MAE143 B - Linear Control - Spring 2019
Final, June 12th

Instructions:

1. This exam is open book. You may use whatever written materials of your choice, including your notes and the book, and a calculator with no communication capabilities.
2. You have 170 minutes. Don't get stuck! Come back later if you have time.
3. Write answers on your "Blue Book" and attach the provided graph sheet.
4. Do not forget to write your name and student ID.
5. The exam has 8 questions for a total of 71 points and 2 bonus points.

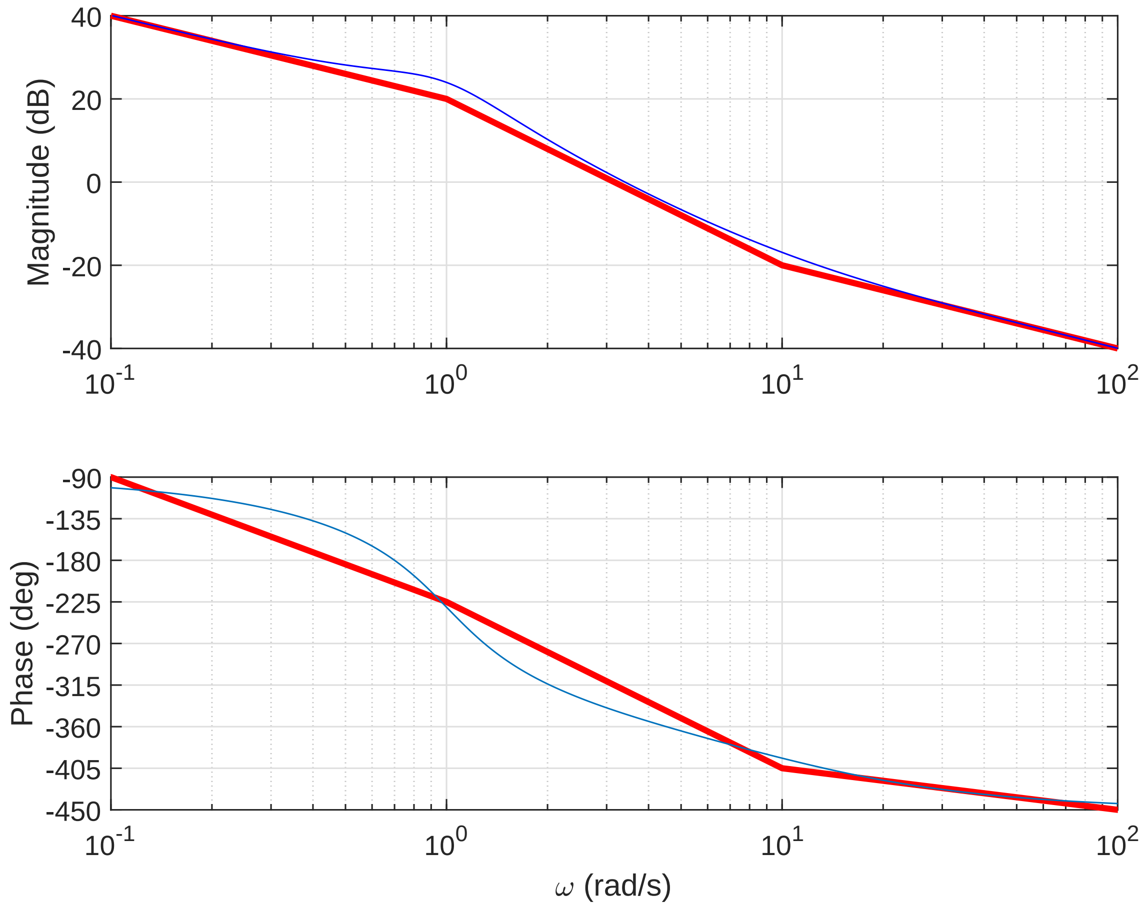


Figure 1: Bode diagrams for Part I

1. **Frequency Domain Analysis and Design.**

   Answer the following questions based on the Bode magnitude and phase diagrams and straight-line approximations shown in Figure 1.

   (a) (2 points) Segment the plots and calculate the slope of the magnitude and phase of the diagrams in each segment.

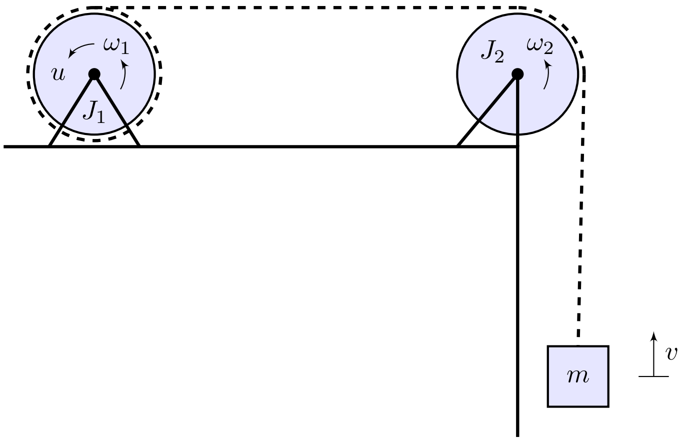
   (b) (5 points) Use the slopes from part (a) to identify poles and zeros of the corresponding transfer-function and indicate whether they are real, complex, or imaginary, and the sign of the corresponding real part. Explain your reasoning.

   (c) (3 points) Use parts (a) and (b) to write down the corresponding transfer-function.

   (d) (1 point) Is the transfer-function you obtained in part (c) asymptotically stable in open-loop? Explain.

   (e) (4 points) Sketch the Nyquist diagram corresponding to the transfer-function you obtained in part (c). Based on this diagram, are there positive values of gain for which this transfer-function is asymptotically stable in closed loop? Explain.

   (f) (4 points) Sketch the root-locus diagram corresponding to the transfer-function you obtained in part (c). Based on this diagram, are there positive values of gain for which this transfer-function is asymptotically stable in closed loop? Explain.



Source: `www.cranestodaymagazine.com`

Figure 2: Winch/pulley/mass systems

For the rest of this exam, consider the winch/pulley/mass system in Fig. 2, in which the winch, with inertia J_1, angular speed ω_1, and radius r_1, is connected to a pulley, with inertia J_2, angular speed ω_2, and radius r_2, which is connected to the mass m by an inextensible and massless cable that is never slack. Consider also that $r_1 = r_2 = r$ and that a torque u is applied at the winch as shown.

In the Midterm 1 you showed that the motion of this system could be approximately modeled by the ordinary differential equation

$$J_r\,\dot{v}(t) + b_r v(t) = r(u(t) + w(t)), \qquad w(t) = -mgr, \qquad v(t) = r\,\omega_1(t) = r\,\omega_2(t), \tag{1}$$

where $J_r = J_1 + J_2 + mr^2$, $b_r = b_1 + b_2$, and b_1 and b_2 represent viscous damping at the winch and pulley.

$\boxed{\text{For the rest of the exam assume that } J_r = b_r = 1,\ m = g = 10, \text{ and } r = 0.1.}$

You have also shown that the Laplace transform of the above signals are related by

$$V(s) = G_v(s)(U(s) + W(s)), \qquad G_v(s) = \frac{r/J_r}{s + b_r/J_r} = \frac{0.1}{s+1},$$

and, in the Midterm 2, that

$$X(s) = G_x(s)(U(s) + W(s)), \qquad G_x(s) = \frac{r/J_r}{s(s + b_r/J_r)} = \frac{0.1}{s^2+s},$$

where

$$x(t) = x(0) + \int_0^t v(\tau)\,d\tau.$$

2. **Open-loop System for Speed Control.**
   (a) (1 point) Is $G_v(s)$ asymptotically stable? Explain.
   (b) (3 points) Sketch the magnitude and phase Bode plots of $G_v(s)$.
   (c) (2 points) Sketch the Nyquist plot associated with $G_v(s)$.

3. **Speed Control I**

One student suggested that in order to design a feedback controller for controlling the speed of the mass, one can simply compensate for the gravitational force by offsetting the winch torque to be

$$u(t) = \tilde{u}(t) + mgr. \tag{2}$$

From this perspective, answer the following questions.

(a) (1 point) Show that the model (1) with (2) is equivalent to

$$J_r\, \dot{v}(t) + b_r v(t) = r\tilde{u}(t).$$

(b) (2 points) Is it possible to design a proportional controller

$$\tilde{u}(t) = K_p\, e(t), \qquad e(t) = \bar{v}(t) - v(t), \tag{3}$$

that would be able to asymptotically stabilize the closed-loop system and ensure that

$$\lim_{t\to\infty} v(t) = \bar{v} \tag{4}$$

when $\bar{v}(t) = \bar{v}$, $t \geq 0$? If not possible, explain why. If possible, calculate a suitable gain.

(c) (2 points) Use a root-locus or a Nyquist diagram to support your argument about asymptotic closed-loop stability in part (b). If you have already used one such argument then skip to the next question.

(d) (2 points) Draw a block-diagram for the closed-loop system resulting from using the controller in part (b) in term of the signals v, $\bar{v}$, e, u, and $\tilde{u}$.

(e) (1 point) Would your answer to part (b) change if the value of g deviated from the value used to design the controller? Explain.

4. **Speed Control II**

Another student suggested that in order to design a feedback controller for controlling the speed of the mass, one should leave model (1) as is and treat the gravitational force as a constant disturbance $w(t) = \bar{w} = -mgr = -10$ Nm. From this perspective, answer the following questions.

(a) (2 points) Draw a block-diagram for a standard closed-loop system in which $u = Ke$, $e = \bar{v} - v$, in terms of the signals v, $\bar{v}$, e, u, and w.

(b) (2 points) Is it possible to design a proportional controller

$$u(t) = K_p\, e(t), \qquad e(t) = \bar{v}(t) - v(t), \tag{5}$$

that would be able to asymptotically stabilize the closed-loop system and ensure that

$$\lim_{t\to\infty} v(t) = \bar{v} \tag{6}$$

when $\bar{v}(t) = \bar{v}$, $t \geq 0$? If not possible, explain why. If possible, calculate a suitable gain.

(c) (2 points) Use a root-locus or a Nyquist diagram to support your argument about asymptotic closed-loop stability in part (b). If you have already used one such argument then skip to the next question.

(d) (1 point) Would your answer to part (b) change if the value of g deviated from the value used to design the controller? Explain.

(e) (1 point) Compare and constrast the solution from Question 3 with the solution from Question 4.

5. **Speed Control III**

Consider the same setup as in Question 4 and answer the following questions.

(a) (2 points) Is it possible to design a proportional-integral controller

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\, d\tau, \quad e(t) = \bar{v}(t) - v(t), \tag{7}$$

that would be able to asymptotically stabilize the closed-loop system and ensure that

$$\lim_{t \to \infty} v(t) = \bar{v} \tag{8}$$

when $\bar{v}(t) = \bar{v}$, $t \geq 0$? If not possible, explain why. If possible, calculate suitable gains.

(b) (2 points) Use a root-locus or a Nyquist diagram to support your argument about asymptotic closed-loop stability in part (b). If you have already used one such argument then skip to the next question.

(c) (1 point) Would your answer to part (b) change if the value of g deviated from the value used to design the controller? Explain.

(d) (1 point) Compare and constrast the solution from Question 4 with the solution from Question 5.

6. **Open-loop System for Position Control.**

(a) (1 point) Is $G_x(s)$ asymptotically stable? Explain.

(b) (3 points) Sketch the magnitude and phase Bode plots of $G_x(s)$.

(c) (2 points) Sketch the Nyquist plot associated with $G_x(s)$.

7. **Position Control I**

A third student interjected that in many cases it is the position of the mass and not the velocity that might need to be controlled. From this perspective, answer the following questions.

(a) (2 points) Draw a block-diagram for a standard closed-loop system in which $u = Ke$, $e = \bar{x} - x$, in terms of the signals x, $\bar{x}$, e, u, and w.

(b) (2 points) Is it possible to design a proportional controller

$$u(t) = K_p\, e(t), \qquad e(t) = \bar{x}(t) - x(t), \tag{9}$$

that would be able to asymptotically stabilize the closed-loop system and ensure that

$$\lim_{t \to \infty} x(t) = \bar{x} \tag{10}$$

when $\bar{x}(t) = \bar{x}$, $t \geq 0$? If not possible, explain why. If possible, calculate a suitable gain.

(c) (2 points) Use a root-locus or a Nyquist diagram to support your argument about asymptotic closed-loop stability in part (b). If you have already used one such argument then skip to the next question.

(d) (1 point) Would your answer to part (b) change if the value of g deviated from the value used to design the controller? Explain.

8. **Position Control II**

After implementing a proportional controller as in Problem 7, a student found out that starting from $x(0) = \dot{x}(0) = 0$ and for values of reference position $0 \leq \bar{x} \leq 10$m that a) the system response display no oscillations, b) the mass would not reach the desired reference position $\bar{x}$, and c) the mass would actually come down instead of coming up toward $\bar{x}$. With that in mind, assume that $\bar{x}(t) = \bar{x}$, $t > 0$, and answer the following questions.

(a) (1 point (bonus)) Show that

$$x = H\bar{x} + Dw, \qquad u = Q\bar{x} - Hw,$$

where

$$H = \frac{0.1K_p}{s^2 + s + 0.1K_p}, \qquad D = \frac{0.1}{s^2 + s + 0.1K_p}, \qquad Q = \frac{K_p(s^2 + s)}{s^2 + s + 0.1K_p}.$$

(b) (2 points) Calculate the range of $K_p > 0$ for which the closed-loop response to a constant reference position displays no oscillations.

(c) (2 points) Use the frequency-response method to show that

$$\lim_{t\to\infty} x(t) = \bar{x} + \bar{w}/K_p = \bar{x} - 10/K_p, \qquad \text{and} \qquad \lim_{t\to\infty} u(t) = -\bar{w} = 10.$$

(d) (2 points) Use the initial-value theorem

$$\lim_{t\to 0^+} f(t) = \lim_{|s|\to\infty} sF(s)$$

to show that

$$\lim_{t\to 0^+} u(t) = K_p\,\bar{x}.$$

(e) (2 points) Use the above facts to infer what was the student's gain. Explain what is happening to the student's closed-loop control system.

(f) (3 points) Knowing that the maximum torque available at the winch is 400 Nm, can you propose a value of gain that: a) would not lead to closed-loop oscillations, b) the mass would not come down when $\bar{x} \geq 1$ m, and c) would not saturate the motor when $\bar{x} < 10$m?

(g) (1 point (bonus)) Can you add to the requirements of part (f) that the closed-loop system d) would have a steady-state error of less than 1% when 1 m $\leq \bar{x} \leq 100$ m?