MAE 280A – Homework #1

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1. Is the system

$$\begin{aligned} \dot{x}(t) &= A(t)x(t) + B(t)u(t), & x(t_0) &= x_0 \\ y(t) &= C(t)x(t) + D(t)u(t), & t &\geq t_0 \end{aligned} \tag{1}$$

   linear? Do you need any assumptions on the model, i.e. the $A(t)$, $B(t)$, $C(t)$, $D(t)$ matrices, or otherwise to reach your conclusion?

2. Can you solve the linear differential equation

$$\dot{x}(t) = \frac{x(t)}{1-t}, \qquad x(t_0) = x_0, \qquad t \geq t_0$$

   What happens around $t = 1$?

3. Can you solve the non-linear differential equation

$$\dot{y}(t) = \frac{\sin(2y(t))}{2(1-t)}, \qquad y(t_0) = y_0, \qquad t \geq t_0$$

   What happens around $t = 1$?

4. Show that if $x(t)$ and $y(t)$ are such that

$$\begin{aligned} \dot{x}(t) &= \frac{x(t)}{1-t}, & x(t_0) &= x_0, \\ y(t) &= \arctan(x(t)), & t &\geq t_0 \end{aligned}$$

   then $x(t)$ and $y(t)$ solve the differential equations in problems 2. and 3. Is this system linear? Is it time-invariant?

5. Consider systems that process the input as follows:

$$\begin{aligned} L_1(u(t)) &= \int_{t_0}^{t} u(\tau)\, d\tau \\ L_2(u(t)) &= \frac{1}{t-t_0} \int_{t_0}^{t} u(\tau)\, d\tau \\ L_3(u(t)) &= \frac{1}{\delta} \int_{t-\delta}^{t} u(\tau)\, d\tau, \qquad \delta > 0 \end{aligned}$$

   where $t \geq t_0$. For each system L_i, $i = 1, 2, 3$, answer the following questions:

   (a) Is the system linear?
   (b) Is the system time-invariant?
   (c) What does the system do to the input $u(t)$?
   (d) Find a representation in state-space form (1).
   (e) What is the dimension of the state?
   (f) What is the impact of the initial state in the solution?