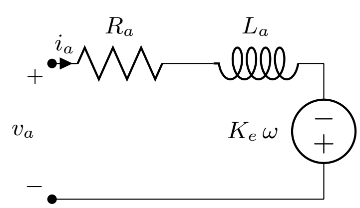
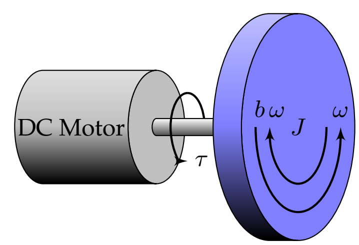
MAE143 B - Linear Control - Spring 2020
Midterm 1, January 28th

Instructions:

1. This exam is **open book**. You can consult any printed or written material of your liking.
2. You have 70 minutes.
3. Most questions show you the answer, and you're asked to prove it. Move on. Do not get stuck!
4. Write your answers on your "Blue Book".
5. Do not forget to write your name and student number.
6. This exam has 4 questions, 28 total points and 2 bonus points.
7. Good luck!

Consider the DC motor electrical and mechanical schematics:





You have shown in the homework that the motion of the rotor of a DC motor shown schematically above can be described by the differential equation:

$$J\,\dot{\omega} + \left(b + \frac{K_e K_t}{R_a}\right)\omega = \frac{K_t}{R_a} v_a$$

where v_a is the motor's input voltage and all other constants are positive.

Answer the following questions:

Questions:

1. **Modeling and Open-Loop Response I**

   (a) (2 points) Calculate the transfer-function, $G_\omega(s)$, from $u(t) = v_a(t)$ to $y(t) = \omega(t)$.

   (b) (2 points) Calculates the poles and zeros of $G_\omega(s)$. Is $G_\omega(s)$ asymptotically stable?

   (c) (2 points) Show that if $y(0) = y_0$ then

   $$y(t) = y_0\, e^{-\lambda t} + \tilde{y}\,(1 - e^{\lambda t}),$$
   $$\tilde{y} = \frac{V_a K_t / R_a}{b + K_e K_t / R_a}, \qquad \lambda = -\frac{b}{J} - \frac{K_e K_t}{J R_a},$$

   is the response to a voltage step input $u(t) = v_a(t) = V_a 1(t)$.

   (d) (1 point) Sketch the response from part (c).

2. **Proportional Velocity Control**

   In this question, consider the DC motor from Question 1 and assume that

   $$J = b = K_t = K_e = R_a = 1.$$

   Consider the proportional control law

   $$u(t) = K e(t), \quad e(t) = \bar{y}(t) - y(t)$$

   where $\bar{y}(t)$ is a reference angular velocity, $\bar{\omega}(t)$.

   (a) (1 point) Draw a block diagram of the corresponding closed-loop system showing the relationship between G, K and the signals $\bar{y}$, e, u, and y.

   (b) (2 points) Calculate the closed-loop transfer-functions from $\bar{y}$ to e and $\bar{y}$ to y.

   (c) (2 points) What are the values of K for which the closed-loop system is asymptotically stable?

   (d) (2 points) Does the signal $y(t)$ converge to $\bar{y}$ when $\bar{y}(t) = \bar{y}$ is a non-zero constant? Explain.

3. **Tracking Design**

   Consider the proportional control law from Question 2 in which K is chosen so that the closed-loop is asymptotically stable.

   (a) (2 points) Calculate the steady-state error in response to a reference angular velocity $\bar{y}(t) = \bar{y}\cos(t)$.

   (b) (3 points) Show how the parameters a, b, and K of the controller with transfer-function

   $$K(s) = \frac{K(s+a)^2}{s^2 + b^2}$$

   can be selected so that the closed-loop is asymptotically stable and the steady-state error in response to the reference angular velocity from part (a) is zero.

   (c) (2 points) Select suitable a, b, and K and calculate the corresponding sensitivity transfer-function $S(s) = (1 + K(s)G(s))^{-1}$. Calculate also its poles and zeros.

4. **Proportional Position Control**

   (a) (2 points) Calculate the transfer-function, $G_\theta(s)$, from $u(t) = v_a(t)$ to

   $$y(t) = \theta(t) = \int_0^t \omega(\tau)\, d\tau.$$

   (b) (2 points) Calculates the poles and zeros of $G_\theta(s)$. Is $G_\theta(s)$ asymptotically stable?

   Assume that

   $$J = b = K_t = K_e = R_a = 1,$$

   and consider the proportional control law

   $$u(t) = K e(t), \quad e(t) = \bar{y}(t) - y(t)$$

   where $\bar{y}(t)$ is a reference angular position, $\bar{\theta}(t)$.

   (c) (2 points) What are the values of K for which the closed-loop system is asymptotically stable?

   (d) (1 point) Pick a stabilizing value of K and calculate the corresponding closed-loop poles.

   (e) (2 points (bonus)) Does the signal $y(t)$ converge to $\bar{y}$, when $\bar{y}(t) = \bar{y}$ is a non-zero constant? Explain.