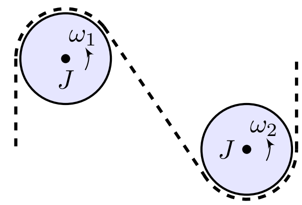
MAE143 B - Linear Control - Spring 2018
Midterm, May 3rd

Instructions:



1. This exam is **open book**. You can consult any printed or written material of your liking.
2. You have 70 minutes.
3. Most questions show you the answer, and you're asked to prove it. Use the given answers to move on to the next question. Do not get stuck!
4. Write your answers on your "Blue Book".
5. Do not forget to write your name and student number.
6. This exam has 2 questions, 25 total points and 2 bonus points.
7. Good luck!

Questions:

The motion of two identical rollers on a segment of a paper milling machine can be approximately described by the differential equations:

$$J\dot{\omega}_1(t) + b\,\omega_1(t) = c\,u_1(t) + \tau_1(t), \qquad J\dot{\omega}_2(t) + b\,\omega_2(t) = c\,u_2(t) + \tau_2(t), \tag{1}$$

where u_1 and u_2 are control commands for the motors of each roller and τ_1 and τ_2 are torque disturbances. Assume that all constants are positive. A basic requirement is that both rollers should run at the exact same velocity, that is $\omega_2 = \omega_1$. In this exam you will design controllers that attempt to achieve that goal.

1. **Modeling and Open-Loop Response**

   (a) (1 point) Show that the difference between the speeds of the rollers, $\delta_\omega = \omega_2 - \omega_1$, satisfy the differential equation:

   $$J\dot{\delta}_\omega(t) + b\,\delta_\omega(t) = c\,(\delta_u(t) + \delta_\tau(t)), \tag{2}$$

   where $\delta_u = u_2 - u_1$, and $\delta_\tau = (\tau_2 - \tau_1)/c$.

   (b) (3 points) Assume zero-initial conditions and show that the Laplace transforms of the signals in (2) are related through:

   $$\Delta_\omega(s) = G(s)(\Delta_u(s) + \Delta_\tau(s)), \qquad G(s) = \frac{c/J}{s + b/J}.$$

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† https://commons.wikimedia.org/wiki/File:Papermaking_machine_at_a_paper_mill_near_Pensacola.jpg

Draw a block diagram of the corresponding open-loop system showing the relationship between G and the signals δ_ω, δ_u and δ_τ.

(c) (3 points) Assume zero initial conditions and that $\delta_u(t) = 0$, and use the Laplace transform to calculate the response $\delta_\omega(t)$ to a disturbance $\delta_\tau(t) = \bar{\delta}_\tau \neq 0$, $t \geq 0$. Sketch the response and identify the transient and steady-state components of $\delta_\omega(t)$.

(d) (2 points) If possible, use the frequency response method to calculate the steady-state component of $\delta_\omega(t)$ you calculated in part (c). If not possible, explain why.

2. **Controller Design**

(a) (2 points) Consider the proportional control law

$$\delta_u(t) = Ke(t), \quad e(t) = \bar{\delta}_\omega(t) - \delta_\omega(t) \tag{3}$$

where $\bar{\delta}_\omega(t)$ is a reference velocity. Draw a block diagram of the corresponding closed-loop system showing the relationship between G, K and the signals δ_ω, δ_u, δ_τ, $\bar{\delta}_\omega$, and e.

(b) (3 points) Calculate the closed-loop transfer-functions from $\bar{\delta}_\omega$ to e, from δ_τ to e, and from $\bar{\delta}_\omega$ to δ_ω.

(c) (3 points) Assume that $\delta_\tau(t) = 0$ and $\bar{\delta}_\omega(t) = \bar{\delta}_\omega \neq 0$, $t \geq 0$, and show that

$$\delta_\omega(t) = H(0)(1 - e^{-\lambda t})\bar{\delta}_\omega + e^{-\lambda t}\delta_\omega(0), \quad t \geq 0, \qquad \lambda = \frac{b}{J} + K\frac{c}{J}, \tag{4}$$

where $H(s)$ is the transfer function from $\bar{\delta}_\omega$ to δ_ω you calculated in part (b).

(d) (1 point) Identify the transient and steady-state components of $\delta_\omega(t)$ in part (c).

(e) (2 points) Under what conditions in K does the closed-loop signal $\delta_\omega(t)$ in part (c) converge to $\bar{\delta}_\omega$ when $\bar{\delta}_\omega = 0$?

(f) (2 points) Assume that $\delta_\tau(t) = \bar{\delta}_\tau \neq 0$, select a proper K and use the frequency response method to calculate the steady-state error to a constant reference input $\bar{\delta}_\omega(t) = \bar{\delta}_\omega \neq 0$. Does the closed-loop signal $\delta_\omega(t)$ converge to $\bar{\delta}_\omega$?

(g) (3 points) Modify the feedback law (3) so that $\delta_\omega(t)$ converges to zero when $\bar{\delta}_\omega(t) = 0$ and $\delta_\tau(t) = \bar{\delta}_\tau \neq 0$, $t \geq 0$. Make sure to select a feedback gain so that the closed-loop is asymptotically stable and clearly explain the requirements and your reasoning.

(h) (1 point (bonus)) Does your controller in part (g) perform a pole-zero cancellation? Explain why your closed-loop system is internally stable.

(i) (1 point (bonus)) What happens in part (g) if $\bar{\delta}_\omega(t) = \bar{\delta}_\omega \neq 0$, $t \geq 0$?