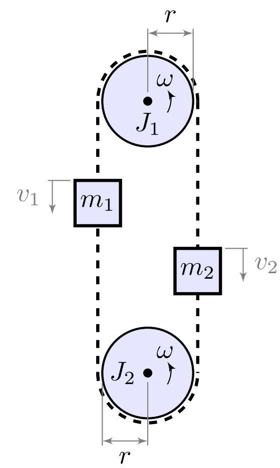
MAE143 B - Linear Control - Spring 2017
Midterm, May 5th



Instructions:

1. This exam is **not open book**. The only material you can consult is your one-page cheat sheet.
2. You have 70 minutes.
3. Most questions show you the answer, and you're asked to prove it. Use the given answers to move on to the next question. Do not get stuck!
4. Write your answers on your "Blue Book".
5. Do not forget to write your name and student number.

Questions:

1. **Modeling**

   The differential equation

$$\big(J_1 + J_2 + r^2(m_1 + m_2)\big)\, \dot{\omega}(t) + (b_1 + b_2)\, \omega(t) = \tau(t) + g\, r(m_1 - m_2), \tag{1}$$

   is a simplified description of the motion of the elevator schematically depicted on the top of the page. In this model $\tau(t)$ is the torque applied by the driving motor on the inertia J_1, and b_1 and b_2 are viscous friction torque coefficients at the inertias J_1 and J_2. The vertical velocities are $v_1(t) = r\,\omega(t)$ and $v_2(t) = -r\,\omega(t)$.

   (a) (1 point) If a motor is used to drive the elevator then the torque can be approximated by

$$\tau(t) = \alpha v(t) - \beta\omega(t), \tag{2}$$

   where α and β are parameters of the motors and $v(t)$ is a voltage applied to the motor. Combine (1) and (2) to show that

$$a\, \dot{\omega}(t) + b\, \omega(t) = c\, v(t) + c\, w(t)$$

   where

$$a = J_1 + J_2 + r^2(m_1 + m_2), \qquad b = b_1 + b_2 + \beta, \qquad c = \alpha$$

   and

$$w(t) = \frac{g\, r}{\alpha}(m_1 - m_2)$$

   can be seen as a disturbance.

(b) (2 points) Assume zero-initial conditions and show that the Laplace transforms of the voltage $v(t)$, the disturbance $w(t)$, and the vertical velocity $v_1(t)$ are related through:

$$V_1(s) = G(s)(V(s) + W(s)), \qquad\qquad G(s) = \frac{rc/a}{s + b/a}.$$

(c) (2 points) Assume that $m_1 = m_2$ and that all constants are positive and use the frequency response method to show that if a constant voltage $v(t) = \tilde{v}$, $t \geq 0$, is applied to the motor then

$$v_1^{\mathrm{ss}}(t) = \tilde{v}_1 = \frac{rc}{b}\tilde{v}.$$

(d) (2 points) Assume that $m_1 = m_2$ and zero-initial conditions and use the inverse Laplace transform to show that the complete response to a constant input voltage $v(t) = \tilde{v}$, $t \geq 0$, is

$$v_1(t) = \tilde{v}_1(1 - e^{-(b/a)t}), \quad t \geq 0.$$

(e) (2 points) Assume that $m_1 = m_2$, $r = 1$ m, $b/a = 1\ \mathrm{s}^{-1}$, $c/a = 0.02\ \mathrm{s}^{-1}/\mathrm{V}$. Sketch the complete response to a constant input voltage $v(t) = \tilde{v} = 100\mathrm{V}$ using the formula you calculated in part (d). What is the time-constant of the system?

2. **Closed-loop control**

Consider the proportional control law

$$v(t) = Ke(t), \quad e(t) = \bar{v}_1(t) - v_1(t), \tag{3}$$

where $\bar{v}_1(t)$ is a reference velocity and answer the following questions.

(a) (3 points) Calculate the closed-loop transfer-functions from $\bar{v}_1$ to e, from w to e, and from $\bar{v}_1$ to v_1.

(b) (1 point) For what positive values of K is the closed-loop asymptotically stable?

(c) (3 points) Select a proper K and substitute the values from Question 1(e) to calculate the steady-state error to a constant reference input $\bar{v}_1(t) = \bar{v}_1 = 1\mathrm{m/s}$. Repeat for $\bar{v}_1(t) = \bar{v}_1 = -1\mathrm{m/s}$. Explain any differences. Does the closed-loop system achieve asymptotic tracking?

(d) (3 points) Repeat part (c) when $m_1 = 1000\mathrm{kg}$ and $m_2 = 800\mathrm{kg}$. Assume $\alpha = 10\ \mathrm{Nm/V}$, $g = 10\mathrm{kg/(m\ s^2)}$. Does the closed-loop system achieve asymptotic disturbance rejection?

(e) (3 points) Modify the feedback law (3) so that the closed-loop can achieve asymptotic tracking of a constant reference $\bar{v}_1(t) = \bar{v}_1$, $t \geq 0$. Does the resulting closed-loop achieve asymptotic disturbance rejection of a constant disturbance? Clearly explain the requirements and your reasoning.