MAE143 B - Linear Control - Winter 2022
Final, March 17th

Instructions:

1. This exam is open book. You may use whatever written materials of your choice, including your notes and the book, and a calculator with no communication capabilities.
2. You have 120 minutes. Don't get stuck! Come back later if you have time.
3. Write the answers on your "Blue Book."
4. Do not forget to write your name and student ID.
5. The exam has 6 questions for a total of 36 points.

Questions:

1. **Root Locus I.**

   Consider the loop transfer-function

   $$L(s) = \frac{s+10}{s(s+1)}$$

   and answer the following questions.

   (a) (4 points) Sketch the root-locus diagram for $L(s)$.

   (b) (1 point) Based on your root-locus diagram, what is the range of values of $\alpha > 0$ such that the zeros of $1 + \alpha L(s)$ are asymptotically stable? Explain.

   (c) (1 point) Based on your root-locus diagram, is there a value of $\alpha > 0$ for which all zeros of $1 + \alpha L(s)$ are real and have negative real part less than -10? Explain.

2. **Root Locus II.**

   Consider the loop transfer-function

   $$L(s) = \frac{s+10}{s(s+1)^2}$$

   and answer the following questions.

   (a) (4 points) Sketch the root-locus diagram for $L(s)$.

   (b) (1 point) Based on your root-locus diagram, are there values of $\alpha > 0$ for which at least one zeros of $1 + \alpha L(s)$ is *not* asymptotically stable? Explain.

   (c) (1 point) Based on your root-locus diagram, is there a value of $\alpha > 0$ for which all zeros of $1 + \alpha L(s)$ are real and asymptotically stable? Explain.

3. **Bode and Nyquist Plots.**

   Consider the loop transfer-function

   $$L(s) = \frac{s+10}{s^2-1}$$

   and answer the following questions.

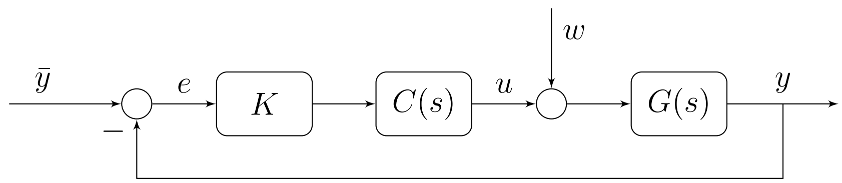

Figure 1

(a) (4 points) Sketch the magnitude and phase of the Bode plots for $L(s)$.

(b) (4 points) Sketch the Nyquist plot for $L(s)$.

(c) (2 points) Based on the Nyquist diagram, determine the range of values of $\alpha > 0$ for which the zeros of $1 + \alpha L(s)$ are asymptotically stable. Explain.

4. **Feedback Analysis I.**

Consider the feedback diagram in Fig. 1 in which

$$G(s) = \frac{s+10}{s+1}, \qquad C(s) = \frac{1}{s}, \qquad L(s) = C(s)\,G(s) = \frac{s+10}{s(s+1)},$$

and answer the following questions. *HINT: use your answers from Question 1!*

(a) (2 points) Assume $w(t) = 0$. Is the closed-loop capable of asymptotically tracking a constant non-zero reference input $\bar{y}$? Explain.

(b) (2 points) Assume $w(t) = \bar{w} \neq 0$. Is the closed-loop capable of asymptotically tracking a constant non-zero reference input $\bar{y}$? Explain.

(c) (2 points) Assume $\bar{y}(t) = 0$ and $w(t) = \bar{w} \neq 0$. Calculate the steady-state value of $u(t)$.

5. **Feedback Analysis II.**

Now let

$$G(s) = \frac{1}{s}, \qquad C(s) = \frac{s+10}{s+1}, \qquad L(s) = C(s)\,G(s) = \frac{s+10}{s(s+1)},$$

and answer the following questions. *HINT: use your answers from Question 1!*

(a) (2 points) Assume $w(t) = 0$. Is the closed-loop capable of asymptotically tracking a constant non-zero reference input $\bar{y}$? Explain.

(b) (2 points) Assume $w(t) = \bar{w} \neq 0$. Is the closed-loop capable of asymptotically tracking a constant non-zero reference input $\bar{y}$? Explain.

6. **Feedback Design.**

Consider the feedback diagram in Fig. 1 in which

$$G(s) = \frac{1}{s}.$$

(a) (4 points) Design a controller transfer-function $C(s)$ so that the closed-loop system is capable of asymptotically tracking a constant non-zero reference input $\bar{y}$ while rejecting a constant non-zero disturbance w. Use a graphical method (root-locus or Nyquist) to justify the existence of a stabilizing gain. You do not need to explicitly calculate a value of gain, but you need to show that a suitable choice exists.