

HOMEWORK #1

1. Calculate by hand the matrix/vector products:

$$\begin{bmatrix} 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 4 & 5 \end{bmatrix}, \quad \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \\ 2 & -2 \end{bmatrix}$$

2. Use Matlab/Octave to compute the products in 1. (attach your input/output).
 3. Verify that $AB \neq BA$ for

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

but that $AB = BA$ for

$$A = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & -1 \\ -1 & 5 \end{bmatrix}.$$

4. Classify the set of vectors $X = \{v_1, v_2, v_3\}$ as linearly dependent or linearly independent for:

$$v_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, v_2 = \begin{pmatrix} 3 \\ 4 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, v_2 = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}.$$

5. Calculate the p -norm of the vectors v_1, v_2 for $p = 1, 2, \infty$ and determine the angle between v_1 and v_2 and whether they are orthogonal or not for

$$v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, v_2 = \begin{pmatrix} 3 \\ 4 \\ 5 \end{pmatrix}, \quad v_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, v_2 = \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$$

6. Write a script/function in Matlab/Octave that computes the result of the recursive equation:

$$x_{k+1} = Ax_k, \quad A = \begin{bmatrix} 1.1 & -1 \\ -1 & 1.1 \end{bmatrix}.$$

Use your script/function to compute x_k for $k = 20$ when

$$x_0 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad x_0 = \begin{pmatrix} 1 \\ 1.001 \end{pmatrix}.$$

Comment your findings!

7. Prove that if non-zero vectors v_1 and v_2 are orthogonal then they are also linearly independent.

Notes:

- 1) Clearly justify all your answers!
- 2) You may use Matlab/Octave for calculations in all questions except when noted that you should perform the calculation by hand.