MAE143 B - Linear Control - Winter 2020
Midterm 2, February 18th

Instructions:

1. This exam is **open book**. You can consult any printed or written material of your liking.
2. You have 70 minutes.
3. Most questions show you the answer, and you're asked to prove it. Move on. Do not get stuck!
4. Write your answers on your "Blue Book".
5. Do not forget to write your name and student number.
6. This exam has 4 questions, 24 total points and 3 bonus points.
7. Good luck!

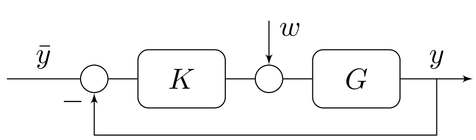

Figure 1: Feedback system diagram

Questions:

1. **Feedback Design I**

   Consider the standard feedback diagram from Fig. 1 in which

   $$G(s) = \frac{1}{s(s+1)}.$$

   Consider the proportional controller

   $$K(s) = K_p$$

   and answer the following questions:

   (a) (2 points) Calculate the closed-loop transfer functions

   $$S = \frac{1}{1+KG}, \qquad Q = \frac{K}{1+KG},$$
   $$D = \frac{G}{1+KG}, \qquad H = \frac{KG}{1+KG}.$$

   (b) (2 points) For what values of $K_p > 0$ is the closed-loop system internally stable?

   (c) (1 point) Can this closed-loop system asymptotically track a constant reference input $\bar{y}$? Explain.

   (d) (1 point) Can this closed-loop system asymptotically reject a constant input disturbance w? Explain.

2. **Feedback Design II**

Consider the standard feedback diagram from Fig. 1 in which

$$G(s) = \frac{s}{s+1}.$$

Consider the proportional-integral controller

$$K(s) = K_p + K_i s^{-1}$$

and answer the following questions:

(a) (2 points) Calculate the closed-loop transfer functions

$$S = \frac{1}{1+KG}, \qquad Q = \frac{K}{1+KG},$$
$$D = \frac{G}{1+KG}, \qquad H = \frac{KG}{1+KG}.$$

(b) (2 points) For what values of $K_p > 0$ and $K_i > 0$ is the closed-loop system internally stable?

(c) (1 point) Can this closed-loop system asymptotically track a constant reference input $\bar{y}$? Explain.

(d) (1 point) Can this closed-loop system asymptotically reject a constant input disturbance w? Explain.

3. **System Realization and State Space**

In the Midterm I you solved a question involving a controller of the form

$$K(s) = \frac{K(s+a)^2}{s^2+b^2}.$$

(a) (3 points) Draw a block-diagram using only integrators, amplifiers and summers that can realize $K(s)$.

(b) (3 points) (+1 bonus point) Write state-space equations associated with the block-diagram you obtained in part (a).

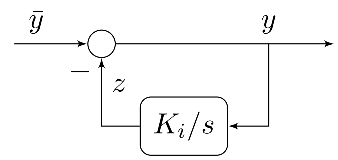

Figure 2: System for Question 4

4. **Filter**

Answer the following questions to see how the feedback system in Fig. 2 can be used as a filter.

(a) (2 points) Calculate the transfer-functions from the input $\bar{y}$ to the outputs y and z.

(b) (1 point) For what values of $K_i > 0$ is the system asymptotically stable?

(c) (3 points) Use the frequency-response method to explain why when $\omega \gg K_i$ and

$$\bar{y}(t) = a + b\cos(\omega t)$$

that

$$y_{ss}(t) \approx b\cos(\omega t), \qquad z_{ss}(t) \approx a.$$

(d) (2 points (bonus)) Explain how one could use the above system to remove the DC component of a signal and, at the same time, produce an estimate of that DC component.