MAE143 B - Linear Control - Spring 2019
Midterm, May 24th

Instructions:

1. This exam is **open book**. You can consult any printed or written material of your liking.
2. You have 50 minutes.
3. Most questions show you the answer, and you're asked to prove it. Use the given answers to move on to the next question. Do not get stuck!
4. Write your answers on your "Blue Book".
5. Do not forget to write your name and student number.
6. This exam has 2 questions, 19 total points and 2 bonus points.
7. Good luck!

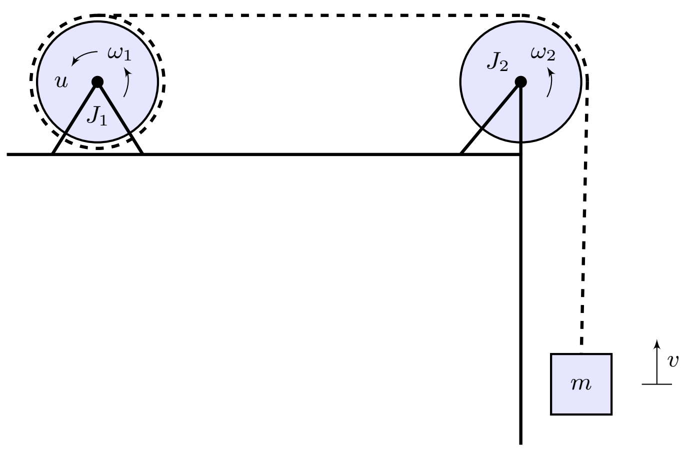

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Figure 1: Winch/pulley/mass systems

Consider the winch/pulley/mass system in Fig. 1, in which the winch, with inertia J_1, angular speed ω_1, and radius r_1, is connected to a pulley, with inertia J_2, angular speed ω_2, and radius r_2, which is connected to the mass m by an inextensible and massless cable that is never slack. Consider also that $r_1 = r_2 = r$ and that a torque u is applied at the winch as shown.

Questions:

In the first midterm you showed that the motion of this system could be approximately modeled by the ordinary differential equation

$$J_r\,\dot v(t) + b_r v(t) = r(u(t) + w(t)), \qquad w(t) = -mgr, \qquad v(t) = r\,\omega_1(t) = r\,\omega_2(t), \tag{1}$$

where $J_r = J_1 + J_2 + mr^2$, $b_r = b_1 + b_2$, and b_1 and b_2 represent viscous damping at the winch and pulley.

$$\boxed{\text{For the rest of the exam assume that } J_r = b_r = 1,\ m = g = 10, \text{ and } r = 0.1.}$$

1. **Modeling**

   (a) (1 point) Show that

   $$G(s) = \frac{r/J_r}{s + b_r/J_r} = \frac{0.1}{s+1} \tag{2}$$

   is the open-loop transfer-function associated with (1) from u to v and draw a block-diagram showing the relationship between G and the signals u, w, and v.

2. **Proportional-Integral Speed Control**

   (a) (2 points) Consider the proportional plus integral control law

   $$u(t) = K_p e(t) + K_i \int_0^t e(\tau)\,d\tau, \quad e(t) = \bar v(t) - v(t) \tag{3}$$

   where $\bar v(t)$ is a reference velocity. Calculate the open-loop transfer-function $K(s)$ from e to u and draw a closed-loop block-diagram showing the relationship between G, K and the signals u, w, e, v, and $\bar v$.

   (b) (2 points) Calculate the closed-loop transfer-functions from $\bar v$ to e and from w to e.

   (c) (4 points) Calculate K_p and K_i so that the closed-loop system has $\omega_n = 1\text{rad/s}$ and $\zeta = 1$. What is the transfer-function of the corresponding controller? Where are the closed-loop poles located?

   (d) (2 points) For the K_p and K_i you calculated in part (c) is the closed-loop internally stable? Explain.

   (e) (4 points) Explain how can you use the root-locus method to calculate values of K_p and K_i for which the closed-loop system is asymptotically stable. Locate the points in your root-locus diagram that correspond to the K_p and K_i you calculated in part (c).

   (f) (1 point) Does the signal $v(t)$ converge to a given constant $\bar v$ when $w(t) = -mgr$ and K_p and K_i are as in part (c)? Explain.

   (g) (3 points) Use the frequency response method to calculate the steady-state value of $u(t)$ when $v(t) = \bar v$ and $w(t) = -mgr$. What does the signal $u(t)$ converge to when $\bar v = 0$? Explain.

   (h) (1 point (bonus)) What happens in part (g) if $\bar v < -1\text{m/s}$? Explain.

   (i) (1 point (bonus)) Show that when $\bar v(t) = 0$ and K_p and K_i are as in (c), the closed-loop transfer-function from $w(t)$ to

   $$x(t) = x(0) + \int_0^t v(\tau)\,d\tau$$

   is equal to

   $$F(s) = \frac{0.1}{(s+1)^2}$$

   Sketch the complete response $x(t)$ when $\bar v = 0$, $x(0) = v(0) = 0$, and $w(t) = -mgr$. Explain.