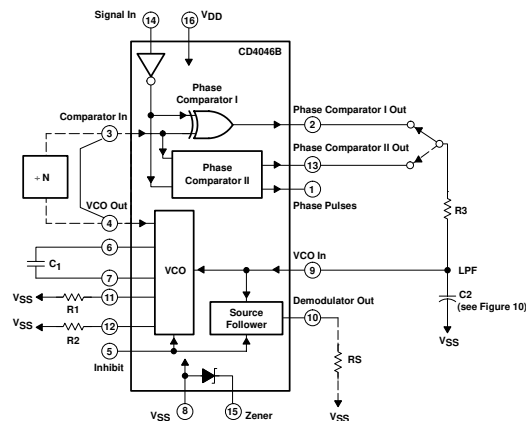
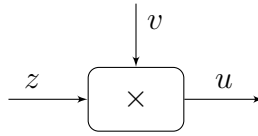


Figure 2. CD4046B Block Diagram

Figure 1: PLL TI Application Note

Questions:

1. **Phase Comparator.** The product block:



is a simple *phase-comparator*.

(a) (1 point) Show that if

$$z(t) = A \cos(2\pi f_c t + \bar{y}(t) + \phi), \quad v(t) = -\sin(2\pi f_c t + y(t)), \quad A > 0$$

then

$$u(t) = z(t)v(t) = \frac{A}{2} \sin(\bar{y}(t) + \phi - y(t)) + w(t) \quad (1)$$

where

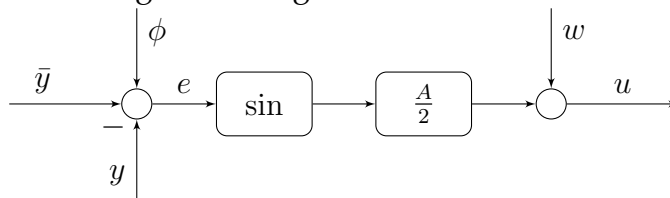
$$w(t) = -\frac{A}{2} \sin(4\pi f_c t + \bar{y}(t) + \phi + y(t))$$

Hint: $2 \sin a \cos b = \sin(a + b) + \sin(a - b)$.

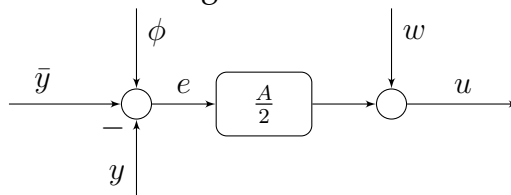
Solution: Calculating

$$\begin{aligned} u(t) &= z(t)v(t) \\ &= -A \sin(2\pi f_c t + y(t)) \cos(2\pi f_c t + \bar{y}(t) + \phi) \\ &= -\frac{A}{2} \sin(2\pi f_c t + \bar{y}(t) + \phi + 2\pi f_c t + y(t)) - \frac{A}{2} \sin(2\pi f_c t + y(t) - 2\pi f_c t - \bar{y}(t) - \phi) \\ &= -\frac{A}{2} \sin(y(t) - \bar{y}(t) - \phi) - \frac{A}{2} \sin(2\pi f_c t + \bar{y}(t) + \phi + 2\pi f_c t + y(t)) \\ &= \frac{A}{2} \sin(\bar{y}(t) + \phi - y(t)) + w(t) \end{aligned} \quad (+1 \text{ point})$$

(b) (2 points) The relationship between y , $\bar{y}$, ϕ , w and u expressed in (1) can be represented by the following block-diagram:



A student claims that if e is small then this block-diagram can be well approximated by the following linear block-diagram:



Justify this approximation.

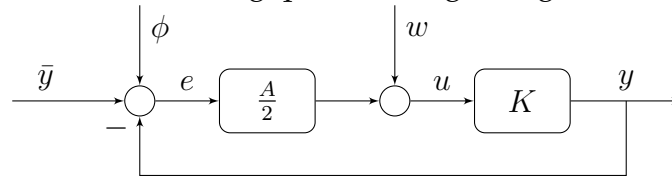
Solution: If e is small one can expand $\sin$ in its Taylor series about the origin:

$$\sin(e) = \sin(0) + \cos(0)e - \frac{1}{2!}\sin(0)e^2 - \frac{1}{3!}\cos(0)e^3 + \dots \quad (+1 \text{ point})$$

The approximation correspond to keeping only the first-order term

$$\sin(e) \approx e. \quad (+1 \text{ point})$$

2. **Controller A.** Answer the following questions regarding the closed-loop diagram:



with the controller

$$K(s) = \frac{K}{s}.$$

The controller $K(s)$ correspond to the combination of the low-pass-filter (LPF) + voltage-controller oscillator (VCO) in Fig. 1.

(a) (1 point) For what values of $K \geq 0$ is the closed-loop asymptotically stable?

Solution: This is a standard feedback diagram in which $G = K$ and $K = A/2$ will be asymptotically stable if the pole of

$$S = \frac{1}{1 + GK} = \frac{1}{1 + \frac{KA}{2}s} = \frac{s}{s + \frac{KA}{2}} \quad (+0.5 \text{ point})$$

is negative. That is, if

$$\frac{KA}{2} > 0 \implies K > 0. \quad (+0.5 \text{ point})$$

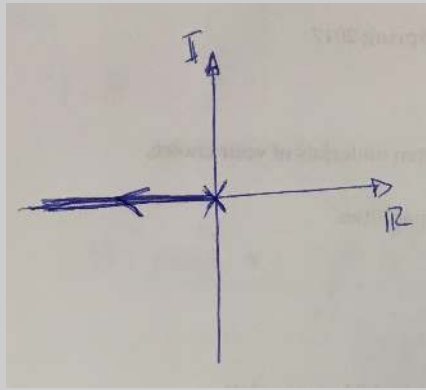
(b) (3 points) Sketch the root-locus diagram associated with the loop transfer-function:

$$L(s) = \frac{1}{s}.$$

Explain how this plot can be used to determine the values of $K > 0$ that stabilize the above closed-loop diagram and justify your answer to part (a).

Solution:

A sketch of the root-locus should look like the figure:



(+1 point)

The root-locus plot describes the roots of the closed-loop negative feedback connection of the loop transfer-function L with a gain $\alpha > 0$. In this case $\alpha = KA/2$.

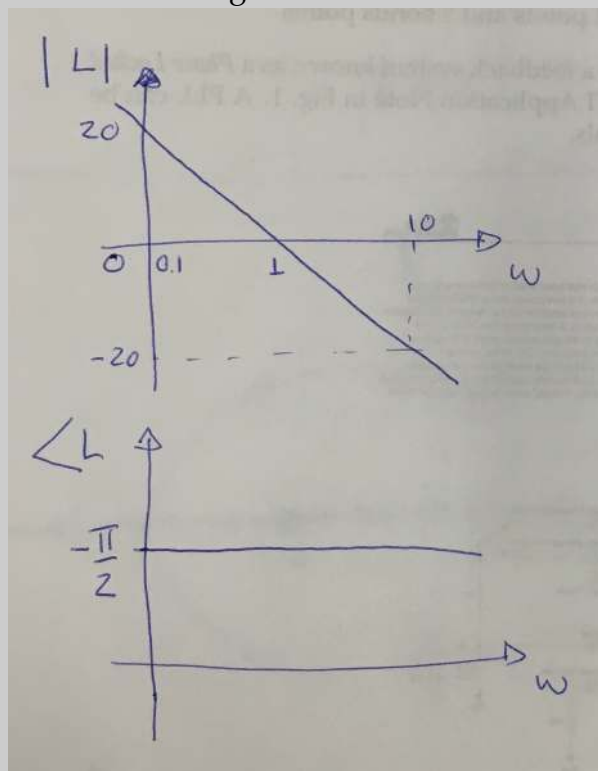
(+1 point)

The locus correspond to the closed-loop poles for $\alpha > 0$. As in part (a), $\alpha > 0$ is equivalent to $K > 0$. Because the locus never goes to the negative side of the complex-plane, the closed-loop system is asymptotically stable for all $K > 0$, which agrees with part (a).

(+1 point)

- (c) (3 points) Sketch the magnitude and phase of the Bode diagram associated with the loop transfer-function $L(s)$.

Solution: Should look like in the figure:

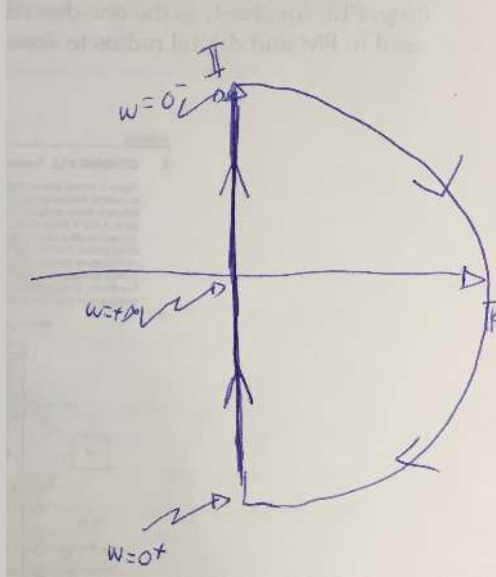


NOTE TO GRADERS: Use the 3 points to distribute partial credit.

- (d) (3 points) Sketch the Nyquist plot of the loop transfer-function $L(s)$. Explain how

this plot can be used to determine the values of $K > 0$ that stabilize the above closed-loop diagram and justify your answer to part (a).

Solution: Should look like in the figure:



(+2 points)

Because $L(s)$ does not have any poles on the right-hand side of the complex plane and the Nyquist plot does not encircle the point $-1/\alpha$ for any $\alpha = AK/2 > 0$ the closed-loop is stable for all $K > 0$, which agrees with part (a). (+1 point)

(e) (4 points) Assume for now that $w = 0$ and show that

$$\bar{y} - y = (1 - H)\bar{y} - H\phi \quad e = S(\bar{y} + \phi), \quad (2)$$

where

$$H(s) = \frac{\rho}{s + \rho}, \quad S(s) = \frac{s}{s + \rho},$$

and $\rho = KA/2$.

Solution:

NOTE TO GRADERS: There are many correct ways to approach this question.

NOTE TO GRADERS: In the original version of the exam the first equation was written as $\bar{y} - y = S\bar{y} - H\phi$ which holds only for Question 2.

Assuming $w = 0$ and calculating:

$$y = \frac{KA}{s} \frac{1}{2} e = \frac{1}{s} \rho (\bar{y} + \phi - y)$$

from which

$$(s + \rho)y = \rho(\bar{y} + \phi)$$

or

$$y = \frac{\rho}{s + \rho}(\bar{y} + \phi) = H(\bar{y} + \phi)$$

Consequently

$$\bar{y} - y = (1 - H)\bar{y} - H\phi \quad (+2 \text{ points})$$

Similarly for e :

$$e = \bar{y} + \phi - y = \bar{y} + \phi - \frac{K A}{s} \frac{1}{2} e = \bar{y} + \phi - \frac{1}{s} \rho e$$

from which

$$(s + \rho)e = s(\bar{y} + \phi)$$

or

$$e = \frac{s}{s + \rho}(\bar{y} + \phi) = S(\bar{y} + \phi). \quad (+2 \text{ points})$$

- (f) (2 points (bonus)) The goal of this system is to produce an output y which is as close as possible to the original $\bar{y}$ while keeping e small. Show that when the closed-loop is stable, the steady-state response to the inputs

$$\phi(t) = \bar{\phi}, \quad \bar{y}(t) = \bar{y} \cos(2\pi f_s t), \quad w(t) = 0$$

where $\bar{\phi}, \bar{y} > 0$, $\bar{w} > 0$, and $\bar{\psi}$ are constants, is such that:

$$\bar{y}(t) - y_{ss}(t) = |1 - H(j2\pi f_s)| \bar{y}(t - \tau_s) - H(0) \phi(t), \quad (3)$$

where $\tau_s = -(2\pi f_s)^{-1} \angle(1 - H(j2\pi f_s))$ and

$$|e_{ss}(t)| \leq \bar{y} |S(j2\pi f_s)| + |S(0) \bar{\phi}|. \quad (4)$$

Solution:

NOTE TO GRADERS: In the original version of the exam equation (3) was written as $\bar{y}(t) - y_{ss}(t) = |S(j2\pi f_s)| \bar{y}(t - \tau_s) - H(0) \phi(t)$.

Use the frequency response method and the transfer-functions from part (e) to calculate:

$$\begin{aligned} \bar{y}(t) - y_{ss}(t) &= \bar{y} |1 - H(j2\pi f_s)| \cos(2\pi f_s t + \angle(1 - H(j2\pi f_s))) - H(0) \bar{\phi} \\ &= \bar{y} |1 - H(j2\pi f_s)| \cos(2\pi f_s t - 2\pi f_s \tau_s) - H(0) \phi(t) \\ &= |1 - H(j2\pi f_s)| \bar{y}(t - \tau_s) - H(0) \phi(t). \end{aligned} \quad (+1 \text{ point})$$

Similarly

$$e_{ss}(t) = S(0) \bar{\phi} + \bar{y} |S(j2\pi f_s)| \cos(2\pi f_s t + \angle S(j2\pi f_s))$$

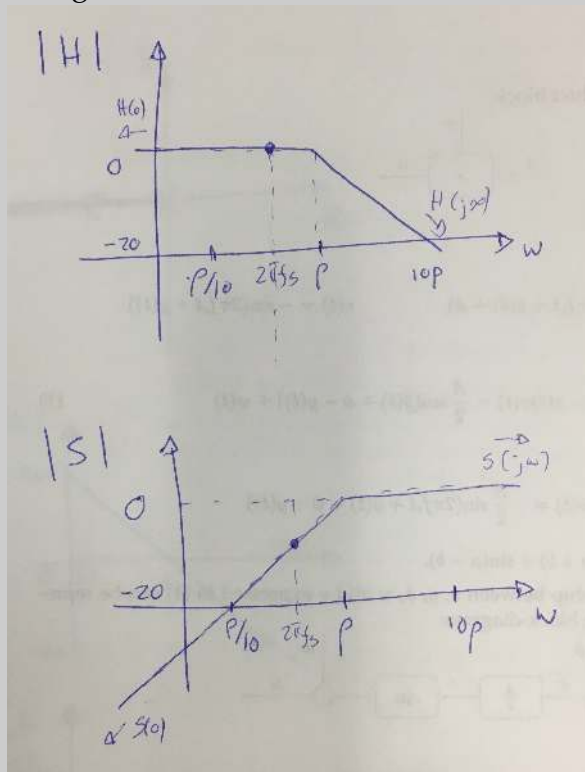
and

$$|e_{ss}(t)| \leq |S(0) \bar{\phi}| + \bar{y} |S(j2\pi f_s)|. \quad (+1 \text{ point})$$

- (g) (3 points) Sketch only the Bode magnitude diagrams of the frequency-response of the closed-loop transfer-functions $S(s)$ and $H(s)$.

Solution:

Should look like in the figure:



NOTE TO GRADERS: Use the 3 points to distribute partial credit.

- (h) (3 points) Calculate

$$S(0), \quad H(0), \quad |S(j2\pi f_s)|, \quad \angle S(j2\pi f_s), \quad S(j\infty), \quad H(j\infty).$$

Locate those points on your Bode diagrams from part (g).

Solution:

Calculating:

$$S(0) = 0, \quad H(0) = 1, \quad S(j\infty) = 1, \quad H(j\infty) = 0 \quad (+2 \text{ points})$$

For

$$S(j2\pi f_s) = \frac{j2\pi f_s}{j2\pi f_s + \rho}$$

we have

$$|S(j2\pi f_s)| = \frac{|j2\pi f_s|}{|j2\pi f_s + \rho|} = \frac{|2\pi f_s|}{\sqrt{4\pi^2 f_s^2 + \rho^2}} \quad (+0.5 \text{ point})$$

and

$$\angle S(j2\pi f_s) = \angle \frac{j2\pi f_s}{j2\pi f_s + \rho} = \angle j2\pi f_s - \angle(j2\pi f_s + \rho) = \pi/2 - \tan^{-1} 2\pi f_s / \rho$$

(+0.5 point)

when $f_s > 0$.

- (i) (3 points) The goal of this system is to achieve $y_{ss}(t) = \bar{y}(t)$, in which case the system output y has successfully *demodulated* the original signal $\bar{y}$. On the other hand, e should remain small, say $e_{ss} = 0$, for the linear approximation to hold. Can **Controller A** asymptotically achieve these two goals? Explain.

Hint: Use the formulas (3) and (4) and the Bode plot and values from parts (g) and (h).

Solution:

The answer is no. Using (3), (4) and part (h):

$$\bar{y}(t) - y_{ss}(t) = |1 - H(j2\pi f_s)| \bar{y}(t - \tau_s) - \phi(t) = |S(j2\pi f_s)| \bar{y}(t - \tau_s) - \phi(t),$$

(+1 point)

and

$$|e_{ss}(t)| \leq \bar{y} |S(j2\pi f_s)|.$$

(+1 point)

which are not both zero.

The conclusion is that the system will not asymptotically track the signal $\bar{y}(t)$ without some error.

(+1 point)

- (j) (3 points) How can you measure the effect of on the closed-loop performance of a non-zero *disturbance*:

$$w(t) = -\frac{A}{2} \sin(2\pi f_c t + \bar{y}(t) + \phi + y(t))$$

in which $f_c \gg f_s$? Can you modify the controller $K(s)$ in order to completely asymptotically reject the disturbance w ? Explain.

Solution: Because $f_c \gg f_s$ one can think of this disturbance as a sinusoid of frequency f_c . Because our system is based on the standard feedback diagram with $K = A/2$ and $G = K$ we can think of w as an input disturbance. The transfer-function from w to the error e is given by the transfer-function

$$D(s) = G(s)S(s) = \frac{Ks}{s + \rho} = \frac{2}{A} \frac{\rho s}{s + \rho}$$

(+1 point)

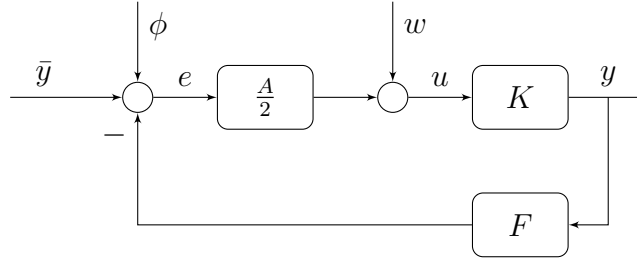
Because $f_c \gg f_s$ is large, which should be of order we expect $|D(j2\pi f_c)| \approx K$, hence the gain K has to be kept small if the high-frequency disturbance is not to have an impact on the performance of the system.

(+1 point)

The only way to reject this disturbance is to have a high-gain at the frequency f_s on the “controller” block $K(s) = A/2$, which in this case is not possible since “plant” $G(s) = K$ is all that we have access to.

(+1 point)

3. **Controller B.** Answer the following questions regarding the closed-loop diagram:



with the controller

$$K(s) = K, \quad F(s) = \frac{1}{s}.$$

The block $K(s)$ correspond to the low-pass-filter (LPF) and the block $F(s)$ correspond to the voltage-controller oscillator (VCO) in Fig. 1.

(a) (1 point) A student claims that the same loop transfer-function used in Question 2:

$$L(s) = \frac{1}{s}.$$

can be used to determine the values of $K > 0$ that stabilize the above closed-loop diagram. Explain why is he or she correct.

Solution: They are correct because by splitting the gain K from the integrator $1/s$ into the block F one is not affecting the loop but only the output y . (+1 point)

(b) (4 points) Show that if

$$S(s) = \frac{s}{s + \rho}, \quad H(s) = \rho S(s), \quad \rho = \frac{KA}{2} \quad (5)$$

then the formulas (2) are correct.

Solution:

NOTE TO GRADERS: There are many correct ways to approach this question.

Assuming $w = 0$ and calculating:

$$e = \bar{y} + \phi - \frac{1}{s}y = \bar{y} + \phi - \frac{KA}{2s}e = \bar{y} + \phi - \frac{1}{s}\rho e$$

from which

$$(s + \rho)e = s(\bar{y} + \phi)$$

or

$$e = \frac{s}{s + \rho}(\bar{y} + \phi) = S(\bar{y} + \phi). \quad (+2 \text{ points})$$

Likewise:

$$y = K \frac{A}{2} e = \rho(\bar{y} + \phi - \frac{1}{s}y)$$

from which

$$(s + \rho)y = \rho s(\bar{y} + \phi)$$

or

$$y = \frac{\rho s}{s + \rho}(\bar{y} + \phi) = H(\bar{y} + \phi)$$

Consequently

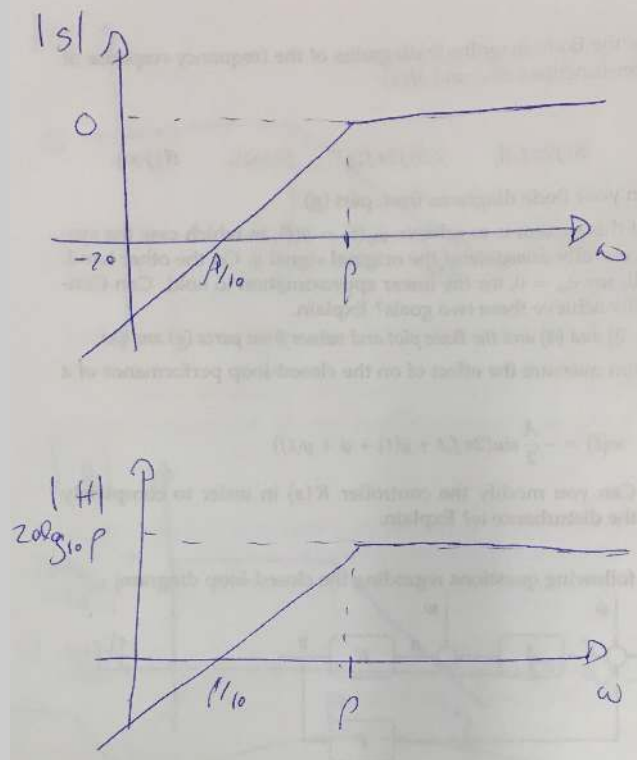
$$\bar{y} - y = (1 - H)\bar{y} - H\phi. \quad (+2 \text{ points})$$

NOTE TO GRADERS: Here it is important to have $1 - H \neq S!$

- (c) (3 points) Sketch only the Bode magnitude diagrams of the frequency-response of the closed-loop transfer-functions $S(s)$ and $H(s)$.

Solution:

Should look like in the figure:



NOTE TO GRADERS: Use the 3 points to distribute partial credit.

- (d) (3 points) Calculate

$$S(0), \quad H(0), \quad |S(j2\pi f_s)|, \quad \angle S(j2\pi f_s), \quad S(j\infty), \quad H(j\infty).$$

Locate those points on your Bode diagrams from part (c).

Solution:

Calculating:

$$S(0) = 0, \quad H(0) = 0, \quad S(j\infty) = 1, \quad H(j\infty) = \rho \quad (+2 \text{ points})$$

$$S(j2\pi f_s) \text{ is an in Q 2(h).} \quad (+1 \text{ point})$$

- (e) (3 points) The goal of this system is to achieve $y_{ss}(t) = \bar{y}(t)$, in which case the system output y has successfully *demodulated* the original signal $\bar{y}$. On the other hand, e should remain small, say $e_{ss} = 0$, for the linear approximation to hold. Can **Controller B** asymptotically achieve these two goals? Explain.

Hint: Use the formulas (3) and (4) and the Bode plot and values from parts (c) and (d).

Solution:

The answer is no. Using(3), (4) and part(h):

$$\bar{y}(t) - y_{ss}(t) = |1 - H(j2\pi f_s)| \bar{y}(t - \tau_s), \quad (+1 \text{ point})$$

and

$$|e_{ss}(t)| \leq \bar{y} |S(j2\pi f_s)|. \quad (+1 \text{ point})$$

which are not both zero.

Note that $|H(j2\pi f_s)| = \rho |S(j2\pi f_s)|$ hence if $|S(j2\pi f_s)|$ is small $|1 - H(j2\pi f_s)|$ will be close to 1. (+1 point)

The conclusion is that the system will not asymptotically track the signal $\bar{y}(t)$ without some error.

- (f) (1 point) Compare the performance of **Controller A** with **Controller B**.

Solution:

Compared with Controller A, Controller B is able to reject the phase bias ϕ completely but still do not completely tracks the original signal.

NOTE TO GRADERS: Reasonable different reasoning earns a point.

4. **Controller C.** Answer the following questions regarding the same closed-loop diagram as in Question 3 with the controller:

$$K(s) = \frac{K(1 + s/a)^2}{1 + (s/\omega_s)^2}, \quad F(s) = \frac{1}{s}, \quad \omega_s = 2\pi f_s.$$

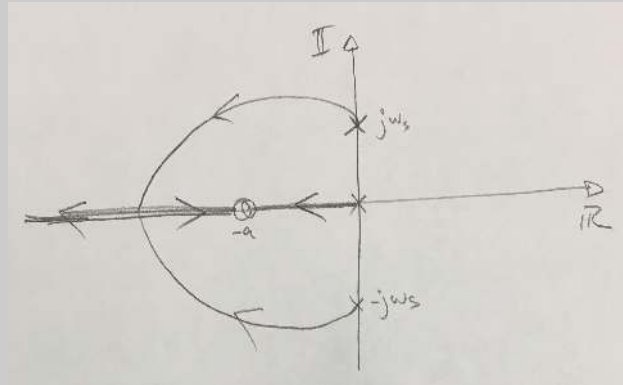
- (a) (3 points) Sketch the root-locus diagram associated with the loop transfer-function:

$$L(s) = \frac{(1 + s/a)^2}{s + s(s/\omega_s)^2}.$$

Explain how this plot can be used to determine values of a and $K > 0$ that stabilize the above closed-loop diagram.

Solution:

Should look like in the figure:



(+2 points)

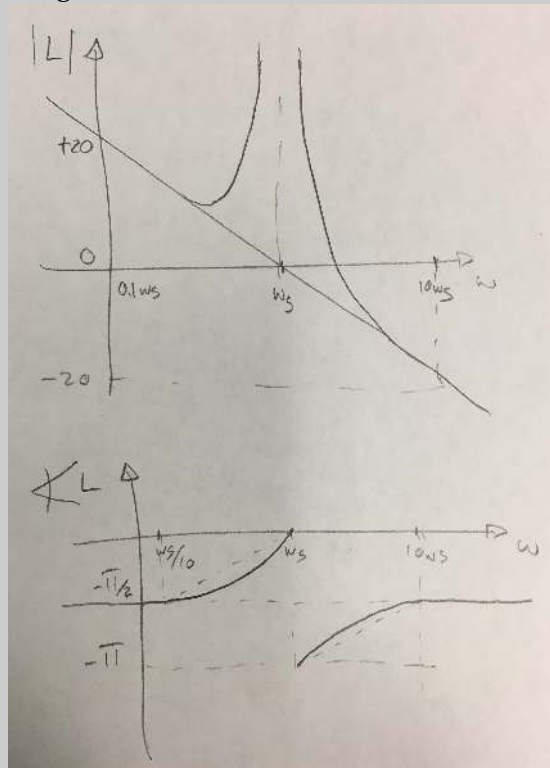
The locus correspond to the closed-loop poles for $\alpha = KA/2 > 0$. As in Q 2, $\alpha > 0$ is equivalent to $K > 0$. Because the locus never goes to the negative side of the complex-plane, the closed-loop system is asymptotically stable for all $K > 0$. (+1 point)

NOTE TO GRADERS: Use the 3 points to distribute partial credit.

- (b) (3 points) Assume that $a = \omega_s = 2\pi f_s$ and sketch the magnitude and phase of the Bode diagram associated with the loop transfer-function $L(s)$.

Solution:

Should look like in the figure:



NOTE TO GRADERS: Students that did not consider $a = \omega_s$ should get full credit

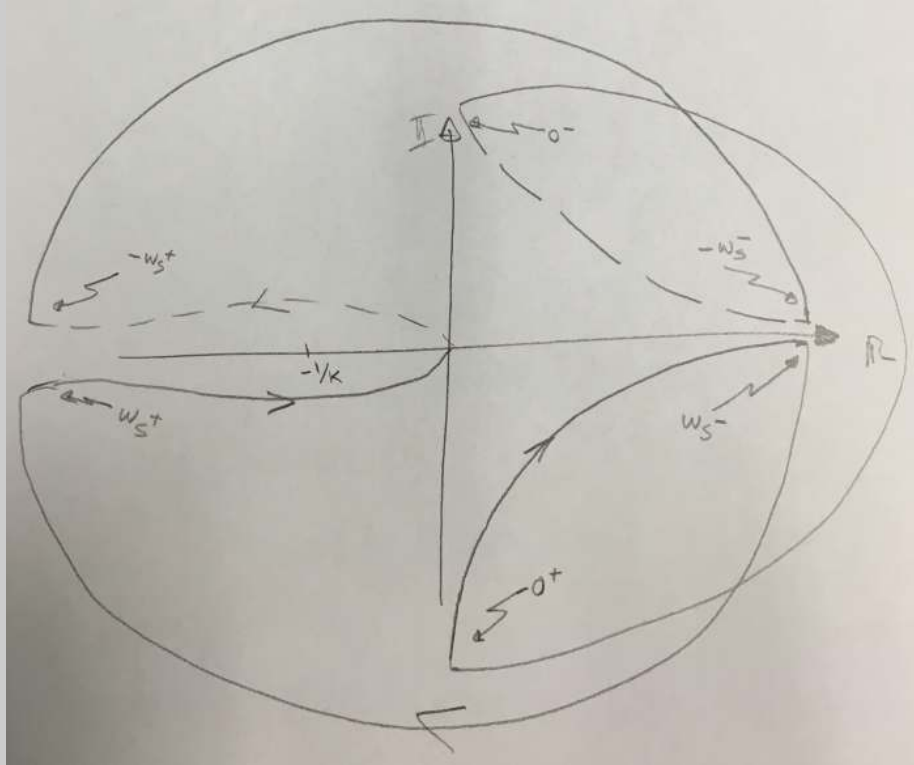
as well.

NOTE TO GRADERS: Use the 3 points to distribute partial credit.

- (c) (3 points (bonus)) Assume that $a = \omega_s = 2\pi f_s$ and sketch the Nyquist plot of the loop transfer-function $L(s)$. Explain how this plot can be used to determine the values of $K > 0$ that stabilize the above closed-loop diagram. Compare with your answer to part (a).

Solution:

Should look like in the figure:



(+2 points)

Because $L(s)$ does not have any poles on the right-hand side of the complex plane and the Nyquist plot does not encircle the point $-1/\alpha$ for any $\alpha = AK/2 > 0$ the closed-loop is stable for all $K > 0$, which agrees with part (a). (+1 point)

- (d) (4 points) Assume that $a = \omega_s = 2\pi f_s$ and show that if

$$S(s) = \frac{s(s^2 + \omega_s^2)}{p(s)}, \quad H(s) = \frac{\rho s(s + \omega_s)^2}{p(s)}, \quad \rho = \frac{KA}{2}$$

where $p(s) = s(s^2 + \omega_s^2) + \rho(s + \omega_s)^2$, then the formulas (2) are correct.

Solution:

Work is the same as in Question 3 since the block diagram did no change. One needs only to calculate:

$$e = \bar{y} + \phi - \frac{1}{s}y = \bar{y} + \phi - \frac{K(1 + s/a)^2}{s(1 + (s/\omega_s)^2)} \frac{A}{2}e$$

from which

$$e = S(\bar{y} + \phi)$$

where

$$\begin{aligned} S(s) &= \frac{1}{1 + K \frac{A}{2} \frac{(1+s/a)^2}{s(1+s^2/\omega_s^2)}} \\ &= \frac{s(1 + s^2/\omega_s^2)}{s(1 + s^2/\omega_s^2) + \rho(1 + s/a)^2} \\ &= \frac{s(s^2 + \omega_s^2)}{s(s^2 + \omega_s^2) + \frac{\omega_s^2}{a^2} \rho(s + a)^2} \\ &= \frac{s(s^2 + \omega_s^2)}{p(s)} \end{aligned} \quad (+2 \text{ points})$$

when $a = \omega_s$.

Likewise

$$y = K \frac{A}{2} \frac{(1 + s/a)^2}{(1 + s^2/\omega_s^2)} e = \rho \frac{(1 + s/a)^2}{(1 + s^2/\omega_s^2)} S(s)(\bar{y} + \phi)$$

from which

$$\begin{aligned} H(s) &= \rho \frac{(1 + s/a)^2}{(1 + s^2/\omega_s^2)} \frac{s(s^2 + \omega_s^2)}{p(s)} \\ &= \frac{\omega_s^2}{a^2} \rho \frac{(s + a)^2}{(s^2 + \omega_s^2)} \frac{s(s^2 + \omega_s^2)}{p(s)} \\ &= \frac{\omega_s^2}{a^2} \frac{\rho s(s + a)^2}{p(s)} \\ &= \frac{\rho s(s + \omega_s)^2}{p(s)} \end{aligned} \quad (+2 \text{ points})$$

when $a = \omega_s$.

(e) (3 points) Assume that $a = \omega_s = 2\pi f_s$ and calculate

$$p(0), \quad p(j2\pi f_s), \quad S(0), \quad H(0), \quad |S(j2\pi f_s)|, \quad \angle S(j2\pi f_s), \quad S(j\infty), \quad H(j\infty).$$

Solution:

Calculating:

$$p(0) = \rho \omega_s^2, \quad p(j\omega_s) = \rho \omega_s^2(1 + j)^2 = 2\rho \omega_s^2 j \quad (+1 \text{ point})$$

and

$$S(0) = 0, \quad H(0) = 0, \quad S(j\infty) = 1, \quad H(j\infty) = \rho. \quad (+1 \text{ point})$$

For

$$S(j2\pi f_s) = S(j\omega_s) = 0. \quad (+1 \text{ point})$$

- (f) (3 points) The goal of this system is to achieve $y_{ss}(t) = \bar{y}(t)$, in which case the system output y has successfully *demodulated* the original signal $\bar{y}$. On the other hand, e should remain small, say $e_{ss} = 0$, for the linear approximation to hold. Can **Controller C** asymptotically achieve these two goals? Explain.

Hint: Use the formulas (3) and (4) and the values from part (e).

Solution:

The answer is no. Using (3), (4) and part (h):

$$|e_{ss}(t)| \leq \bar{y} |S(j2\pi f_s)| = 0. \quad (+1 \text{ point})$$

However, because

$$\begin{aligned} H(j2\pi f_s) &= H(j\omega_s) \\ &= \frac{\rho j\omega_s (j\omega_s + \omega_s)^2}{p(j\omega_s)} \\ &= \frac{\rho j\omega_s (j\omega_s + \omega_s)^2}{\rho (j\omega_s + \omega_s)^2} \\ &= j\omega_s \neq 1 \end{aligned}$$

the term

$$\bar{y}(t) - y_{ss}(t) = |1 - H(j2\pi f_s)| \bar{y}(t - \tau_s), \quad (+1 \text{ point})$$

will not be zero.

The conclusion is that the system still will not asymptotically track the signal $\bar{y}(t)$ without some error.

- (g) (1 point) Compare the performance of **Controllers A, B** and **C**.

Solution:

A remarkable feature of the closed-loop with Controller C is that the error signal asymptotically converges to zero. This is essential for the operation of the system. However, the analysis of the signal $y(t)$ might suggest that it is not demodulating $\bar{y}(t)$. Note however that

$$\begin{aligned} y_{ss}(t) &= |H(j2\pi f_s)| \bar{y} \cos(2\pi f_s t + \angle H(j2\pi f_s)) - H(0)\phi(t) \\ &= -|2\pi f_s| \bar{y} \sin(2\pi f_s t + \pi/2) \\ &= \frac{d}{dt} \bar{y} \cos(2\pi f_s t) = \frac{d}{dt} \bar{y}(t) \end{aligned}$$

That is, the system is tracking the derivative of $\bar{y}$ rather than $\bar{y}$! This can be used to demodulate FM radio. (+1 point)

NOTE TO GRADERS: Reasonable different reasoning earns a point.