MAE143 B - Linear Control - Spring 2018
Final, June 14th

Instructions:

1. This exam is open book. You may use whatever written materials of your choice, including your notes and the book, and a calculator with no communication capabilities.
2. You have 170 minutes. Don't get stuck! Come back later if you have time.
3. Write answers on your "Blue Book" and attach the provided graph sheet.
4. Do not forget to write your name and student ID.
5. The exam has 9 questions for a total of 81 points and 2 bonus points.

Part I

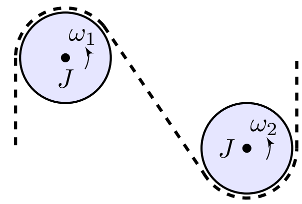
The diagrams in Fig. 1 show pairs of open-loop pole-zero maps and their corresponding Nyquist plots. Assume that the associated transfer-function is of the form:

$$G(s) = G_0 \frac{(s - z_1) \cdots (s - z_m)}{(s - p_1) \cdots (s - p_n)}.$$

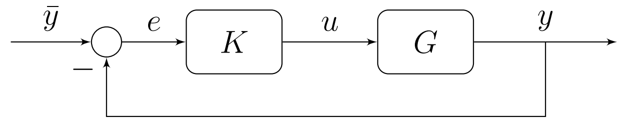
For each pair, answer the following questions: NOTE: Points shown are total for all 3 plots.

1. **Open-loop System.**
   (a) (3 points) Write down the transfer-function for each system.
   (b) (3 points) Is it asymptotically stable in open-loop? Explain.
2. **Root-Locus.** **IMPORTANT: Use the plots provided!**
   (a) (9 points) Sketch the corresponding root-locus diagram.
   (b) (3 points) Are there positive values of feedback gain for which the closed-loop system is asymptotically stable? Explain using your root-locus diagram.
3. **Nyquist Criterion.** **IMPORTANT: Use the plots provided!**
   (a) (3 points) Can a proportional controller with unit gain stabilize the system in closed-loop? Explain using the Nyquist Criterion.
   (b) (3 points) What are the positive values of gain for which the closed-loop system is asymptotically stable? Approximate values are fine.
   (c) (3 points) What are the positive values of gain for which the closed-loop system is not asymptotically stable? How many unstable closed-loop poles there are?
4. **Bode Plots.** **IMPORTANT: Use the plots provided!**
   (a) (9 points) Sketch the straight-line approximations for the corresponding Bode plots on top of the (exact) Bode plots provided in Fig. 2.
   (b) (3 points) Annotate the plots to show the points critical for the calculation of the gain-margin and phase-margin. Provide estimates for these quantities based on your reading of the plots.

(a) Paper mill†



(b) Two rollers



(c) Feedback diagram

† https://commons.wikimedia.org/wiki/File:Papermaking_machine_at_a_paper_mill_near_Pensacola.jpg

Figure 1: Figures for Part II

Part II

The motion of two identical rollers on a segment of a paper milling machine can be approximately described by the differential equations:

$$J\dot{\omega}_1(t) + b\,\omega_1(t) = c\,u_1(t) + \tau_1(t), \qquad J\dot{\omega}_2(t) + b\,\omega_2(t) = c\,u_2(t) + \tau_2(t), \tag{1}$$

where u_1 and u_2 are control commands for the motors of each roller and τ_1 and τ_2 are torque disturbances. Assume that all constants are positive.

In the midterm 1 you showed that the difference between the speeds of the rollers, δ_ω, satisfy the differential equation:

$$J\dot{\delta}_\omega(t) + b\,\delta_\omega(t) = c\,(\delta_u(t) + \delta_\tau(t)), \tag{2}$$

and that the Laplace transforms of the signals in (2) are related through:

$$\Delta_\omega(s) = G_\omega(s)(\Delta_u(s) + \Delta_\tau(s)), \qquad G_\omega(s) = \frac{c/J}{s + b/J}.$$

In the midterm 2 you showed that the transfer-function between the phase difference of the rollers, δ_θ, and the signals δ_u and δ_τ is given by:

$$\Delta_\theta(s) = G_\theta(s)(\Delta_u(s) + \Delta_\tau(s)), \qquad G_\theta(s) = \frac{c/J}{s(s + b/J)}.$$

The signals mentioned above are

$$\delta_\omega = \omega_2 - \omega_1, \quad \delta_u = u_2 - u_1, \quad \delta_\tau = \frac{\tau_2 - \tau_1}{c}, \quad \delta_\theta = \theta_2 - \theta_1, \quad \theta_k(t) = \int_0^t \omega_k(\sigma)d\sigma, k = 1, 2 \tag{3}$$

THROUGHOUT THE REST OF THIS EXAM:

Assume that $b/J = c/J = 1$.

Consider feedback connections as in Fig. 1c

1. **Open-loop System**
   (a) (1 point) Is $G_\omega(s)$ asymptotically stable? Explain.
   (b) (3 points) Sketch the magnitude and phase Bode plots of $G_\omega(s)$.
   (c) (2 points) Sketch the Nyquist plot associated with $G_\omega(s)$.
   (d) (1 point) Is $G_\theta(s)$ asymptotically stable? Explain.
   (e) (3 points) Sketch the magnitude and phase Bode plots of $G_\theta(s)$.
   (f) (2 points) Sketch the Nyquist plot associated with $G_\theta(s)$.

2. **Integral Control**

   Consider the feedback controller with transfer-function

   $$K(s) = \frac{K_i}{s}$$

   and answer the following questions:

   (a) (3 points) Use a root-locus or a Nyquist diagram to show that *it is not possible* to select $K_i > 0$ so that the closed-loop connection of K and G_θ is asymptotically stable.
   (b) (3 points) Use a root-locus or a Nyquist diagram to show that *it is possible* to select $K_i > 0$ so that the closed-loop connection of K and G_ω is asymptotically stable.
   (c) (1 point) Is the closed-loop in part (b) internally stable? Explain.
   (d) (1 point) Does the closed-loop in part (b) achieve asymptotic tracking of a constant reference angular velocity $\bar{\delta}_\omega$? Explain.
   (e) (1 point) Does the closed-loop in part (b) achieve asymptotic rejection of a constant torque disturbance δ_τ? Explain.

3. **Proportional-Integral Phase Control**

   Consider the feedback controller with transfer-function

   $$K(s) = \frac{K_p s + K_i}{s} = K_p \frac{s+z}{s}, \quad z = K_i/K_p.$$

   and answer the following questions:

   (a) (3 points) Use a root-locus or a Nyquist diagram to show that *it is not possible* to select $K_p > 0$ and $z = b/J$ so that the closed-loop connection of K and G_θ is asymptotically stable.
   (b) (3 points) Use a root-locus or a Nyquist diagram to show that *it is possible* to select $K_p > 0$ and $0 < z < b/J$ so that the closed-loop connection of K and G_θ is asymptotically stable.
   (c) (1 point) Is the closed-loop in part (b) internally stable? Explain.
   (d) (1 point) Does the closed-loop in part (b) achieve asymptotic tracking of a constant reference phase $\bar{\delta}_\theta$? Explain.
   (e) (1 point) Does the closed-loop in part (b) achieve asymptotic rejection of a constant torque disturbance δ_τ? Explain.

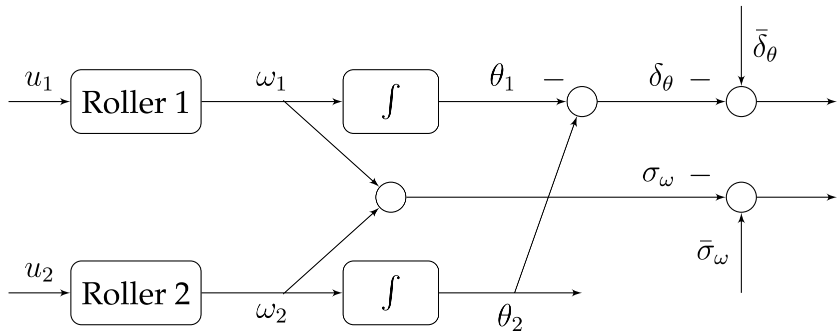


Figure 2: Block Diagram for Question II-5

4. **Speed Control**

   So far you have been controlling the phase of the rollers. You also need to control the speed of the rollers.

   (a) (1 point) Show that the sum of the speed of the rollers, $\sigma_\omega = \omega_1 + \omega_2$, satisfy the differential equation:

   $$J\dot{\sigma}_\omega(t) + b\,\sigma_\omega(t) = c\,(\sigma_u(t) + \sigma_\tau(t)), \tag{4}$$

   where $\sigma_u = u_1 + u_2$, and $\sigma_\tau = (\tau_1 + \tau_2)/c$.

   (b) (4 points) Design a feedback controller that can achieve closed-loop asymptotic tracking of a constant reference speed $\bar{\sigma}_\omega$ and asymptotic rejection of a constant disturbance σ_τ.

5. **Speed and Phase Control**

   Assume now that you have designed a phase controller

   $$\Delta_u(s) = K_\delta(s)(\bar{\Delta}_\theta - \Delta_\theta)$$

   and a speed controller

   $$\Sigma_u(s) = K_\sigma(s)(\bar{\Sigma}_\omega - \Sigma_\omega)$$

   (a) (2 points (bonus)) Show that

   $$u_1 = \frac{1}{2}(\sigma_u - \delta_u), \qquad u_2 = \frac{1}{2}(\sigma_u + \delta_u)$$

   are the corresponding control inputs to the individual rollers.

   (b) (3 points) Assume $\tau_1 = \tau_2 = 0$ and complete the block-diagram in Fig. 2 to show the relationship between the signals shown in the figure and δ_u, σ_u.

   (c) (4 points) Explain what is necessary to ask of K_δ and K_σ if the phases of the rollers are to asymptotically converge, i.e. $\theta_1 \to \theta_2$, and the speed of the rollers are to asymptotically converge to $\omega_1 \to \omega_2 \to \bar{\omega}$. What are the corresponding references $\bar{\delta}_\theta$ and $\bar{\sigma}_\omega$?

WRITE YOUR NAME AND ID AND
DO NOT FORGET TO RETURN THIS PAGE WITH YOUR EXAM!

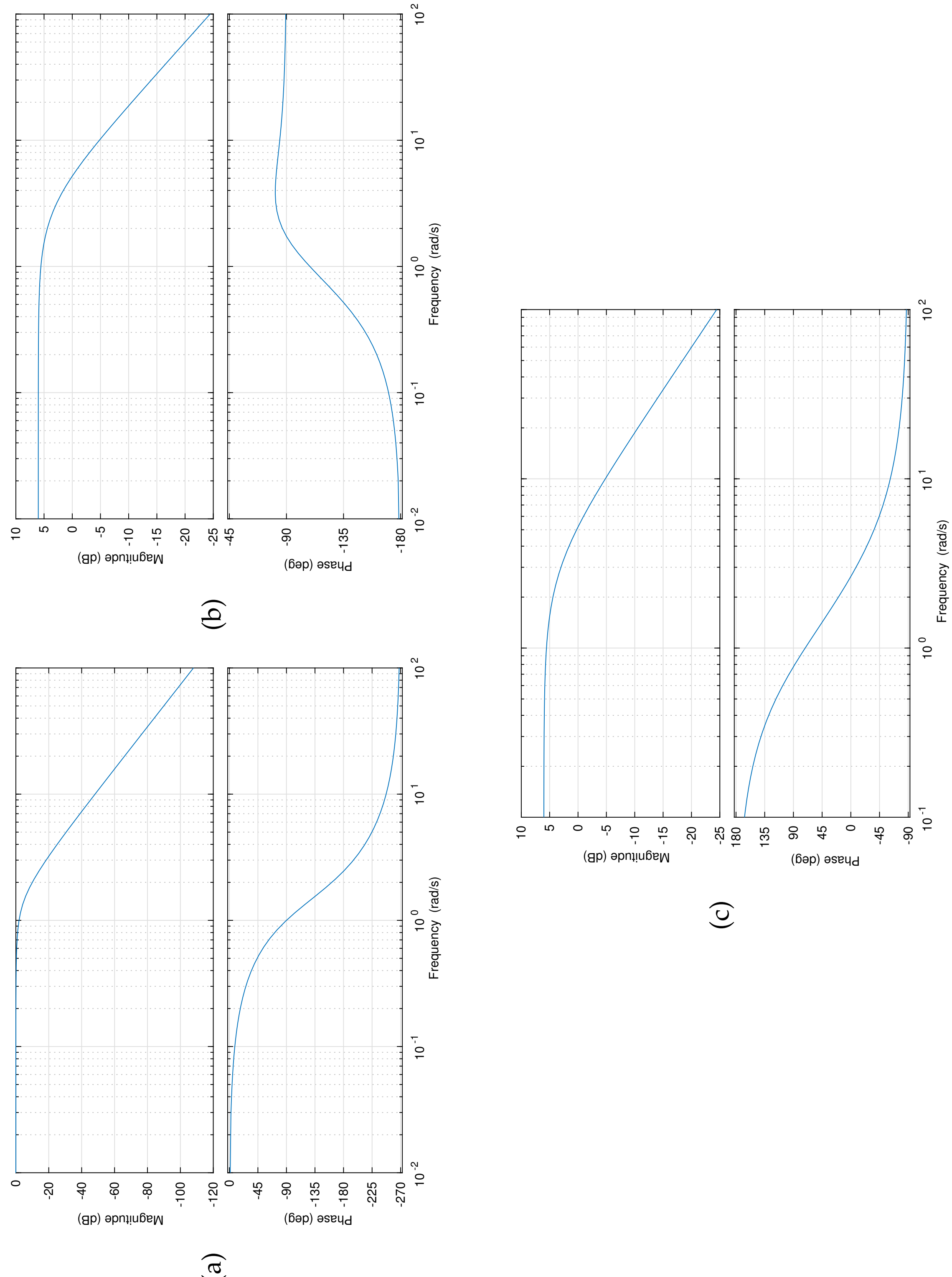


Figure 2: Diagrams for Question I-4

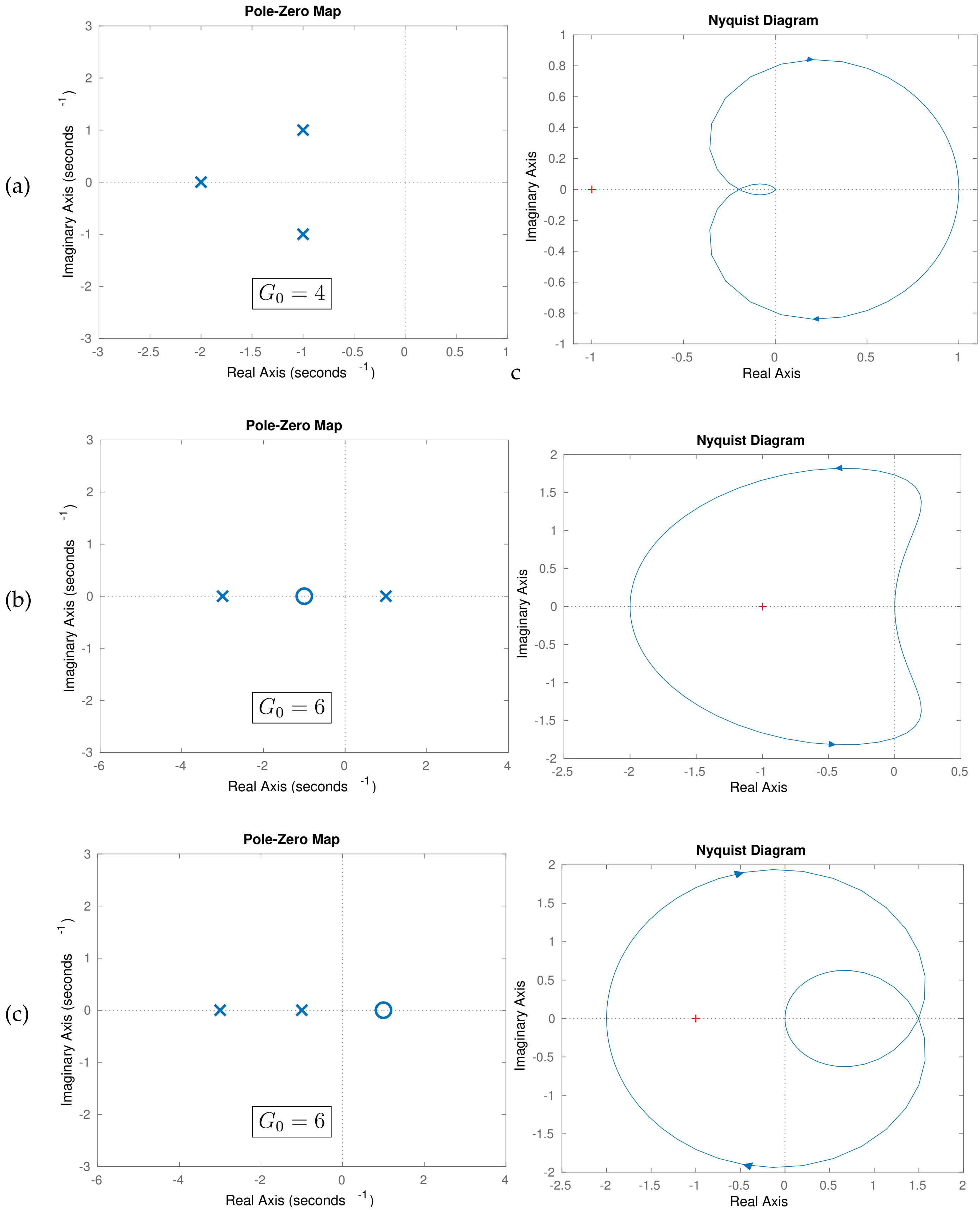


Figure 1: Diagrams for Questions I-1 through I-3