Midterm

I. Answer the following questions related to the transfer functions:

a) $H_1(s) = \dfrac{s-1}{s^2+5s+6}$

b) $H_2(s) = \dfrac{s+1}{s^2+s-2}$

c) $H_3(s) = \left[\begin{array}{cc|cc} 0 & 4 & 2 & 1 \\ 1 & 0 & 1 & 0 \\ \hline 1 & 2 & 0 & 0 \end{array}\right]$

d) $H_4(s) = \left[\begin{array}{ccc|cc} -1 & -2 & -3 & 1 & 2 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 \\ \hline 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 2 \end{array}\right]$

1. Compute the poles and zeros.
2. Are the realizations $H_3(s)$ and $H_4(s)$ controllable? Are they observable?
3. Compute the H_2 and H_∞ norms. Use the Lyapunov equation to compute the H_2 norm and a bissection algorithm based on the Riccati equation to compute the H_∞ norm.
4. Compute the $L_2(j\mathbb{R})$ and $L_\infty(j\mathbb{R})$ norms. Use the same algorithms as in the previous question.
5. Compute stable coprime factorizations. Verify the Bezout equation for all computed factorizations.
6. For $H_4(s)$, find a controller parameter $Q(s) \in RH_\infty$ that stabilizes $H_4(s)$ by feedbacking the first output to the first input only and, at the same time, places the closed loop poles at $\{-1, -2 \pm j1, -3, -4 \pm j2\}$. Compute the H_∞ norm of the closed loop transfer function.

II. Solve problem 12.1, p. 246 from Zhou.