MAE 280A – Homework #3

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1. The transfer-function between the voltage, v, and the angular velocity, ω, of a D.C. motor can be approximated by:

$$\frac{\Omega(s)}{V(s)} = \frac{k}{\tau s + 1}, \qquad k = 10 \text{ rad/(V s)}, \quad \tau = 0.1 \text{ s}$$

   Answer the following questions:

   (a) Calculate a state-space representation for the system with input equal to the motor voltage, v, and output equal to the motor angular position, θ. Is this realization in controller canonical form?

   (b) Use Matlab or any other software you would like to compute the transfer function of the state-space system you obtained. Validate that transfer-function with a frequency-domain calculation.

   (c) From this transfer-function, construct a controller canonical state-space realization.

   (d) Use this realization to determine state feedback gains that place the closed-loop poles at $p_1 = p_2 = -1 < 0$.

   (e) Interpret the feedback control law you designed in part (d).

   (f) Is is possible to design a state-feedback controller that uses only the velocity with which both p_1 and p_2 have negative real part? Explain.

2. Consider a sensor that measures the angular velocity of the D.C. motor in p.1 which is modeled by the transfer-function:

$$\frac{\hat{\Omega}(s)}{\Omega(s)} = \frac{2\pi f_c}{s + 2\pi f_c}, \qquad f_c = 12 \text{ Hz}$$

   Answer the following questions:

   (a) Use state-space formulas to calculate a state-space representation for a system with input equal to the motor voltage, v, and output equal to the estimated motor angular position, $\hat{\theta} = \int_0^t \hat{\omega}(\tau)\, d\tau$. Is this realization in controller canonical form?

   (b) Use Matlab or any other software you would like to compute the transfer function of the state-space system you obtained. Validate that transfer-function with a frequency-domain calculation.

   (c) Determine state feedback gains that place the closed-loop poles at $p_1 = p_2 = p_3 = -1 < 0$.

   (d) Interpret the feedback control law you designed in part (c).

   (e) How could you implement this controller? What is required of the gains to make this controller implementable using only knowledge of the measurement $\hat{\omega}$?