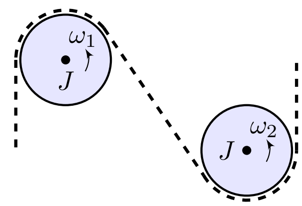
MAE143 B - Linear Control - Spring 2018
Midterm #2, May 22nd

Instructions:

1. This exam is **open book**. You can consult any printed or written material of your liking.
2. You have 70 minutes.
3. Many questions show you the answer. Use the given answers to move on to the next question. Do not get stuck!
4. Write your answers on your "Blue Book".
5. Do not forget to write your name and student ID.
6. This exam has 2 questions, 26 total points and 2 bonus points. Good luck!



Questions:

The motion of two identical rollers on a segment of a paper milling machine can be approximately described by the differential equations:

$$J\dot{\omega}_1(t) + b\,\omega_1(t) = c\,u_1(t) + \tau_1(t), \qquad J\dot{\omega}_2(t) + b\,\omega_2(t) = c\,u_2(t) + \tau_2(t), \tag{1}$$

where u_1 and u_2 are control commands for the motors of each roller and τ_1 and τ_2 are torque disturbances. Assume that all constants are positive.

In the first midterm you showed that the difference between the speeds of the rollers, $\delta_\omega = \omega_2 - \omega_1$, satisfy the differential equation:

$$J\dot{\delta}_\omega(t) + b\,\delta_\omega(t) = c\,(\delta_u(t) + \delta_\tau(t)), \tag{2}$$

where $\delta_u = u_2 - u_1$, $\delta_\tau = (\tau_2 - \tau_1)/c$, and that the Laplace transforms of the signals in (2) are related through:

$$\Delta_\omega(s) = G(s)(\Delta_u(s) + \Delta_\tau(s)), \qquad G(s) = \frac{c/J}{s + b/J}.$$

1. **Angular Velocity Control**

   (a) (2 points) Consider the proportional-integral control law

$$\delta_u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau, \quad e(t) = \bar{\delta}_\omega(t) - \delta_\omega(t) \tag{3}$$

† `https://commons.wikimedia.org/wiki/File:Papermaking_machine_at_a_paper_mill_near_Pensacola.jpg`

where $\bar{\delta}_\omega(t)$ is a reference velocity. Calculate the transfer-function $K(s)$ from e to δ_u and draw a block diagram of the corresponding closed-loop system showing the relationship between G, K and the signals δ_ω, δ_u, δ_τ, $\bar{\delta}_\omega$, and e.

(b) (2 points) Let $K_i/K_p = b/J$. Under what conditions on K_p is the closed-loop asymptotically stable?

(c) (2 points) Repeat part (b) using a root-locus diagram. Clearly identify the loop transfer-function $L(s)$ and α.

(d) (2 points) If the closed-loop is asymptotically stable is it also internally stable? Explain why or why not.

(e) (2 points) Does $\delta_\omega(t)$ converge to $\bar{\delta}_\omega \neq 0$ when $\bar{\delta}_\tau(t) = 0$ and $\bar{\delta}_\omega(t) = \bar{\delta}_\omega$, $t > 0$? Explain.

(f) (2 points) Does $\delta_\omega(t)$ converge to $\bar{\delta}_\omega \neq 0$ when $\bar{\delta}_\tau(t) = \bar{\delta}_\tau \neq 0$ and $\bar{\delta}_\omega(t) = \bar{\delta}_\omega$, $t > 0$? Explain.

2. **Phase Synchronization**

A stronger requirement is that both rollers should run at the exact same phase, that is $\theta_2 = \theta_1$, where $\theta_k(t) = \int_0^t \omega_k(\sigma)d\sigma$, $k = 1, 2$.

(a) (2 points) Represent in a block-diagram using integrators the relationship between

$$\delta_\theta = \theta_2 - \theta_1$$

and the signals δ_ω, δ_u, and δ_τ.

(b) (2 points) Assume zero-initial conditions and show that the Laplace transforms of the signals in (2) and δ_θ, are related through:

$$\Delta_\theta(s) = G(s)(\Delta_u(s) + \Delta_\tau(s)), \qquad G(s) = \frac{c/J}{s(s + b/J)}.$$

(c) (2 points) Consider the proportional control law

$$\delta_u(t) = K_\theta e_\theta(t), \quad e_\theta(t) = \bar{\delta}_\theta(t) - \delta_\theta(t) \tag{4}$$

where $\bar{\delta}_\theta(t)$ is a reference phase. Calculate the closed-loop transfer-function from $\bar{\delta}_\theta$ to e_θ.

(d) (2 points) For what values of K_θ is the closed-loop asymptotically stable?

(e) (2 points) Repeat part (d) using a root-locus diagram. Clearly identify the loop transfer-function $L(s)$ and α.

(f) (2 points) Does $\delta_\theta(t)$ converge to $\bar{\delta}_\theta \neq 0$ when $\bar{\delta}_\tau(t) = 0$ and $\bar{\delta}_\theta(t) = \bar{\delta}_\theta$, $t > 0$? Explain.

(g) (2 points) How can you modify the controller so that $\delta_\theta(t)$ converge to $\bar{\delta}_\theta \neq 0$ when $\bar{\delta}_\tau(t) = \bar{\delta}_\tau \neq 0$ and $\bar{\delta}_\theta(t) = \bar{\delta}_\theta$, $t > 0$? Explain.

(h) (2 points (bonus)) Analyze closed-loop stability for the controller you proposed in part (g) using the root-locus method.